SU(5) AND THE MAGNETIC MOMENT OF $(\frac{1}{2})^+$ 40-PLET (MIXED) BARYONS

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The magnetic moment of $(1/2)^+$ beautiful baryons are predicted using the Gelfand and Zetlin technique. The magnetic moment sum rules are also derived.

INTRODUCTION

THE recently observed narrow dimuon resonance at 9.4 (GeV) (Herb, 1977) has been widely interpreted as the bound state of a new heavy quark b (beauty) and its antiquark $\bar{b}$ analogous to the Charmonium interpretation of J/ψ.

The magnetic moment calculations have been carried out by other authors (Singh, 1977) but the technique that is presented here is assumed to be elegant.

In this paper, the authors present the SU(5) model, whose fifth quark carries charge $= -1/3$. This model is compatible with the branching ratio calculation done for the Υ particle presuming that it is a bound state of a new quark b and $\bar{b}$.

The magnetic moment of charmed and beautiful baryons that belong to 40-plet mixed of SU (5) is derived here. Also the SU(3) results that are derived here are in excellent agreement with the experiment. The Gelfand and Zetlin technique is made use of (Gelfand *et al.*, 1963; M. Ahmad *et al.*, 1978) to this end.

MAGNETIC MOMENT AND GELFAND-ZETLIN TECHNIQUE

A unique pattern is associated with every state by imposing the condition of lexical ordering. All the states of SU(5) can uniquely be represented by I, I_3, Y, C, and Beauty quantum number B, according to the Gelfand patterns. The irreducible representations of SU (5) can be described by a set of five integers (m_{15}) with $t=1.......5$. The states of a given representation can be expressed in an economical way by the scheme of Gelfand-Zetlin. In this scheme (Gelfand *et al.*, 1963) the states aer described by a triangular array of integers,

$$\left(\begin{array}{ccccccccc} m_{15} & & m_{25} & & m_{35} & & m_{45} & & m_{55} \\ & m_{14} & & m_{24} & & m_{34} & & m_{44} & \\ & & m_{13} & & m_{23} & & m_{33} & & \\ & & & m_{12} & & m_{22} & & & \\ & & & & m_{11} & & & & \end{array}\right) \qquad \ldots(1)$$

with the m_{ij} taking all the values of consistent with a "between-ness condition"

$$m_{ij} < m_{i,j+1} < m_{i+1,j}$$

The values of various quantum numbers in terms of the m_{ij} are thus written as,

$$I = \tfrac{1}{2}(m_{13} - m_{12});$$

$$Y = m_{13} - [(m_{14} - \tfrac{1}{8}(m_{15} + m_{25} + m_{35} + m_{45}))];$$

$$C = m_{14} - \tfrac{1}{4}(m_{15} + m_{25} + m_{35} + m_{45});$$

and

$$B = \tfrac{3}{8}(m_{15} + m_{25} + m_{35} + m_{45}) - m_{14} \qquad \ldots(2)$$

We then write the mass operator for SU(5) as,

$$\begin{aligned} M = \; & M_0 + M_1 < m \mid A_8 \mid m > + M_2 < m \mid D_8 \mid m > \\ & + M_3 < m \mid A_{15} \mid m > + M_4 < m \mid D_{15} \mid m > \\ & + M_5 < m \mid A_{24} \mid m > + M_6 < m \mid D_{24} \mid m >, \end{aligned} \qquad \ldots(3)$$

where m is the Gelfand pattern specifying the states with M_0, M_1, M_2, M_3, M_4, M_5 and M_6 as constants. In eqn. (3), A_8 and D_8 single out hypercharge A_{15} and D_{15} singles out charm while A_{24} and D_{24} single out Beauty.

In SU(3), the magnetic moment of hadron depends on the charge and U-spin, where U-spin connects two quarks (d and s). In SU(4), the magnetic moment of charmed hadrons depends on L-spin, connecting u and c quarks. Similarly in SU(5), there is another spin called T-spin which connects c and b quarks.

Then the magnetic moment operator for SU(4) is extended to SU(5) and is written as,

$$< m \mid \mu_{op} \mid m > = a\mathrm{F}_Q + bD_Q + dA_Q + eH_Q + gE_Q + kZ_Q, \qquad \ldots(4)$$

where A_Q corresponds to A_{15}, H_Q corresponds to D_{15}, E_Q corresponds to A_{24} while as Z_q corresponds to D_{24}.

Using the mass formula of SU(5)

$$\begin{aligned} M = M_0 + \tfrac{1}{2}M_1 Y + \frac{M_2}{6}[I(I+1) - \tfrac{1}{4}Y^2] + aC^2 + fC(C+1) \\ + [(^*xB^2 + yB(B+1)] + F. \end{aligned} \qquad \ldots(5)$$

Then the magnetic moment operator for SU(5) is

$$\begin{aligned} < m \mid \mu_{0p} \mid m > = \tfrac{1}{2}aQ + \frac{b}{6}[U(U+1) - \tfrac{1}{4}Q^2 - I_2] - \frac{b}{6}C^2 \\ + \lambda C - \frac{b}{6}B^2 + \lambda' B = aQ/6 + \tfrac{1}{2}b[U(U+1) - \tfrac{1}{4}Q^2 - I_2] \\ - B\frac{b}{6}(C^2 + B^2) + \lambda C + \lambda' B \end{aligned} \qquad \ldots(6)$$

We now find the magnetic moment of all the 40-plet beautiful baryons in terms of the magnetic moments of proton and neutron. From eqn. (6), we have,

$$\begin{aligned} < m \mid \mu_0 P \mid m > = \tfrac{1}{2}(2\mu_p + 3\mu_n)Q + \mu_n[U(U+1) - \tfrac{1}{4}Q^2 - I_2] \\ - 5/2\,\mu_n C + 3/2\,\mu_n(C^2 + B^2) + (3/2\,\mu_p - 15/4\,\mu_n)B, \end{aligned} \qquad \ldots(7)$$

where μ_n and μ_p stand for the magnetic moment of neutron and protron respectively.

Then the above equation yields the conventional SU(5) results as (Ahmad Zadoo, 1977)

$$\mu_{\Sigma^+} = \mu_p,$$

$$\mu_{\Xi^\circ} = \mu_n,$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = (\mu_p + \mu_n)$$

and

$$\mu_{\Omega^-} = -\mu_p.$$

Magnetic Moment of (1/2)$^+$ *Charmed Baryons* (Ahmad & Zadoo, 1977)

From eqn. (7), we get the SU(4) results as,

$$\mu_{\Delta_C^{++}} = 2\mu_p - \mu_n;\ \mu_{\Delta_C^{o}} = \mu_n;\ \mu_{\Delta_C^{*o}} = \mu_n;$$

$$\mu_{\Delta_C^+} = \mu_p;\ \mu_{\Sigma_C^o} = \mu_n$$

$$\mu_{\Sigma_{CC}^+} = (\mu_p + \mu_n);\ \mu_{\Sigma_C^{++}} = 2(\mu_p + \mu_n)$$

$$\mu_{\Xi_{CC}^+} = (\mu_p + \mu_n)$$

For SU(4), the following sum=rule holds good.

$$\mu_{\Delta_C^{++}} - \mu_{\Delta_C^+} = \mu_{\Delta_C^+} - \mu_{\Delta_C^{*o}}$$

Magnetic Moment of $(\frac{1}{2})^+$ *Beautiful Baryons :*

$$\mu_{uub} = \tfrac{5}{2}\mu_p - 3\mu_n;\quad \mu_{ddb} = \tfrac{1}{2}(5\mu_p + u_n);$$

$$\mu_{udb} = \tfrac{3}{2}\mu_p - 2\mu_n;$$

$$\mu_{usb} = \tfrac{3}{2}\mu_p - 2\mu_n;$$

$$\mu_{ucb} = \tfrac{5}{2}\mu_p - 3\mu_n;$$

$$\mu_{ubb} = \tfrac{1}{4}(6\mu_p + \mu_n);$$

$$\mu_{dsb} = \tfrac{1}{2}(\mu_p - 4\mu_n);$$

$$\mu_{dcb} = \tfrac{1}{2}(3\mu_p - 5\mu_n);$$

$$\mu_{dbb} = \tfrac{1}{2}(4\mu_p - 5\mu_n);$$

$$\mu_{ssb} = \tfrac{1}{2}(\mu_p - 4\mu_n);$$

$$\mu_{scb} = \tfrac{1}{2}(3\mu_p - 5\mu_n);$$

$$\mu_{ccb} = \tfrac{5}{2}\mu_p.$$

From SU(5) results the sum rules are

$$\mu_{uub} - \mu_{dbb} = \mu_{dbb} - \mu_{udb}$$

$$\mu_{dbb} = \tfrac{1}{2}\,(\mu_{uub} + \mu_{udb})$$

or $\quad \mu_{dbb} = \tfrac{1}{2}\,(4\,\mu_p - 5\,\mu_n).$

Conclusion

Only the sum rules for $(\frac{1}{2})^+$ beautiful 40-plet baryons from the known results of SU(3) and SU (4) (Vonsovsky, 1975; Ahmad & Zadoo, 1977) are derived by making some modest assumptions with the implications for the old physics, in respect of its magnetic moment properties. The SU(3) and SU(4) results are obtained correctly. In particular, the SU(5) results and their sum rules are derived.

References

Ahmad, M., and Zadoo, T. K. (1977). SU(4) symmetry and the magnetic moment of 20-plet baryons. *Phys. Rev.,* **D15**, 2483–2485.

Ahmad, M. Sofi, G. Q., Shafi, M., and Shah, N. A. (1978). Beauty quark and new hadrons. *Proc. INSA*, **44**, A, 5, 308–316.

Herb, S., Hom, D. C., Lederman, L. M., Sen, J. A., Brown, B. C., Brown, C. N., Innes, W. R., Ueno, K., Yamanouchi, T., Ito, A. S., Jostlein, H., Kaplan, D. M., and Kephart, R. D. (1977). Observation of a dimuon resonances at 9.5 GeV in 400 GeV proton-nucleus collisions. *Phys. Rev. Lett.,* **39**, 5, 252–255.

Gelfand, I. M., Minos, P. A., Shapies, Z. Ya. (1963). *Representation of the Rotation Groups and their Applications* Pergamon Press Ltd., Oxford, London, The Macmillan Company, New York, 1963.

Singh, L. P. (1977). Quark additivity and the hadron magnetic moment. *Phys. Rev.* **D16**, 158–159.

Vonsovsky, S. V. (1975). *Magnetism of Elementary Particles.* Mir Publishers, Mcscow, 1975.