MATRIX ELEMENTS FOR CALCULATING TRANSITIONS AMONG THE 1s, 2s, 2p, 3s, 3p and 3d STATES OF ATOMIC HYDROGEN BY THE FADDEEV APPROACH*

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Matrix elements which are useful for calculating the electron impact transitions among the 1s, 2s, 2p, 3s, 3p and 3d states of atomic hydrogen have been presented systematically. Methods for checking the accuracy of the tabulated matrix elements are given, and the relevance of these methods for further calculation of matrix elements is discussed.

INTRODUCTION

THE approximate form of Faddeev equations (Faddeev, 1960, 1961) formulated by Sloan and Moore (1968) has extended the application of Faddeev equations to atomic systems. Saha *et al.* (1975) have used this approximation for obtaining electron impact excitation cross sections between the 1s, 2s and 2p states of atomic hydrogen. Their results show an excellent agreement of Faddeev calculations with experimental results. Using the same approximation method, Lal & Narain (1977) (hereafter will be referred to as LN) have derived general expressions for the partial transition matrix in electron-hydrogen collisions. The expressions of LN are lengthy and complicated.

At present, there is a need of Faddeev calculations for the electron impact excitation cross sections of the 1s→3s, 1s→3p and 1s→3d excitations of hydrogen atom, since the calculations of these excitation cross sections in other well known methods are in disagreement with the experimental findings of Mahan *et al.* (1976). Even the adequacy of close coupling method for these excitations is doubtful (cf. Mahan *et al.,* 1976). But the expressions of matrix elements derived by LN are so complicated and lengthy that it is considerably more laborious to obtain values of matrix elements from them. The present paper has two purposes : First, it presents values of some matrix elements, calculated from the formulae of LN which may be immediately used to calculate transitions among the 1s, 2s, 2p, 3s, 3p and 3d states of atomic hydrogen and, secondly, it provides some methods for checking the values of the matrix elements calculated from the formulae of LN, thereby lending confidence to our attempts to obtain further values of the matrix elements.

TABULATION OF MATRIX ELEMENTS

Let the principal and azimuthal quantum numbers of the bound states be denoted respectively by n_i and l_i, and the wave number and azimuthal quantum number of

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i	π_i	i	α_i
1	$(1k_1, j, 0)$	6	$(3k_3, j+1, 1)$
2	$(2k_2, j, 0)$	7	$(3k_3, j-1, 1)$
3	$(2k_2, j+1, 1)$	8	$(3k_3, j+2, 2)$
4	$(2k_2, j-1, 1)$	9	$(3k_3, j, 2)$
5	$(3k_3, j, 0)$	10	$(3k_3, j-2, 2)$

the free state be k_i and L_i. Then the states $n_ik_iL_il_i$ of the total system $(e^- + H)$ are conveniently represented by π_i, according to the scheme.

However, for simplicity, in Tables I–III, we shall use α for α_i and α' for α_j.

Let k_1^2 be the energy (in rydberg) of the incident electron when the hydrogen atom is in its 1s state. k_2, k_3 etc., are then given by

$$k_n^2 = k_1^2 - \left(1 - \frac{1}{n^2}\right). \qquad ...(1)$$

The partial transition matrix $T^{J\pm}(\alpha_j, \alpha_i)$ is written as

$$T^{J\pm}(\alpha_j,\alpha_i) = B^{J\pm}(\alpha_j, \alpha_i) + \frac{i}{4\pi}\sum_{\alpha_m} B^{J\pm}(\alpha_j, \alpha_m)\ T^{J\pm}(\alpha_m, \alpha_i), \qquad ...(2)$$

where the $B^{J\pm}$ are two body matrices. $B^{J\pm}$ may be written in terms of the matrices B_D^J and B_E^J which correspond to the direct and exchange scattering, respectively.

$$B^{J\pm} = B_D^J \pm B_E^J. \qquad ...(3)$$

General expressions for the B_D^J and B_E^J have been derived in LN. The expression of B_D^J [eqn. (15) of LN] is lengthy and complicated, while that for B_E^J expressed in terms of the integrals $I_{ll'}^{nn'}$ and $I_{\lambda LL'}$ [eqns. (28) and (29) of LN], is simple and short. Moreover, the integrals S_α^λ and $I_{ll'}^{nn'}$ also have complicated expressions, while, $I_{yLL'}$ does not. So, for numerical computation, need arises for explicit values of the matrix elements $B_D^J(\alpha_j, \alpha_i)$ and of the integrals S_α^λ and $I_{ll'}^{nn'}$.

For calculating transitions among the 1s, 2s, 2p, 3s, 3p and 3d states of hydrogen atom one needs hundred $B_D^J(\alpha_j, \alpha_i)$ for $i, j = 1, 2, ...10$. But the symmetry relation

$$B_D^J(\alpha_j, \alpha_i) = B_D^{J*}(\alpha_i, \alpha_j), \qquad ...(4)$$

where * denotes complex conjugate, reduces this number to fifty five. These fifty five elements have been tabulated in Table I in terms of the integrals $S_\alpha^\lambda(n'l', nl)$. The arguments $(n'l', nl)$ of S_α^λ have been omitted since there is no possibility of confusion regarding them.

TABLE I

Values of the matrix elements $B^J_D(\alpha', \alpha)$ for $l, l' \leqslant 2$

$$B^J_D(n'k', J, 0; nk, J, 0) = 4\pi (kk')^{-1/2} Q_J(X_0)\, \delta_{nn'} - 4\pi (kk')^{1/2} S^0_J$$

$$B^J_D(n'k', J+1, 1; nk, J, 0) = -4\pi \left(\frac{3kk'}{2J+1}\right)^{1/2} \left[{}_1W^1_J(k', k)\right]$$

$$B^J_D(n'k', J-1, 1; nk, J, 0) = -4\pi \left(\frac{3kk'}{2J+1}\right)^{1/2} \left[{}_1W^1_J(k, k')\right]$$

$$B^J_D(n'k', J+2, 2; nk, J, 0) = -2\pi \left(\frac{30kk'}{2J+1}\right)^{1/2} \left[{}_1W^2_J(k, k')\right]$$

$$B^J_D(n'k', J, 2; nk, J, 0) = 4\pi (5kk')^{1/2} \frac{J(J+1)}{(2J-1)(2J+3)} \left[{}_2W^2_J(k, k')\right]$$

$$B^J_D(n'k', J-2, 2; nk, J, 0) = -2\pi \left(\frac{30kk'}{2J+1}\right)^{1/2} \left[{}_1W^2_{J+2}(k', k)\right]$$

$$B^J_D(n'k', J+1, 1; nk, J+1, 1) = 4\pi (kk')^{-1/2} Q_{J+1}(X_0)\, \delta_{nn'} - 4\pi (kk')^{1/2} S^0_{J+1}$$
$$- 4\pi (kk')^{1/2} \left(\frac{J+2}{2J+1}\right) \left[{}_2W^2_{J+1}(k, k')\right]$$

$$B^J_D(n'k', J+1, 1, nk, J-1, 1) = -3\pi \left(\frac{kk'}{2(2J+1)}\right)^{1/2} \left[{}_1W^2_{J-1}(k, k')\right]$$

$$B^J_D(n'k', L, 2; nk, L-1, 1) = -12\pi (2kk')^{1/2} \begin{Bmatrix} L & 1 & L-1 \\ 1 & J & 2 \end{Bmatrix} \left[{}_1W^1_{L-1}(k', k)\right]$$
$$+ 6\pi (42kk')^{1/2} \begin{Bmatrix} L & 3 & L-1 \\ 1 & J & 2 \end{Bmatrix} \left[{}_2W^3_L(k, k')\right], \quad (L = J+2, J)$$

$$B^J_D(n'k', L, 2; nk, L+1, 1) = -12\pi (2kk')^{1/2} \begin{Bmatrix} L & 1 & L+1 \\ 1 & J & 2 \end{Bmatrix} \left[{}_1W^1_L(k, k')\right]$$
$$+ 6\pi (42kk')^{1/2} \begin{Bmatrix} L & 3 & L+1 \\ 1 & J & 2 \end{Bmatrix} \left[{}_2W^3_{L+1}(k', k)\right], \quad (L = J, J-2)$$

$$B^J_D(n'k', L, 2; nk, L+3, 1) = 6\pi (70kk')^{1/2} \begin{Bmatrix} L & 3 & L+3 \\ 1 & J & 2 \end{Bmatrix} \left[{}_1W^3_L(k, k')\right]$$

$$B^J_D(n'k', L, 2; nk, L-3, 1) = 6\pi (70kk')^{1/2} \begin{Bmatrix} L & 3 & L-3 \\ 1 & J & 2 \end{Bmatrix} \left[{}_1W^3_{L-3}(k', k)\right]$$

(contd. on next page)

$$B_D^J\,(n'k', L, 2; nk, L, 2) = 4\pi\,(kk')^{-1/2}\,Q_L(X_0)\,\delta_{nn'} - 4\pi\,(kk')^{1/2}\,S_L^0$$

$$-40\sqrt{\frac{5}{14}}\,\pi\,(kk')^{1/2}\begin{Bmatrix} L & 2 & L \\ 2 & J & 2 \end{Bmatrix}\sqrt{\frac{L(L+1)(2L+1)}{(2L-1)(2L+3)}}\Big[\,{}_2W_L^2(k, k')\,\Big]$$

$$-108\pi\left(\frac{5kk'}{14}\right)^{1/2}\begin{Bmatrix} L & 4 & L \\ 2 & J & 2 \end{Bmatrix}(2L+1)\sqrt{\frac{(L+2)\,!\,(2L-5)\,!\,!}{(L-2)\,!\,(2L+5)\,!\,!}}$$

$$\times\Big[(k^4+k'^4)\,S_L^4 - 2kk'(k^2+k'^2)\,\frac{(2L+5)\,S_{L-1}^4 + (2L-3)\,S_{L+1}^4}{(2L+1)}$$

$$+ k^2k'^2\left\{\frac{(2L-3)(2L-1)}{(2L+3)(2L+1)}\,S_{L+2}^4 + \frac{4(2L+5)(2L-3)}{(2L+3)(2L-1)}\,S_L^4\right.$$

$$\left. + \frac{(2L+5)(2L+3)}{(2L+1)(2L-1)}\,S_{L-2}^4\right\}\Big]$$

(In this expression $L = J + 2,\ J,\ J - 2$)

$$B_D^J(n'k', L, 2; nk, L+2, 2) = 20\pi\left(\frac{15kk'}{7}\right)^{1/2}\begin{Bmatrix} L & 2 & L+2 \\ 2 & J & 2 \end{Bmatrix}\Big[\,{}_1W_L^2\,(k', k)\Big]$$

$$+ 180\pi\left(\frac{kk'}{7}\right)^{1/2}\begin{Bmatrix} L & 4 & L+2 \\ 2 & J & 2 \end{Bmatrix}\sqrt{\frac{(L+3)(L+2)(L+1)L}{(L2+7)(2L+3)(2L-1)}}$$

$$\times\Big[\,k^4 S_L^4 - k^3 k'\,\frac{(2L+7)\,S_{L-1}^4 + 3\,(2L-1)\,S_{L+1}^4}{(2L+1)}$$

$$+ 3k^2k'^2\,\frac{(2L+7)\,S_L^4 + (2L-1)\,S_{L+2}^4}{(2L+3)}$$

$$- kk'^3\,\frac{3\,(2L+7)\,S_{L+1}^4 + (2L-1)\,S_{L+3}^4}{(2L+5)} + k'^4\,S_{L+2}^4\Big]$$

(In this expression $L = J,\ J - 2$)

$$B_D^J\,(n'k', L, 2; nk, L+4, 2) = -90\pi\,(kk')^{1/2}\begin{Bmatrix} L & 4 & L+4 \\ 2 & J & 2 \end{Bmatrix}\Big[\,{}_1W_L^4\,(k, k')\Big]$$

The ${}_{\nu}W^{\lambda}_{\alpha}$'s in Table I, denote following expressions :

$$_1W^{\lambda}_J(k,k') = \sqrt{\frac{(j+\lambda)!\,(2J+1)!!}{J!(2J+2\lambda-1)!!}} \sum_{b=0}^{\lambda} k^{\lambda-b}(-k')^b S^{\lambda}_{J+b}, \quad \ldots(5)$$

$$_2W^2_J(k,k') = (k^2+k'^2)\,S^2_J - \frac{kk'}{(2J+1)}\Big\{(2J+3)\,S^2_{J-1} + (2J-1)\,S^2_{J+1}\Big\}, \quad \ldots(6)$$

$$_2W^3_J(k,k') = \sqrt{\frac{(J+1)\,J\,(J-1)}{(2J+3)(2J-3)}}\Big[k^3 S^3_J - \Big(\frac{k^2k'}{2J+1}\Big)\Big\{(2\,(2J+3)\,S^3_{J-1} + (2J-3)\,S^3_{J+1}\Big\} + \Big(\frac{kk'^2}{2J-1}\Big) \times\Big\{(2J+3)\,S^3_{J-2} + 2(2J-3)\,S^3_J\Big\} - k'^3 S^3_{J-1}\Big]. \quad \ldots(7)$$

Values of the intergrals S^{λ}_{α} $(n'l', nl)$, for n, $n' \leqslant 3$ and $l, l' \leqslant 2$, have been tabulated in Table II. These values are symmetric in (nl) and $(n'l')$, that is

$$S^{\lambda}_{\alpha}(n'l', nl) = S^{\lambda}_{\alpha}(nl, n'l'). \quad \ldots(8)$$

TABLE II

Values of $S^{\lambda}_{\alpha}(n'l', nl)$ for $n.n' \leqslant 3$ and $l, l' \leqslant 2$. Argument of Q^b_{α} is X_{α}

$$S^0_{\alpha}(10,10) = \frac{1}{kk'}\,U_{\alpha}(1,a)$$

$$S^0_{\alpha}(20,10) = (\tfrac{1}{2})^{3/2}\frac{1}{k^3k'^3}\,Q^2_{\alpha}$$

$$S^0_{\alpha}(30,10) = -\frac{3\sqrt{3}}{16\,kk'}\Big[U_{\alpha}(1,a) - 2a^2 Q^2_{\alpha} - \frac{1}{6}a^3 Q^3_{\alpha}\Big]$$

$$S^0_{\alpha}(20,20) = \frac{1}{kk'}\Big[U_{\alpha}(1,a) - \tfrac{1}{2}a^2 Q^2_{\alpha} - a^3 Q^3_{\alpha}\Big]$$

$$S^0_{\alpha}(30,20) = \frac{\sqrt{3}}{9\sqrt{2}\,k^3k'^3}\Big[Q^2_{\alpha} + \frac{344}{225}a\,Q^3_{\alpha} + \frac{32}{75}a^2 Q^4_{\alpha}\Big]$$

(*contd. on next page*)

$$S^0_\alpha(30, 30) = \frac{1}{kk'}\left[U_\alpha(1, a) + a^2 Q^2_\alpha + \frac{13}{3} a^3 Q^3_\alpha + \frac{8}{3} a^4 Q^4_\alpha + \frac{4}{3} a^5 Q^5_\alpha \right]$$

$$S^1_\alpha(21, 10) = \frac{i}{kk'} \frac{2^7\sqrt{2}}{3^5\sqrt{3}} U_\alpha(2, a)$$

$$S^1_\alpha(31, 10) = \frac{i}{kk'} \frac{9\sqrt{6}}{2^7}\left[U_\alpha(2, a) - \tfrac{1}{3} a^3 Q^3_\alpha \right]$$

$$S^1_\alpha(21, 20) = -\frac{i\sqrt{3}}{kk'}\left[U_\alpha(2, a) + \tfrac{1}{3} a^3 Q^3_\alpha \right]$$

$$S^1_\alpha(31, 20) = \frac{i}{kk'} \frac{2^9.3\sqrt{3}}{5^6}\left[U_\alpha(2, a) - \tfrac{1}{3} a^3 Q^3_\alpha - \frac{1}{24} a^4 Q^4_\alpha \right]$$

$$S^1_\alpha(21, 30) = -\frac{i}{kk'} \frac{2^7\sqrt{2}\ 3^6}{5^7}\left[U_\alpha(2. a) + \frac{4}{9} a^3 Q^3_\alpha + \frac{4}{27} a^4 Q^4_\alpha \right]$$

$$S^1_\alpha(31, 30) = -\frac{i}{kk'} \frac{129}{2}\left[U_\alpha(2, a) + \frac{23}{129} a^3 Q^3_\alpha - \frac{25}{516} a^4 U^4_\alpha + \frac{41}{3096} a^5 Q^5_\alpha \right]$$

$$S^0_\alpha(21, 21) = \frac{1}{kk'}\left[U_\alpha(2, a) + \tfrac{1}{3} a^3 Q^3_\alpha \right]$$

$$S^0_\alpha(31, 21) = \frac{1}{kk'} \frac{2^9 3^2}{5^6} a^3\left[Q^3_\alpha + \tfrac{2}{3} a\, Q^4_\alpha \right]$$

$$S^0_\alpha(31, 31) = \frac{1}{kk'}\left[U_\alpha(2, a) + \frac{5}{6} a^3 Q^3_\alpha + \frac{8}{9} a^4 Q^4_\alpha + \frac{2}{9} a^5 Q^5_\alpha \right]$$

$$S^2_\alpha(21, 21) = -\frac{2}{kk'} U_\alpha(3, a)$$

$$S^2_\alpha(31, 21) = \frac{1}{kk'} \frac{2^{13}\ 3^4}{5^8}\left[U_\alpha(3, a) - \frac{1}{6} a^4 Q^4_\alpha \right]$$

$$S^2_\alpha(31, 31) = -\frac{12}{kk'}\left[U_\alpha(3, a) + \frac{1}{48} a^4 Q^4_\alpha + \frac{1}{24} a^5 Q^5_\alpha \right]$$

$$S^2_\alpha(32, 10) = -\frac{1}{kk'} \frac{3^3\sqrt{30}}{5.2^8} U_\alpha(3, a)$$

$$S^2_\alpha(32, 20) = \frac{1}{kk'} \frac{2^{16}3^3\sqrt{15}}{5^9}\left[U_\alpha(3, a) - \frac{1}{16} a^4 Q^4_\alpha \right]$$

$$S^2_\alpha(32, 30) = -\frac{3\sqrt{10}}{kk'}\left[U_\alpha(3, a) - \frac{1}{60} a^4 Q^4_\alpha + \frac{1}{30} a^5 Q^5_\alpha \right]$$

(contd. on next page)

$$S^1_\alpha(32, 21) = \frac{i}{kk'} \frac{2^{11}3^3}{5^6\sqrt{5}} \left[U_\alpha(3, a) - \frac{1}{15} a^4 Q^4_\alpha \right]$$

$$S^1_\alpha(32, 31) = -\frac{i}{kk'} \frac{3\sqrt{5}}{2} \left[U_\alpha(3, a) + \frac{2}{45} a^5 Q^5_\alpha \right]$$

$$S^3_\alpha(32, 21) = -\frac{i}{kk'} \frac{2^{16}3^5}{5^9\sqrt{5}} U_\alpha(4, a)$$

$$S^3_\alpha(32, 31) = \frac{i\, 9\sqrt{5}}{kk'} \left[U_\alpha(4, a) + \frac{1}{60} a^5 Q^5_\alpha \right]$$

$$S^0_\alpha(32, 32) = \frac{1}{kk'} \left[U_\alpha(3, a) + \frac{2}{45} a^5 Q^5_\alpha \right]$$

$$S^2_\alpha(32, 32) = -\frac{42}{5kk'} \left[U_\alpha(4, a) + \frac{1}{84} a^5 Q^5_\alpha \right]$$

$$S^4_\alpha(32, 32) = \frac{27}{kk'} U_\alpha(5, a)$$

In Table II we have used a notation $U_\alpha(p, a)$ to denote the following expression :

$$U_\alpha(p, a) = Q_\alpha(X_0) - \sum_{b=0}^{p} \frac{(-a)^b}{b!} Q^b_\alpha(X_a), \qquad \ldots(9)$$

where Q^b_α is the bth derivative with respect to X_a of the Legendre polynomial of second kind $Q_\alpha(X_a)$, and

$$\left.\begin{aligned} a &= (n^{-1} + n'^{-1})^2/2kk', \\ X_0 &= \frac{k^2 + k'^2}{2kk'} \text{ and } X_a = X_0 + a \end{aligned}\right\} \qquad \ldots(10)$$

In using Tables I and II, there will be a need to find values of some linear combinations of $Q_\alpha(1)$. This may be done by using the formula

$$Q_\alpha(1) - Q_{\alpha+1}(1) = \frac{1}{\alpha+1}\,. \qquad \ldots(11)$$

Values of the integrals $I^{nn'}_{ll'}$ (expression 28 of LN) have been tabulated in Table III in terms of two integrals $M^n_l(k')$ and $N^{n'}_{l'}(k)$:

$$I^{nn'}_{ll'}(k, k') = M^n_l(k')\, N^{n'}_{l'}(k). \qquad \ldots (12)$$

Tables I and II contain sufficient matter for computing direct partial transition matrix (with $B^J_E = 0$). To include exchange effects, expression (17) and (29) of LN, respectively, for the exchange matrix B^J_E and the integrals $I_{\lambda LL'}$, must be used directly for numerical computation, while for $I^{nn'}_{ll'}$ use should be made of Table III.

Accuracy of the Matrix Elements

Unless methods to check the accuracy of the matrix elements presented in Tables I–III are given, their accuracy becomes questionable. We, therefore, give here some methods for checking the tabulated matrix elements. To check the B^J_D's one should see whether or not the values of the linear combinations in the square brackets (of Table I) vanish if all the S^λ_α's are replaced by unity, and k is taken equal to k'. The expressions of the B^J_D's are correct if the above values vanish. This is applicable to all B^J_D's.

Simple methods to check the accuracy of the S^λ_α's are available only in two cases. First, when $n = l+1$, $n' = l'+1$ and $\lambda = l + l'$. In this case the expressions of the

TABLE III

Values of the integrals $M^n_l(k')$ and $N^{n'}_{l'}(k)$ for $n, n' \leqslant 3$ and $l, l' \leqslant 2$

$M^1_0(k') = \dfrac{4}{(1+k'^2)^2}$	$N^1_0(k) = \dfrac{2}{(1+k^2)}$
$M^2_0(k') = -\dfrac{16\sqrt{2}\,(1-4k'^2)}{(1+4k'^2)^3}$	$N^2_0(k) = -\dfrac{2\sqrt{2}\,(1-4k^2)}{(1+4k^2)^2}$
$M^3_0(k') = \dfrac{12\sqrt{3}}{(1+9k'^2)^2}\left[1-\dfrac{6-18k'^2}{1+9k'^2}+\dfrac{8-72k'^2}{(1+9k'^2)^2}\right]$	
	$N^3_0(k) = \dfrac{2\sqrt{3}\,(1-3k^2)(1-27k^2)}{(1+9k^2)^3}$
$M^2_1(k') = \dfrac{2^7 k'}{\sqrt{6}\,(1+4k'^2)^3}$	$N^2_1(k) = \dfrac{16k}{\sqrt{6}\,(1+4k^2)^2}$
$M^3_1(k') = -\dfrac{2^4 3^2\sqrt{6}\,k'(1-9k'^2)}{(1+9k'^2)^4}$	$N^3_1(k) = -\dfrac{8\sqrt{6}\,k(1-9k^2)}{(1+9k^2)^3}$
$M^3_2(k') = \dfrac{2^5 3^3\sqrt{30}\,k'^2}{5(1+9k'^2)^4}$	$N^3_2(k) = \dfrac{2^4 3\sqrt{30}\,k^2}{5(1+9k^2)^3}$

S_α^λ contain only one set of the summation variables and, therefore, these may be checked easily. The second case occurs when $n'=n+1$, $l'=l$ and $\lambda=0$. In this case first few U_α's corresponding to $b = 0, 1, \ldots . p_{min}$, where $p_{min} < p$, do not appear in the value of S_α^λ. One can use this fact to test the accuracy of the S_α^λ , s by realizing that this is a consequence of the magnitudes of the coefficients of $\frac{a^b}{b_1} \; Q_\alpha^b \; (X_a)$ [see eqn. (9) and Table II].

The integrals M_l^n and $N_{l'}^{n'}$ may be checked easily by letting k (or ') tend to zero. In this limiting case, one must have

$$(2l+1)!! \; M_l^n \; k' = 0 \sim (2k'n)^l n^3 \, A_{nl} \sum_s (-2)^s \, B_{nls} \, (2l+2+s)!,$$

$$(2l'+1)!! \, N_{l'}^{n'} \; k=0 \sim (2kn')^{l'} \; n'^2 \, A_{n'l'} \sum_s (-2)^s \, B_{n'l's} \; (2l'+1+s)!,$$

where

$$n^2 A_{nl} = 2\,\{(n-l-1)!\,(n+l)!\}^{1/2} \text{ and } B_{nls} = \overline{s!\,(2l+1+s)!\,(n-l-1-s)!}$$

Conclusions

In view of the methods given above for checking the tabulated matrix elements, one may use the tables with confidence to start immediate computation of cross sections for the transitions among the 1*s*, 2*s*, 2*p*, 3*s*, 3*p* and 3*d* states of atomic hydrogen. Moreover, since the dependence of B_D^J on n' and n' is contained completely in the S_α^λ s, Table I is useful for all n and n' as far as $l, l' \leqslant 2$. Thus, for example, for calculating transitions among the 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 3*d*, 4*s*, 4*p*, and 4*d* states of atomic hydrogen one need only extend the Tables II and III, Table I being applicable without requiring extension.

Furthermore, the methods of checking the matrix elements are also helpful in making further calculations to obtain values of $B_D^J(\alpha_f, \alpha_i)$, S_α^λ $(n_f l, n_i l_i)$ and $I_{ll'}^{nn'}$ (from the lengthy and complicated formulae of LN) for $n, n' > 3$ and $l, l' > 2$, thereby facilitating the computation of cross sections for transitions among $n, n' = 1$ through $n, n' > 3$ states also. (While using expression 28 of LN, one should note that 28 is in error by a phase factor $(-1)^{s+s'}$ in its summand. So one should multiply the summand of 28 of LN by this phase factor.)

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