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I. PHYSICS

Fluid Dynamics

FREE CONVECTION LAMINAR FLOW OF AN INCOMPRESSIBLE VISCOELASTIC FLUID

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Free convection laminar flow of an incompressible Rivlin-Ericksen Viscoelastic fluid past a porous vertical plate with fluctuating suction has been studied. The suction velocity normal to the plate oscillates in magnitude but not in direction about a non-zero constant and the free stream velocity oscillates in magnitude about a non-zero constant. The temperature difference between the constant plate temperature $\bar{T}_w$ and the stream temperature $\bar{T}_\infty$ causes the free convection currents in the boundary layer. Expressions for velocity distribution, rate of heat transfer, Nusselt number, temperature distribution and skin-friction have been derived and plotted graphically. Velocity, temperature and rate of heat transfer from the plate are found to be affected by the variable suction parameter, elastic parameter, Prandtl number and forcing frequency. It has been shown that for cooling of the plate $(G > 0)$, the mean velocity increases with increasing elasticity of the fluid whereas an increase in Prandtl number leads to a decrease in the mean velocity. The fluctuating parts of temperature are found to be decreasing with an increase in Prandtl number and forcing frequency whereas an increase in suction parameter leads to an increase in fluctuating parts of temperature. Transient temperature decreases with increasing Prandtl number and suction parameter whereas an increase in forcing frequency leads to an increase in transient temperature. Amplitude of heat transfer increases with increasing Prandtl number and frequency. Effect of increasing Prandtl number leads to phase-lead whereas an increase in frequency leads to phase-lag. Effect of elasticity is to increase the skin-friction on the plate. Alongwith elasticity, cross-viscosity also affects the pressure.

Keywords : Free convection; Viscoelastic; Laminar; Fluctuating; Porous.

INTRODUCTION

THE free convection flow of viscous incompressible fluid past a porous vertical plate with fluctuating suction has been studied by Siddappa and Bujurke and by Soundalgekar in 1972 and 1977. Siddappa and Gundappa (1977) have discussed free-convection laminar flow of Rivlin-Ericksen Viscoelastic incompressible fluid past a porous flat plate with constant suction.

The present study attempts an effort to extend the problem of Siddappa and Gundappa (1977) by taking time-dependent suction and fluctuating free-stream velocity into account. The main motivation of this paper is to find how the flow past an infinite vertical flat plate in the free convection currents is affected by the variable suction.

BASIC EQUATIONS

Let $\bar{X}$-axis be taken along the vertical wall and $\bar{Y}$-axis at right angle to it. We consider the free-convection flow of an incompressible viscoelastic (Rivlin-Ericksen) fluid past a porous vertical wall. The basic equations of motion in the present problem are :

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{V}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{d\bar{u}}{d\bar{t}} + g\beta_1(\bar{T} - \bar{T}_\infty) + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \beta\frac{\partial^2}{\partial \bar{y}^2}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{V}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \qquad ...(1)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + 2(2\beta + \gamma)\frac{\partial \bar{u}}{\partial \bar{y}}\cdot\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \qquad ...(2)$$

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad ...(3)$$

$$\rho C_p\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{V}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = k\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \qquad ...(4)$$

$$\bar{U}(\bar{t}) = (1 + A\epsilon e^{i\bar{\varpi}\bar{t}}) \text{ (free stream velocity)}$$

where ν, β are kinematic viscosity and viscoelasticity, γ the kinematic cross-viscosity, $K = \frac{k}{\rho C_p}$ the thermal diffusivity, ρ the density of the fluid. β_1 is the coefficient of thermal expansion. Equation (3) shows that $\bar{V}$ can at the most be a function of time. Thus, equation (3) integrates to $\bar{V} = -V_\varpi(1 + A\epsilon e^{i\bar{\varpi}\bar{t}})$, where $A\epsilon << 1$ and V_ϖ is mean constant suction velocity. ϖ is the frequency of the oscillatory suction and free stream velocity. Introducing the following non-dimensional quantities :

$$y = V_\varpi \bar{y}/\nu,\ t = \bar{t}V_\varpi^2/\nu,\ u = \bar{u}/U_1,\ U(t) = \frac{\bar{U}(\bar{t})}{U_1},$$

$$v = \bar{V}/U_1,\ \omega = \frac{\varpi\nu}{V_\varpi^2},\ T = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_\varpi - \bar{T}_\infty},\ G = \frac{\nu g\beta_1}{U_1 V_\varpi^2}(\bar{T}_\varpi - \bar{T}_\infty),$$

$$p = \bar{p}/\rho V_\varpi^2,\ P = \frac{\rho\nu C_p}{k},\ S = V_\varpi^2\beta/\nu^2, \qquad ...(5)$$

where U_1 is steady free-stream velocity, P is the Prandtl number, G the Grashoff number and $S = -S_0$ (say) is the visco-elastic parameter, $0 < S_0 \leqslant \frac{1}{4}$. For Rivlin-Ericksen second order fluid the viscoelastic parameter S is necessarily negative. Here $\bar{T}_\varpi$ is the plate temperature and $\bar{T}_\infty$ is the temperature at infinity.

Eqns. (1) and (4) with the help of (5) reduce to

$$\frac{\partial u}{\partial t} - (1 + A\epsilon e^{i\omega t}) \frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} + \frac{dU(t)}{dt} - S_0 \frac{\partial^2}{\partial y^2} \left[\frac{\partial u}{\partial t} - (1 + A\epsilon e^{i\omega t}) \frac{\partial u}{\partial y}\right] \quad ...(6)$$

and

$$P \frac{\partial T}{\partial t} - P(1 + A\epsilon e^{i\omega t}) \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \quad ...(7)$$

In writing eqn. (4), the viscous dissipation frictional heat is assumed to be negligible. This is indeed a permissible simplification in the problem since the velocities usually encountered in natural convection are rather small.

The boundary conditions of the problem are :

$$\left.\begin{array}{ll} \text{When} & y = 0,\ u = 0,\ V = -V_\omega(1 + A\epsilon e^{i\omega t}) \text{ and } T = 1 \\ \text{and} & \\ \text{When} & y \to \infty,\ u = U(t) = (1 + \epsilon e^{i\omega t}) \text{ and } T \to 0 \end{array}\right\} \quad ...(8)$$

Solution to the Problem

Following Stuart (1955), we look for a solution of the form (in the neighbourhood of the plate)

$$u\,(y, t) = U_0(y) + \epsilon U_1(y)\, e^{i\omega t} \quad ...(9)$$

$$T(y, t) = T_0(y) + \epsilon T_1(y)\, e^{i\omega t} \quad ...(10)$$

The boundary conditions in view of (9) and (10) reduce to

$$\left.\begin{array}{l} U_0(0) = 0 = U_1(0) \\ U_0(\infty) = 1,\ U(\infty) = 1 \\ T_0(0) = 1,\ T_1(0) = 0 \\ T_0(\infty) = 0 = T_1\,(\infty) \end{array}\right\} \quad ...(11)$$

On substituting for U and T in eqns. (6) and (7) and equating the harmonic and non-harmonic terms separately to zero, we would get four linear differential equations in U_0, U_1, T_0, T_1, whose solutions under the conditions (11) are given by

$$\left.\begin{array}{l} U_0(y) = 1 - e^{-h_1 y} + b(e^{-Py} - e^{-h_1 y}), \\ U_1(y) = P_1(e^{-h_2 y} - e^{-hy}) + Q_1(e^{-Py} - e^{-h_2 y}) + \\ \qquad R_1(e^{-h_2 y} - e^{-h_1 y}) + (1 - e^{-h_2 y}), \\ T_0(y) = e^{-Py}, \\ T_1(y) = a(e^{-hy} - e^{-Py}), \end{array}\right\} \quad ...(12)$$

where

$$\left.\begin{aligned} & a = \frac{AP}{i\omega},\ b = \frac{G}{SP^3 - P^2 - P},\ h = \frac{P + \sqrt{P^2 + 4i\omega P}}{2}, \\ & h_1 = \frac{1}{2S_0}\left(1 - \sqrt{1 - 4S_0}\right),\ 0 < S_0 \leqslant \tfrac{1}{4}, \\ & -h_2 \text{ is the root of equation} \\ & \qquad S_0 h_2^3 + (1 - i\omega S_0)\, h_2^2 + h_2 - i\omega = 0, \\ & f(x) = S_0 x^3 + (1 - i\omega S_0)\, x^2 + x - i\omega, \\ & P_1 = \frac{aG}{f(-h)}, \\ & Q_1 = \frac{Ga + S_0 AbP^3 + AbP}{f(-P)}, \\ & R_1 = \frac{(1 + b)\, Ah_1 (1 + S_0 h_1^2)}{f(-h_1)}, \end{aligned}\right\} \qquad \ldots(13)$$

$G > 0$ (Corresponds to the cooling of the plate by the free convection currents).

The temperature distribution in the boundary layer is given by

$$\begin{aligned} T &= e^{-Py} + a\epsilon\, e^{i\omega t}(e^{-hy} - e^{-Py}) \\ &= e^{-Py} + \epsilon\, e^{i\omega t}(T_r + iT_i) \end{aligned} \qquad \ldots(14)$$

where

$$T_r = \frac{-AP}{\omega} \sin\left(\frac{y\xi \sin\eta}{2}\right) \exp\left\{-\frac{y}{2}(P + \xi\cos\eta)\right\}, \qquad \ldots(15)$$

$$T_i = \frac{AP}{\omega}\left[e^{-Py} - \cos\left(\frac{y\xi\sin\eta}{2}\right)\exp\left\{-\frac{y}{2}(P + \xi\cos\eta)\right\}\right], \qquad \ldots(16)$$

$$\left.\begin{aligned} \xi\cos\eta &= \left[\frac{P}{2}\left(\sqrt{P^2 + 16\omega^2} + P\right)\right]^{1/2} \\ \xi\sin\eta &= \left[\frac{P}{2}\left(\sqrt{P^2 + 16\omega^2} - P\right)\right]^{1/2} \end{aligned}\right\} \qquad \ldots(17)$$

Here T_r and T_i are the fluctuating parts of temperature field.

The transient temperature $\left(\omega t = \frac{\pi}{2}\right)$ is given by

$$T = \exp(-Py) - \epsilon\, T_i \qquad \ldots(18)$$

T_r, T_i and T have been plotted for various values of $A (= 0.2, 0.3)$, $P (= 2, 4, 6)$, $\omega (= 8, 10)$ and $\epsilon = 0.2$ in Figures (1–3). In general, the fluctuating parts T_i and

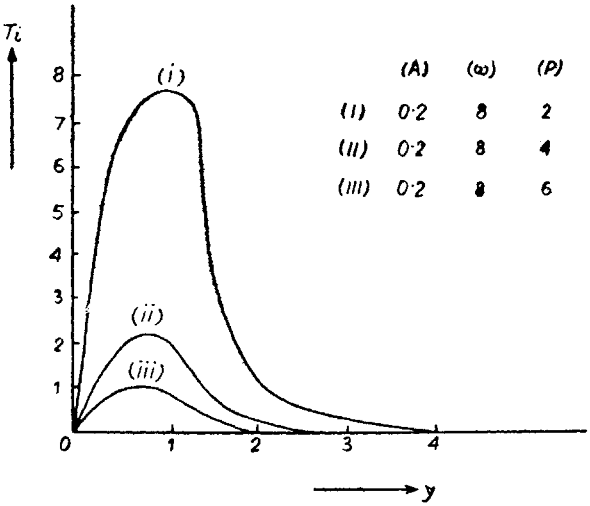

FIG. 1. Fluctuating part of temperature. Graphs have been magnified 10^3 times.

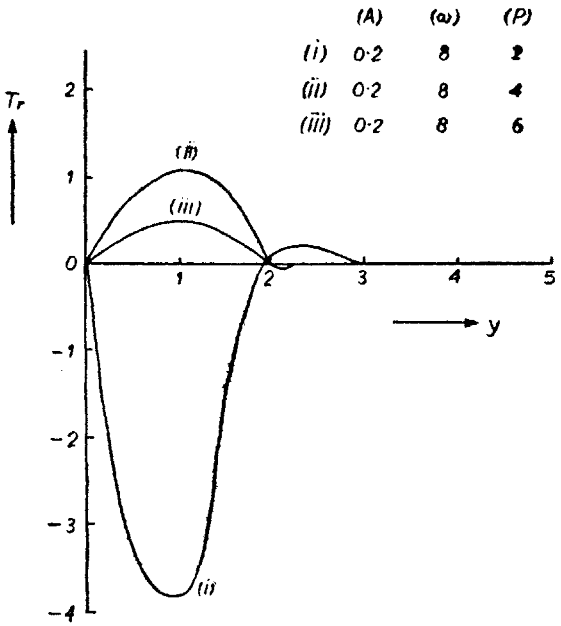

FIG. 2. Fluctuating part of temperature. Graphs have been magnified 10^4 times.

T_r (Figs. 1 & 2) of temperature field show reduction in cooling effect on the plate, for increasing values of Prandtl number P and forcing frequency $\bar{\omega}$; except beyond a certain critical value of $P(2 < P < 4)$, T_r shows heating effect.

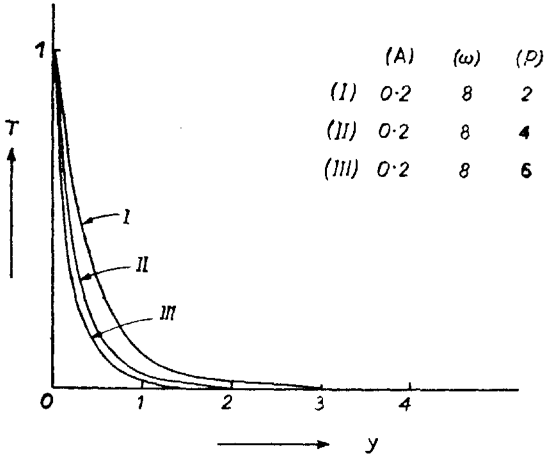

FIG. 3. Transient temperature when $\epsilon = 0.2$.

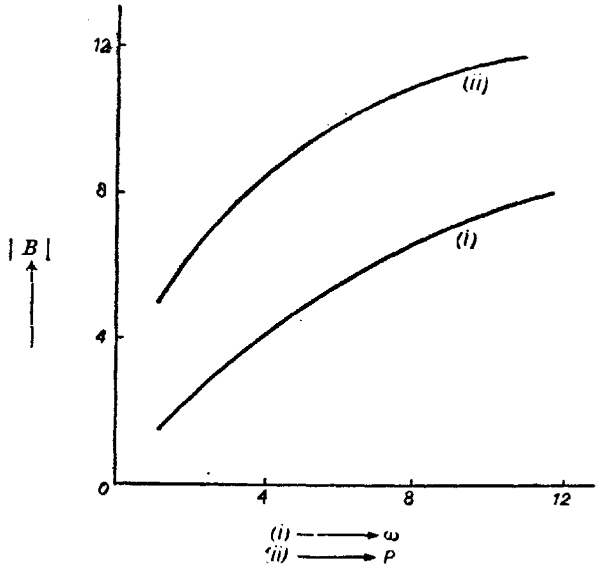

FIG. 4. Amplitude of rate of heat transfer in *BTU*/hr. ft^2.°*F*.

It is evident from graphs that the fluctuating parts of temperature decrease with increasing P and ω and increase with the increase in suction parameter. The transient temperature decreases with increasing P and A whereas an increase in forcing frequency leads to an increase in Transient temperature. Fluctuating part of temperature T_r decreases with increasing P and ω except at $P = 2$ for which T_r increases with increasing ω. Effect of increasing suction parameter A is to increase T_r except at $P = 2$ for which T_r decreases with increasing A.

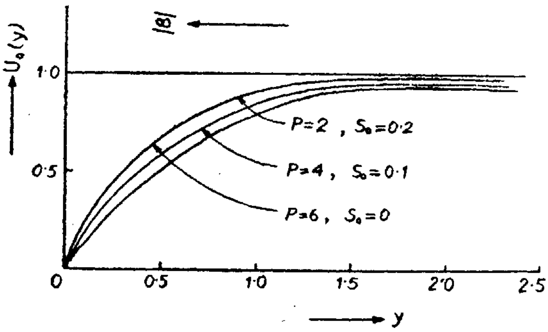

FIG. 5. Mean velocity profiles $U_0(y)$.

The velocity distribution in the boundary layer is given by

$$u = 1 - e^{-h_1 y} + b(e^{-Py} - e^{-h_1 y}) + \epsilon\{(1 - e^{-h_2 y}) + P_1(e^{-h_2 y} - e^{-hy})$$
$$+ Q_1(e^{-Py} - e^{-h_2 y}) + R_1(e^{-h_2 y} - e^{-h_1 y})\} e^{i\omega t} \qquad \text{...(19)}$$

We have drawn mean velocity distribution $U_0(y)$ in Fig. (5) in the two cases, viz., (*i*) S_0 fixed (0.2 say) and P varying and (*ii*) P fixed (2, say) and S_0 varying.

It is clear from Fig. (5) that for $G > 0$ (cooling of the plate) the mean velocity increases with increasing elastic parameter whereas an increase in Prandtl number leads to a decrease in the mean velocity in the boundary layer.

HEAT TRANSFER

The rate of heat transfer from the plate is given by

$$q = -K\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}$$

$$= \frac{GPKV_*^4}{\upsilon^2 g \beta_1}\left[1 + \frac{\epsilon A \mid B \mid}{2\omega} \cos(\omega t + \phi)\right], \qquad \text{...(20)}$$

where

$$\left.\begin{aligned} \mid B \mid &= (\xi^2 + P^2 - 2\xi P \cos\eta)^{1/2} \\ \text{Tan}\,\phi &= (P - \xi\cos\eta)/\xi\sin\eta \end{aligned}\right\} \qquad \text{...(21)}$$

$\xi \cos\eta$ and $\xi \sin\eta$ are given by (17) and $\xi^2 = P\sqrt{P^2 + 16\omega^2}$. From expression (20), it is clear that the variable suction (A) increases the rate of heat transfer. Amplitude $\mid B \mid$ of rate of heat transfer from the plate has been drawn in figure (4) in the two cases, viz., (*I*) P fixed (say 2) and (*II*) ω fixed (say, 8). It is observed that $\mid B \mid$ increases for increasing P and ω. For all $P \neq 0$, there is always phase-lead and for all $\omega \neq 0$, there is always a phase-lag.

The Nusselt number is given by

$$N = \frac{qV_*}{K(\bar{T}_* - \bar{T}_\infty)} = PV_*^2\left[1 + \frac{A \mid B \mid \epsilon}{2\omega}\cos(\omega t + \phi)\right] \qquad \ldots(22)$$

Skin-Friction

The expression for skin-friction on the plate is given by

$$\tau = \left[\left(\upsilon + \beta\frac{\partial}{\partial t}\right)\frac{\partial \bar{u}}{\partial \bar{y}}\right]_{\bar{y}=0}$$

$$= V_*^2\,[(1 + b)\,h_1 - bP + \epsilon\{P_1(h - h_1) + Q_1(h_2 - P) +$$

$$R_1(h_1 - h_2) + h_2\}\,(1 - i\omega S_0)\,e^{i\omega t}] \qquad \ldots(23)$$

The skin-friction corresponding to mean velocity is given by

$$\tau_0 = V_*^2\,(h_1 + bh_1 - bP) \qquad \ldots(24)$$

One obtains the inequality

$$(\tau_0)_{\substack{S_0=0\\P=2}} < (\tau)_{\substack{S_0=0.1\\P=2}} < (\tau_0)_{\substack{S_0=0.2\\P=2}}$$

Hence, it follows that the effect of elasticity is to increase the skin-friction. Solution of (2) on using (5) is given by

$$p = A\epsilon i\omega y(e^{-i\omega t} - 1) + (\psi - 2S_0)\left(\frac{\partial u}{\partial y}\right)^2, \qquad \ldots(25)$$

where $\psi = \frac{\gamma V_*^2}{\nu^2}$ (cross-viscosity). Evidently, along with elasticity cross-viscosity also effects the pressure.

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