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I. PHYSICS

Particle Physics

UPSILON RESONANCE AND BROKEN SU(3) IN NEW QUARK SECTOR

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Using the mass formula of SU(3) in non-strange sector, the magnetic moment, transition moments, decay widths are calculated. Electromagnetic weight diagrams are exploited for mass splitting among baryons.

Keywords : Upsilon resonance; SU(3); Mass sum rules.

INTRODUCTION

IWAO (1977) and Okubo (1977) showed that the available information on charmed particles satisfy approximately the octet version of SU(3) of Gell-Mann-Ne'eman in *u*-, *d*-, and *c*-quark sector. The quark line rule is satisfied strictly for charmed quark. In this paper, a similar problem is examined in *u*-, *d*-, and *b*-quark sector, *b*-quark being referred to as beauty with mass around 5 GeV. Two massive particles $\Upsilon(9.4)$ and Υ' (10.0) were discovered by Herb *et al.* (1977). It was thought that they could be bound states of a quark-antiquark pair in analogy to the J/Ψ (3.1) and Ψ' (3.7) states. In this paper, the magnetic moment, transition moment of mesons under SU(3) are calculated, with quark model *u*-, *d*-, *b*-). The transition moments are exploited to derive the decay widths when a vector meson decays into Pseudo-scalar mesons and vice versa. Weight diagrams are derived to discuss the hadron mass splitting using the Parallelogram rule. Our mass formula gives $\Upsilon(9.46)$, Υ' (10.0) and Υ'' (10.4) GeV.

MASS SPECTRUM

The mass formula under Gelfand-Zetlin technique, in *u*-, *d*-, *b*- quark sector may be written as [Bohm *et al.*, 1978]:

$$M = a + cX + d\left[I(I+1) - \frac{X^2}{4}\right] + F, \qquad \text{...(1)}$$

where a, c, d, and F are arbitrary constants. Here we define X-charge corresponding to the hypercharge as X = (Baryon number + New quantum number, beauty quantum number) analogue of strangeness.

Denoting the meson state corresponding to K-meson by B^{-*} $(b\bar{u})$, we get the sum rule as:

$$m_{B^{-*}} - m_{\rho} = m_{\Upsilon} - m_{B^{-*}} \qquad \text{...(2)}$$

we find $B^{-*} = 5.38$ (GeV) from the mass formula. The mass of the unitary singlet vector meson is

$$m_{\omega_0} = m_\gamma + m_\omega - m_{\omega_8} = 2.526 \text{ GeV}. \quad ...(3)$$

The mass of the vector meson in pure octet state is obtained from eqn. (2) under charge conjugation invariance

$$m_{\omega_8} = 1/3\,(4\, m_{B^{-*}} - m_\rho) = 6.25 \text{ GeV}. \quad ...(4)$$

The mass of the unitary singlet vector meson is

$$m_{\omega_0} = m_\gamma + m_\omega - m_{\omega_8} = 4.07 \text{ GeV} \quad ...(5)$$

The excited states of γ like γ' and γ'' are supposed to have mass 10.0 GeV and 10.4 GeV respectively.

Throughout this manipulation, the wavefunctions ω_8 and ω_0 are defined as usual by

$$\omega_8 = 1/\sqrt{6}\,(u\,\bar{u} + d\,\bar{d} - 2\,b\,\bar{b})$$

$$\omega_0 = 1/\sqrt{3}\,(u\,\bar{u} + d\,\bar{d} + b\,\bar{b}).$$

Magnetic Moment

It is useful to many applications to write down hadron wave functions in terms of quark wave functions. The baryon wave functions are bound state wave functions of a quark-antiquark pair. These wave functions include a spatial part, a spin part, a flavour unitary-symmetry part and a colour unitary symmetry part [Lipkin, 1978].

The spatial part of a hadron wave function cannot be written down explicitly, as its form depends on the unknown details of quark dynamics. However, if we assume that the forces between quarks are basically attractive, the lowest energy state will have symmetric spatial wave functions.

So far as spin part of the wave function is concerned, it is assumed that the lowest lying states have zero orbital angular momentum. Then the spin wave functions of SU(3) are,

$$\chi^{s=1}_{m=1} = \alpha\alpha,\ \chi^{s=0}_{m=0} = \frac{1}{\sqrt{2}}\,(\alpha\beta - \beta\alpha). \quad ...(6)$$

Here α and β denote quark spin functions with the third component 1/2 and $-1/2$, respectively.

We write the magnetic moment operator $\vec{\mu}_q$ of a quark as

$$\vec{\mu}_q = \vec{\mu}_q\,\vec{\sigma} \quad ...(7)$$

where $\vec{\sigma}$ is the pauli spin operator and $\vec{\mu}_q$ is constant which depends on the flavour of quark. It is assumed that the orbital angular momentum of quark is zero, the magnetic moment operator $\vec{\mu}$ of a meson is then written explicitly as

$$\vec{\mu}_M = \sum_{i=1}^{2} \vec{\mu}_q(i)\,\sigma(i) \quad ...(8)$$

where the sum is over the two quarks in a meson. The value of magnetic moment of any meson is the expectation value of Z-component of μ_M with respect to a meson wave function M which is maximally polarized along Z-axis. Thus we write

$$\vec{\mu}_M = (M, \Sigma\, \mu_q(i)\, \sigma_z(i)\, M) \qquad \ldots(9)$$

It is possible to evaluate $\vec{\mu}_M$ in terms of the μ_q for any meson once the flavour and spin wave functions of the mesons are specified. The magnetic moment of a meson in terms of the quark moments, using the simplified meson wave functions, is written as

$$\vec{\mu}_M = (M\chi, \Sigma\, \mu_q(i)\, \sigma_z(i)\, M\chi) \qquad \ldots(10)$$

We evaluate the magnetic moment of B^{*-} $(b\ \bar{u})$ vector meson and write

$$\begin{aligned} \mu_{B^{*-}} &= (b\bar{u}\chi_m, \mu_q(i)\, \sigma_z(i)\, b\bar{u}\chi_m) \\ &= (b\bar{u}\,[\alpha\alpha], \mu_q(b)\, \sigma_z(b)\, b\bar{u}(\alpha\alpha]) \\ &+ (b\bar{u}\,[\alpha\alpha], \mu_q(\bar{u})\, \sigma_z(\bar{u})\, b\bar{u}\,[\alpha\alpha]) \\ &= \mu_b + \mu_{\bar{u}} \end{aligned} \qquad \ldots(11)$$

Similarly, the magnetic moment of other mesons is

$$\mu_{B^{*0}} = \mu_b + \mu_{\bar{d}} \text{ and } \mu_{B^{*+}} = \mu_{\bar{b}} + \mu_u$$

$$\mu_{\bar{B}^{0*}} = \mu_{\bar{b}} + \mu_d \quad \text{and} \quad \mu_{\Upsilon} = \mu_b + \mu_{\bar{b}} \qquad \ldots(12)$$

For Pseudo-scalar mesons, we write the meson wave function as

$$\begin{aligned} \mu_{\bar{B}} &= (b\bar{u}\chi'_m, \mu_q(i)\, \sigma_z(i)\, b\bar{u}\chi'_m) \\ &= (b\bar{u}\chi'_m, \mu_q(b)\, \sigma_z(b)\, b\bar{u}\chi'_m) \\ &\quad + (b\bar{u}\chi'_m, \mu_q(\bar{u})\, \sigma_z(\bar{u}) b\bar{u}\, \chi'_m) \\ &= \tfrac{1}{2}\, (b\bar{u}(\alpha\beta - \beta\alpha), \mu_q(b)\, \sigma_z(b)\, b\bar{u}(\alpha\beta - \beta\alpha)) \\ &+ \tfrac{1}{2}\, (b\bar{u}(\alpha\beta - \beta\alpha), \mu_q(\bar{u})\, \sigma_z(\bar{u})\, b\bar{u}(\alpha\beta - \beta\alpha)) \\ &= 0 \end{aligned} \qquad \ldots(13)$$

Similarly, the magnetic moment of other pseudo-scalar mesons is written as,

$$\mu_{B^0} = \mu_{B^+} = \mu_{\bar{B}^0} = \mu_{\eta_b} = 0 \qquad \ldots(14)$$

The following properties of the operators μ_q and σ_z during the calculation have been used:

$$\mu_q u = \mu_u u,\ \mu_q d = \mu_d d,\ \mu_q b = \mu_b b \qquad \ldots(15.a)$$

$$\sigma_z(\alpha) = \alpha;\ \sigma_z\beta = -\beta \qquad \ldots(15.b)$$

TRANSITION MAGNETIC MOMENTS

The b-quark mesons also seem to possess a sort of inner magnetic moment which manifests itself in transitions to spin-1 states. Thus for transition moments (Singh, 1977)

$$< \phi\,(j = 1, m = 0) \mid \mu \mid \phi\,(j = 0, m = 0) > \qquad ...(16)$$

Using the wave function (6) and eqns. (12) and (14), we write the transition magnetic moment of b-quark mesons as

$$\begin{aligned} &< B^{*0} \mid \mu \mid B^0 > = -(\mu_d + \mu_b) \\ &< B^{*+} \mid \mu \mid B^+ > = -(\mu_b + \mu_u) \\ &< B^{-0*} \mid \mu \mid B^{-0} > = -(\mu_b + \mu_d) \end{aligned} \qquad ...(17)$$

DECAY WIDTHS

The width for the transition $A \rightarrow x + \gamma$ is given by (Boal, 1976)

$$\Gamma = 2.226 \left[\frac{M_A^2 - M_X^2}{M_A M_P}\right]^3 \sum_f \left\langle \left(\frac{\mu_{f_i}}{\mu_q}\right)\right\rangle \text{GeV}. \qquad ...(18)$$

where $\vec{\mu}_q$ is the quark magnetic moment and M_P is the proton mass. The transition moment $\mu_{XA}\,[\equiv (\mu_{f_i})]$ is defined by

$$\begin{aligned} \mu_{XA} &= < A \mid \sum_i \mu_i \mid X > \\ &= \mu_q < A \mid \sum_i q_i \sigma_{iz} \mid X > \end{aligned} \qquad ...(19)$$

where q_i and σ_{iz} are the charge and the z-component of the Pauli spin matrix of the ith quark. Using the eqns. (17), (18) and (19), the following results of decay widths of b-quark mesons when they decay from vector to pseudo-scalar and vice versa form, are obtained as :

$$\begin{aligned} &\Gamma(B^{*-} \rightarrow \bar{B}\,\gamma) = 0.125 \text{ KeV} \\ &\Gamma(\gamma \rightarrow \eta_b \gamma) = 1.68 \text{ KeV}. \end{aligned} \qquad ...(20)$$

ELECTROMAGNETIC WEIGHT DIAGRAMS AND MASS SPLITTING

Under SU(3), the magnetic moment is dependent on charge Q and U-spin if the quark content is u-, d- and s-. But we use u-, d- and b-quark, so instead of U-spin, we assume a new spin i.e., Z-spin which connects d and b-quarks, and have the same value of charge and Z-spin. Hence we assume that the magnetic moment in SU(3) will depend on Z-spin.

We plot two weight diagrams, one between I_3 and Z_3 for $(3/2)^+$ baryons and other between I_3 and Z_3 for $(1/2)^+$ baryons. Besides, weight diagrams between I_3 and X are also plotted (Figs. 1a, b & II a, b).

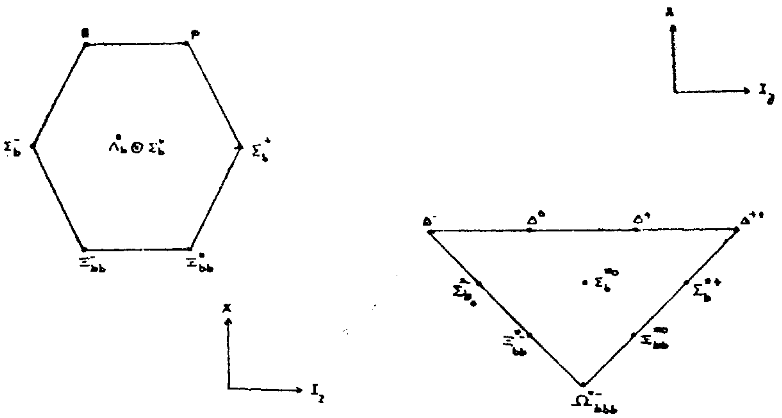
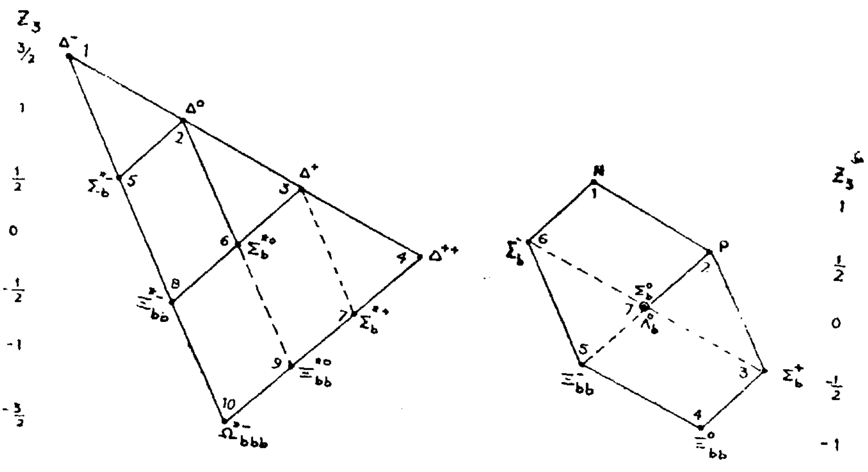


FIG. 1. Weight diagram between I_z and beauty charge-X of $(1/2)^+$ and $(3/2)^+$ beautiful baryons.



(*a*) (*b*)

FIG. 2. (a) Electromagnetic weight diagram used in deriving the parallelogram law for $(3/2)^+$ baryons. (b) Electromagnetic weight diagram used in deriving the parallelogram law for $(1/2)^+$ baryons.

Suppose the Hamiltonian of the hadrons consists of the terms

$$H = H_{vs} + H_{ms} + H_{em} + H_w \qquad ...(21)$$

where H_{vs} is the Hamiltonian of the "very strong" interactions, H_{ms} refers to "medium strong interactions," H_{em} and H_w refer to the electromagnetic and weak interactions. We assume that H_{vs} is SU(3) invariant and neglect H_w.

Take any parallelogram from Fig 2*a*, for example, the parallelogram 2, 5, 6 and 3 formed by the particles Δ^0, Σ_b^{*-}, Σ_b^{*0} and Δ^+. Now if $H_{em} = 0$, then (particle label stands for the mass of the particle)

$$\Delta^+ = \Delta^0,\ \Sigma_b^{*0} = \Sigma_b^{*-}$$

If $H_{ms} = 0$, we would have $\Delta^+ = \Sigma_b^{*0}$ and $\Delta^0 = \Sigma_b^{*+}$

Hence, we can write the parallelogram rule

$$\Delta^+ - \Delta^0 + \Sigma_b^{*0} - \Sigma_b^{*-} = 0, \quad \text{...(22)}$$

which holds even if $H_{em} \neq 0$ and $H_{ms} \neq 0$, provided that we neglect the terms like $< i \mid H_{em} H_{ms} \mid i >$.

From eqn. (22) we write

$$\Delta^0 + \Sigma_b^{*0} = \Sigma_b^{*-} + \Delta^+. \quad \text{...(23)}$$

Therefore, from the above assumptions, it clearly follows that *"for (3/2)+ baryons lying on the four vertices of any parallelogram on the electromagnetic weight diagram, the sum of the masses of the particles lying on the two extremes of one diagonal is equal to the sum of the masses of the particles lying on the extremes of the other."*

Let us apply this rule to some of the parallelograms of Fig. 2*a*. We study the parallelogram 2, 5, 6 and 8 formed by the baryons Δ^0, Σ_b^{*-}, Σ_b^{*0}, and Ξ_{bb}^{*-}. Then the parallelogram rule gives

$$\Delta^0 + \Xi_{bb}^{*-} = \Sigma_b^{*-} + \Sigma_b^{*0} \quad \text{...(24)}$$

Consider the parallelogram 6, 3, 9 and 7 formed by the particles

$$\Sigma_b^{*0},\ \Delta^+\ \Xi_{bb}^{*0} \text{ and } \Sigma_b^{+*}$$

we have,

$$\Sigma_b^{*0} = \Sigma_b^{*+} = \Delta^+ + \Xi_{bb}^{*0} \quad \text{...(25)}$$

For parallelogram 6, 9, 10 and 8 formed by the particles Σ_b^{*0}, Ω_{bbb}^{*-} Ξ_{bb}^{*0} and Ξ_{bb}^{*-}. we write

$$\Sigma_b^{*0} + \Omega_{bbb}^{*-} = \Xi_{bb}^{*0} + \Xi_{bb}^{*-} \quad \text{...(26)}$$

We now apply the parallelogram law for (1/2)+ baryons which, of course, cannot be applied to all the parallelograms of Fig 2*b* for (1/2)+ baryons because of the difficulty that one of the points in a parallelogram is doubly occupied.

Now we take the parallelogram 1, 2, 7 and 6 formed by the particles *N*, *P*, Σ_b^-, (Σ_b^0, Λ_b^0). The point 7 is doubly occupied by the particles (Σ_b^0, Λ_b^0). Hence we cannot apply the parallelogram law directly. we denote these particles by Σ_b^0, Λ_0^b

in I-space and by $\Sigma^0_{b_\mu}$, $\Lambda^0_{b_\mu}$ in Z-space. Then in this case, the parallelogram rule gives

$$P - N + \Sigma^-_b - \Sigma^0_b + \alpha\,(\Sigma^0_b\,,\ \Lambda^0_b) = 0$$

and

$$P - N + \Sigma^-_b - \Sigma^0_{b_\mu} + \alpha'(\Sigma^0_{b_\mu}\,,\ \Lambda^0_\mu) = 0, \qquad \text{...(27)}$$

where (Σ^0_b, Λ^0_b) and ($\Sigma^0_{b_\mu}$, $\Lambda^0_{b_\mu}$) are the transition masses in the I and Z-spin representations respectively (Transition masses are the off diagonal elements of the mass matrix). α and α' are constants and are found by satisfying the identity (27). This relation is of the required type because (Σ^0_b, Λ^0_b) is zero if only H_{ms} is operative and ($\Sigma^0_{b_\mu}$, $\Lambda^0_{b_\mu}$) is zero if only H_{em} is operative. The identity (27) is satisfied by

$$\alpha = \alpha' = \beta$$

giving

$$P - N + \Sigma^-_b - \Sigma^0_b + \beta\,(\Sigma^0_b\,,\ \Lambda^0_b) = 0 \qquad \text{...(28)}$$

Applying the same technique to the parallelogram 5, 4, 3 and 7 formed by the particles Ξ^-_{bb}, Ξ^0_{bb}, Σ^+_b and (Σ^0_b, Λ^0_b). We write

$$\Xi^-_{bb} - \Xi^0_{bb} + \Sigma^+_b - \Sigma^0_b + \beta(\Sigma^0_b\,,\ \Lambda^0_b) = 0 \qquad \text{...(29)}$$

Taking the difference of the relation (28) and (29), we get

$$P - N + \Sigma^-_b - \Xi^-_{bb} + \Xi^0_{bb} - \Sigma^+_b = 0 \qquad \text{...(30)}$$

This is the Colemann-Glashow relation. Since these particles lie on the extremities of a hexagon 1, 2, 3, 4, 5 and 6, so can be written in the hexagon form. According to the hexagon rule, we have

$$m(1) - m(2) + m(3) - m(4) + m(5) - m(6) = 0,$$

where $m(1)$ is the mass of the particle 1, and so on.

Conclusion

The numerical game so far successful is not expected to hold accurately in the *u*-, *d*- and *b*-quark sector, since large mass differences are involved in the problem. The mass spectra, transition probabilities suggests that the SU(3) symmetry in *b*-quark sector is badly broken due to the intrinsic mass difference among the quarks and to their mutual interactions.

However, we have given the sum rules of hadrons and successfully used the electromagnetic weight diagrams for mass splitting. The assumption made by Feldman and Mathew's hold for the above calculations as well. The main assumption made by the present authors is the magnetic moment depends on Z-spin. The stong decay $\gamma \rightarrow B^*B^{*+}$ is allowed according to OZI rule but is forbidden energetically, as suggested by the observed decay $\gamma \rightarrow \mu^+ \mu^-$. For the quark line rule and large mass of γ, it would be safe to assume its branching ratio to $\mu^+ \mu^-$ decay of several per cent. According to our scheme, the mass of γ, γ', γ'' are estimated as 9.46,10.0, 10.4 GeV respectively, and these particles are in 1S, 2S, 3S states i.e. the $n^3S_1Q\bar{Q}$ states ($n = 1, 2, 3$) of a new quark beauty with charge $-\frac{1}{3}$.

The width of upsilon is regarded small because it has hidden beauty in the way in which it is said that psion has small decay width since it has hidden charm.

The transition moments have been successfully derived and then decay widths have been calculated by exploiting these results.

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Appendix I

Quantum numbers of quarks

Quark	u	d	b
I_3-spin	1/2	−1/2	0
X-charge	1/3	1/3	−2/3
Beauty Quantum number B	0	0	−1
Z_3-Spin	0	1/2	−1/2
Z-spin	0	1/2	1/2

Appendix II

Quantum numbers of decuplet (symmetric) b-quark baryons and simplified wave functions

Label	Quark content	I_3	X	B	Z_3
Δ^{++}	$uuu\chi_m$ *	3/2	1	0	0
Δ^{+}	$uud\chi_m$	1/2	1	0	1/2
Δ^{0}	$udd\chi_m$	−1/2	1	0	1
Δ^{-}	$ddd\chi_m$	−3/2	1	0	3/2
Σ_b^{*0}	$udb\chi_m$	0	0	−1	0
Σ_b^{*-}	$ddb\chi_m$	−1	0	−1	1/2
Σ_b^{*+}	$uub\chi_m$	1	0	−1	−1/2
Ξ_{bb}^{*0}	$ubb\chi_m$	1/2	−1	−2	−1
Ξ_{bb}^{*-}	$dbb\chi_m$	−1/2	−1	−2	−1/2
Ω_{bbb}^{*-}	$bbb\chi_m$	0	−2	−3	−3/2

*χ_m is given in Eqn. (6).

Appendix III

Quantum numbers of octet (; 1/2)$^+$ b-quark baryons and simplified wave functions

Label	Quark content	I_3	X	B	Z_3
P	$uud\chi_m$	1/2	1	0	1/2
N	$udd\chi_m$	−1/2	1	0	1
Σ_b^{0}	$udb\chi_m$	0	0	−1	0
Λ_b^{0}	$(ud)_a b\chi'_m$	0	0	−1	0
Σ_b^{-}	$ddb\chi_m$	−1	0	−1	1/2
Σ_b^{+}	$uub\chi_m$	1	0	−1	−1/2
Ξ_{bb}^{0}	$ubb\chi_m$	1/2	−1	−2	−1
Ξ_{bb}^{-}	$dbb\chi_m$	−1/2	−1	−2	−1/2