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I. PHYSICS

Plasma Physics

PLASMA WAVE AND SECOND HARMONIC GENERATION BY A GAUSSIAN EM BEAM IN EXTRAORDINARY/ORDINARY MODE*

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This paper presents an investigation of the generation of an electron plasma wave by a Gaussian EM beam at the pump wave frequency in a homogeneous, collisionless, hot magnetoplasma. The EM beam is assumed to be propagating along the static magnetic field in extraordinary/ordinary mode of propagation. The plasma wave at pump wave frequency again interacting with the pump wave leads to the generation of the second harmonic. When the initial power of the pump wave is more than the critical power for self-focusing, the pump wave gets self-focused and correspondingly the power associated with the plasma wave and second harmonic is also modified. The change in the strength of the static magnetic field affects the propagation of two modes of the EM wave and hence the plasma wave and second harmonic generation is further modified.

Keywords: Gaussian Beams; Plasma Waves; Generation; Harmonic.

INTRODUCTION

PLASMA wave and second harmonic generation have been investigated recently in connection with the heating of plasmas by EM waves (Kruer & Estabrook, 1977; Grebogi *et al.*, 1977; White & Chen, 1974; and Sodha *et al.*, 1976 *a*). The mechanism responsible for the generation of such plasma waves is the usual ($V \times B_0$) force where V is the drift velocity of electron and B_0 is the static magnetic field. For example an EM wave propagating perpendicular to the static magnetic field in extraordinary mode drives a plasma wave at pump wave frequency and this plasma wave is resonantly driven if pump wave frequency is equal to the upper hybrid frequency (Kruer & Extabrook, 1977).

The present paper suggests and analyses a different mechanism of plasma wave and second harmonic generation in a hot collisionless magnetoplasma. This mechanism is applicable only when the incident pump wave is having non-uniform intensity distribution along its wavefront (such beams are generally used in plasma heating experiments). On account of Gaussian intensity distribution of the beam, ponderomotive force becomes finite and leads to the redistribution of the carrier's (Sodha *et al.*, 1976 *a*). Thus the electron density gradient so established contributes to plasma wave generation if a component of the electric vector of the

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EM wave is along this density gradient (Ginzburg, 1964). In addition to this, the gradient of the intensity of EM beam in a transverse plane also contributes to plasma wave generation.

This paper specifically gives an account of the plasma wave generation in a collisionless hot magnetoplasma when the Gaussian beam is propagating along the static magnetic fields in extraordinary/ordinary mode of propagation. The effect of changing the intensity of the d.c. magnetic field is to change the magnitude of the ponderomotive force and hence the electron density gradient is modified. This leads to a change in the self-focusing of the pump wave and the power associated with the plasma wave.

The generated plasma wave again interacts with the pump wave and leads to the generation of the second harmonic. The second harmonic power also gets significantly affected by a change in the strength of the static magnetic field.

The next Section establishes the equation for the generation of the plasma wave and evaluates the power of the generated plasma wave. In the third Section, the pump wave and the second harmonic dynamics are analysed and the expression for the second harmonic power is given. Finally, a brief discussion of results is presented.

Excitation of Plasma Waves

We consider here the propagation of a two dimensional intense Gaussian electromagnetic beam along the direction of magnetic field in a magnetoplasma. Choosing Z-axis along the direction of magnetic field, the two modes $A_{0+} = E_{0x} + i\,E_{0y}$ and $A_0 = E_{0x} - i\,E_{0y}$ propagate independently as long as the electron cyclotron frequency is less than the pump frequency. The intensity distribution of the two modes at $z = 0$ in the direction transverse to the propagation (X-axis) are given by

$$A_{0\pm}\,A^{*}_{0\pm}\;\Big|_{z=0} = A^{2}_{00\pm}\exp\left[-\,x^2/r_0^2\right], \qquad \ldots(1)$$

where $A_{00\pm}$ and r_0 are the axial amplitudes and the initial beam width respectively. It must be mentioned here that this type of distribution corresponds to a two dimensional beam (x, z) and $\dfrac{\partial}{\partial y} \equiv 0$. By making use of the (Brueckner & Jorna, 1974; and Sodha *et al.*, 1976*b*, *c*)

(*i*) *equation of continuity* :

$$\frac{\partial N}{\partial t} + \nabla \cdot [Nv] = 0;$$

(*ii*) *momentum transfer equation* :

$$m\left[\frac{\partial v}{\partial t} + (v \cdot \nabla)\,v\right] = -\,eE - \frac{e}{c}\,v \times (B_0 + B) - 2\Gamma_e m v -$$

$$\gamma\,K_B T_e\,\frac{\nabla N}{N};$$

and (*iii*) *Poisson's equation* :

$$\nabla . E = - 4\pi e N$$

and following Sodha *et al.* (1976*c*), the equation governing the amplitude of the plasma wave is given by

$$\frac{\partial^2 N}{\partial t^2} + 2\Gamma_e \frac{\partial N}{\partial t} - v_{th}^2 \nabla^2 N - \frac{e}{m} \nabla . (NE) = \nabla . \Big[N v \times \omega_c + \frac{N}{2} \nabla (v . v) - v \frac{\partial N}{\partial t} \Big] \quad ...(2)$$

where $v_{th} = [\gamma K_B T_0/m]^{1/2}$ is the electron thermal speed, K_B is Boltzmann constant, e and m are the electronic charge and mass respectively, E is the sum of electric vectors of EM wave and the self-consistent field, $\omega_c = eB_0/mc$ is the electron cyclotron frequency, c is the velocity of light, N is the electron concentration, v is the oscillation velocity of the electrons in the EM field and self consistent field, γ is the ratio of specific heats. $2\Gamma_e$ is the phenomenological damping factor which arises due to Landau damping in a collisionless magnetoplasma. In the present paper our motivation is to study the plasma wave at the pump wave frequency ω_0 and hence the equation governing the amplitude of the plasma wave at pump wave frequency ⟵⟶ can be written as

$$- \omega_0^2 N_1 + 2i\Gamma_e\omega_0 N_1 - v_{th}^2 \left(\frac{\partial^2 N_1}{\partial x^2} + \frac{\partial^2 N_1}{\partial z^2}\right) + \omega_p^2 \left(\frac{N_{0e}}{N_{00}}\right) N_1$$

$$= \frac{e}{2m} \left[1 - \frac{\omega_c(\omega_c \pm \omega_0)}{(\omega_c^2 - \omega_0^2)}\right] \left[N_{0e} \frac{\partial A_{0\pm}}{\partial x} + A_{0\pm} \frac{\partial N_{0e}}{\partial x}\right] \quad ...(3)$$

It must be mentioned here that eqn. (3) is valid when only one mode is present i.e., $E_{0x} = \frac{A_{0\pm}}{2}$, $E_{0y} = \pm \frac{A_{0\pm}}{2i}$. Moreover in deriving eqn. (3), the oscillation velocity of electron has been evaluated from the momentum balance equation by neglecting the terms $2\Gamma_e m v$ and $\gamma K_B T_e \frac{\nabla N}{N}$; this is justified for high frequency waves when $\omega_0 >> 2\Gamma_e$ and when the phase velocity of the wave is much greater than the electron thermal velocity. Further as the pump wave propagates in the plasma, its self-focusing occurs and at a given x the terms $\frac{\partial A_{0\pm}}{\partial x}$ and $\frac{\partial N_{0e}}{\partial x}$ get affected (cf. eqns. 17 and 4–5). This leads to the considerable modification in the plasma wave generation. Here N_{0e} is the time independent component of the electron density as modified by the main beam and is given by (Sodha *et al.*, 1976*a*)

$$N_{0e} = N_{00} \exp [- \alpha_\pm A_{0\pm} A_{0\pm}^*], \quad ...(4)$$

$$\alpha_\pm = \frac{e^2(2\omega_0 \mp \omega_c)}{16 K_B m \omega_0 (T_e + T_i)(\omega_0 \mp \omega_c)^2}, \quad ...(5)$$

N_{00} is the electron density in the absence of beam and $\omega_p^2 = 4\pi N_{00} e^2/m$. Writing the solution of eqn. (3) as (Sodha *et al.*, 1976 *a*, *b*, *c*)

$$N_1 = N_{100} \exp[-i(kz + S_\pm)] + N_{200} \exp[-i(k_{0\pm}z + S_{0\pm})], \quad ...(6)$$

where $S_\pm$ and $S_{0\pm}$ are the eikonals introduced, $k = \dfrac{[\omega_0^2 - \omega_p^2]^{1/2}}{v_{th}}$ is the propagation vector of the plasma wave supported by the hot plasma and not the propagation vector of the plasma wave generated at frequency ω_0 and

$$k_{0\pm} = \frac{\omega_0}{c}\left[1 - \frac{\omega_p^2}{\omega_0^2\left(1 \mp \dfrac{\omega_c}{\omega_0}\right)}\right]^{1/2}$$

is the propagation vector of the pump wave. We obtain from eqn. (3) after separating the real and imaginary parts (Sodha *et al.*, 1976*a*, *b*, *c*)

$$\left(\frac{\partial S_\pm}{\partial x}\right)^2 + 2k\frac{\partial S_\pm}{\partial z} = \frac{1}{N_{100}}\frac{\partial^2 N_{100}}{\partial x^2} + \frac{\left[\omega_0^2 - k^2 v_{th}^2 - \omega_p^2 \dfrac{N_{0e}}{N_{00}}\right]}{v_{th}^2} \quad ...(7a)$$

$$k\frac{\partial N_{100}^2}{\partial z} + N_{100}^2\frac{\partial^2 S_\pm}{\partial x^2} + \frac{\partial S_\pm}{\partial x}\frac{\partial N_{100}^2}{\partial x} + \frac{2\Gamma_e\omega_0}{v_{th}^2}N_{100}^2 = 0, \quad ...(7b)$$

In eqns. (7*a*) and (7*b*) terms $\dfrac{\partial^2 S_\pm}{\partial z^2}$ and $\left(\dfrac{\partial S_\pm}{\partial z}\right)^2$ are neglected within the WKB approximation (Sodha *et al.*, 1976*a*, *b*, *c*) and

$$N_{200} \simeq -\frac{e}{2m}\left[1 - \frac{\omega_c(\omega_c \pm \omega_0)}{\omega_c^2 - \omega_0^2}\right]\left[N_{0e}\frac{\partial A_{0\pm}^0}{\partial x} + A_{0\pm}^0\frac{\partial N_{0e}}{\partial x}\right]\Big/D^r, \quad ...(8)$$

where

$$D^r = \omega_0^2 - \omega_p^2\frac{N_{0e}}{N_{00}} - k_{0\pm}^2 v_{th}^2$$

It must be mentioned here that in deriving eqn. (8) from eqn. (3) the expression for $A_{0\pm}^0$ is used consistently with eqn. (17) of the present paper.

Following Sodha *et al.* (1976*a*, *b*, *c*), the solution of eqns. (7*a*) and (7*b*) can be written as

$$S_\pm = \frac{x^2}{2}\beta_\pm(z) + \phi_\pm(z),$$

$$\beta_\pm = \frac{k}{f_\pm}\frac{df_\pm}{dz}$$

and $$N_{100}^2 = \frac{B'^2}{f_\pm} \exp\left[-\frac{x^2}{a_0^2 f_\pm^2}\right] . \exp[-k_i z], \qquad ...(9)$$

where $k_i = \dfrac{2\Gamma_e \omega_0}{k v_{th}^2}$ and $f_\pm$ is the dimensionless plasma wave width parameter governed by

$$\frac{d^2 f_\pm}{dz^2} = \frac{1}{R_d^2 f_\pm^3} - \frac{\omega_p^2 f_\pm \alpha_\pm A_{00\pm}^2}{k^2 v_{th}^2 r_0^2 f_{0\pm}^3} \exp\left[-\alpha_\pm \frac{A_{00\pm}^2}{f_{0\pm}}\right], \qquad ...(10)$$

where $R_d = ka_0^2$ and $f_{0\pm}$ is the beam width parameter consistent with the eqn. (18) of the next section. The initial conditions on $f_\pm$ are $f_\pm = 1$ and $\dfrac{df_\pm}{dz} = 0$ (plane wave front) at $z = 0$, B' and a_0 are constants to be determined by the boundary condition that the amplitude of the generated plasma wave at $z = 0$ is zero. Thus

$$B' = \frac{N_{0e}(z=0)\left[1 - \dfrac{\omega_c(\omega_c \pm \omega_0)}{(\omega_c^2 - \omega_0^2)}\right]\left[2 \mp \dfrac{\omega_c}{\omega_0}\right] e A_{00\pm}\, x V_{00}^2 M_{10}}{16 m v_{th}^2 \left(1 \mp \dfrac{\omega_c}{\omega_0}\right)^2 r_0^2 \left[\omega_0^2 - k_{0\pm}^2 v_{th}^2 - \omega_p^2 \dfrac{N_{0e}}{N_{00}} z = 0)\right]}$$

and

$$a_0 = r_0, \qquad ...(11)$$

where

$$M_{10} = 1 - 8\frac{v_{th}^2}{V_{00}^2} \frac{\left(1 \mp \dfrac{\omega_c}{\omega_0}\right)^2}{\left(2 \mp \dfrac{\omega_c}{\omega_0}\right)}$$

and

$$V_{00}^2 = e^2 A_{00\pm}^2 / m^2 \omega_0^2$$

The electric vector E_1 of the plasma wave generated at frequency ω_0 can be determined by making use of Poisson's equation

$$\nabla . E_1 = -4\pi e N_1,$$

Keeping in mind the fact that the variations of the electron density in the plasma wave are along the Z-axis, (Sodha *et al.* 1976*c*) one can write,

$$E_1 \simeq -4\pi e \int N_1 dz$$

or

$$E_1 = -i\frac{\omega_p^2}{16\omega_0^2}M_{10}\frac{V_{00}^2}{v_{th}^2}\frac{\left(2\mp\frac{\omega_c}{\omega_0}\right)}{\left(1\mp\frac{\omega_c}{\omega_0}\right)^2}A_{00\pm}\left(1-\frac{\omega_c(\omega_c\pm\omega_0)}{\omega_c^2-\omega^2}\right)\times$$

$$\frac{N_{0e}}{N_{00}}(z=0)\frac{x}{r_0}\left[G_1\exp\left(-\frac{x^2}{2a_0^2f_{\pm}^2}\right).\exp(-i(kz+S_\pm))-\right.$$

$$\left.G_2\exp\left(-\frac{x^2}{2a_0^2f_{0\pm}^2}\right).\exp(-i(k_{0\pm}z+S_{0\pm}))\right], \qquad \ldots(12)$$

where

$$G_1 = \frac{\omega_0^2\exp[-k_iz/2]}{r_0f_\pm k\left[\omega_0^2-k_{0\pm}^2v_{th}^2-\omega_p^2\frac{N_{0e}}{N_{00}}(z=0)\right]}$$

and

$$G_2 = \frac{\omega_0^2N_{0e}\left[1-\frac{8v_{th}^2\left(1\mp\frac{\omega_c}{\omega_0}\right)^2f_{0\pm}}{v_{00}^2\left(2\mp\frac{\omega_c}{\omega_0}\right)}\right]}{r_0N_{0e}(z=0)\,k_{0\pm}f_{0\pm}^{7/2}\left[\omega_0^2-k_{0_\pm}^2v_{th}^2-\omega_p^2\frac{N_{0e}}{N_{00}}\right]\left[1-\frac{8v_{th}^2\left(1\mp\frac{\omega_c}{\omega_0}\right)^2}{v_{00}^2\left(2\mp\frac{\omega_c}{\omega_0}\right)}\right]}$$

In the expression for E_1, the first term in the bracket represents the contribution due to the plasma wave supported by hot plasma and the second term arises on account of intensity gradient and density gradient in a direction transverse to the direction of beam propagation.

Further, it is obvious from eqn. (3) that in the absence of an external pump wave ($N_{0e}=N_{00}$, $A_{0\pm}=0$, $\frac{\partial_0}{\partial x}=0$) the dispersion relation of a high frequency plasma wave (frequency $\omega_0 \gg 2\Gamma_e$) propagating along the static magnetic field has the following dispersion relation

$$\omega_0^2 = \omega_p^2 + k^2v_{th}^2$$

The phase velocity of this wave is $v_{th}\left[1-\frac{\omega_p^2}{\omega_0^2}\right]^{-1/2}$ and hence depending on the value of ω_P/ω_0, the phase velocity of this plasma wave is comparable to the thermal speed of the electrons and wave gets severely Landau damped. On the other hand, the phase velocity of the plasma wave arising on account of intensity and density gradients is the same as that of the pump wave and suffers no Landau damping. Therefore, in the sub-critical density region $\omega_P << \omega_0$, $k\lambda_d >> 1$ for the plasma wave supported by hot plasma; the component arising due to density gradient and intensity gradient contribute significantly for the scattering of EM wave.

The fraction of the power associated with the plasma wave can be written as

$$\frac{P}{P_0} = \left[\frac{\omega_P^2}{16\omega_0^2} M_{10} \frac{v_{00}^2}{v_{th}^2} \frac{\left(2 \mp \frac{\omega_c}{\omega_0}\right)}{\left(1 \mp \frac{\omega_c}{\omega_0}\right)^2} \left(1 - \frac{\omega_c(\omega_c \pm \omega_0)}{(\omega_c^2 - \omega_0^2)}\right) \frac{N_{0e}}{N_{00}} (z = 0) \right]^2$$
$$\times \frac{G_2^2}{2} f_{0\pm}^3, \qquad \text{...(13)}$$

where P_0 is the total power of the incident beam.

Second Harmonic Generation

We have seen in the last section that the plasma wave at pump wave frequency is excited on account of non-uniform intensity distribution of the EM wave. This excited plasma wave in turn interacts with the pump wave to generate the second harmonic significantly. To have an estimate of this second harmonic generation, we write the wave equation in the magnetoplasma as (Sodha *et al.*, 1976*a*).

$$\frac{\partial^2 E_\pm}{\partial z^2} + \frac{1}{2} \frac{\partial^2}{\partial x^2} \left(1 + \frac{\epsilon_{0\pm}}{\epsilon_{0zz}}\right) E_\pm = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_\pm + \frac{4\pi}{c^2} \frac{\partial J_\pm}{\partial t}, \qquad \text{...(14)}$$

where $E_\pm$ is the total electric vector of the EM waves (pump + second harmonic),

$$\epsilon_{0\pm} = 1 - \frac{\omega_P^2}{\omega_0^2 \left(1 \mp \frac{\omega_c}{\omega_0}\right)},$$

$$\epsilon_{0zz} = 1 - \frac{\omega_P^2}{\omega_0^2}$$

and $J_\pm$ is the total current density in the pump wave and second harmonic field. Further,

$$E_\pm = A_{0\pm} \exp [i\omega_0 t] + A_{2\pm} \exp [2i\omega_0 t],$$

and substituting this in eqn. (14), we obtain the equations governing $A_{0\pm}$ and $A_{2\pm}$ as

$$\frac{\partial^2 A_{0\pm}}{\partial z^2} + \frac{1}{2} \frac{\partial^2}{\partial x^2} \left(1 + \frac{\epsilon_{0\pm}}{\epsilon_{0zz}}\right) A_{0\pm} + \frac{\omega_0^2}{c^2} \epsilon_\pm A_{0\pm} = 0, \qquad \text{...(15a)}$$

and

$$\frac{\partial^2 A_{2\pm}}{\partial z^2} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(1 + \frac{\epsilon_{0\pm}}{\epsilon_{0zz}}\right) A_{2\pm} + \frac{4\omega_0^2}{c^2} A_{2\pm}\left[1 - \frac{N_{0e}}{N_{00}} \frac{\omega_P^2}{4\omega_0^2\left(1 - \frac{\omega_c}{2\omega_c}\right)}\right]$$

$$= \frac{\omega_P^2}{c^2} \frac{N_1}{N_{00}} \frac{\omega_0 A_{0\pm}}{(\omega_0 \mp \omega_c)}, \quad \ldots(15b)$$

where

$$\epsilon_{\pm} = \epsilon_{0\pm} + \epsilon_{2\pm}(A_{0\pm} \cdot A_{0\pm}^*)$$

$$\epsilon_{2\pm} = \frac{\omega_P^2}{\omega_0^2\left(1 \mp \frac{\omega_c}{\omega_0}\right)} [1 - \exp(-\alpha_{\pm} A_{0\pm} A_{0\pm}^*)], \quad \ldots(16)$$

Following Sodha *et al.* (1976*a*), the solution of eqn. (15*a*) can be written as

$$A_{0\pm} = A_{0\pm}^0 \exp[-i(k_{0\pm} z + S_{0\pm})]$$

$$[A_{0\pm}^0]^2 = \frac{A_{00\pm}^2}{f_{0\pm}} \exp[-x^2/r_0^2 f_{0\pm}^2]$$

$$S_{0\pm} = \frac{x^2}{2} \beta_{0\pm}(z) + \phi_{0\pm}(z)$$

$$\beta_{0\pm} = \frac{1}{f_{0\pm}} \frac{df_{0\pm}}{dz} \frac{2k_{0\pm}}{\left[1 + \frac{\epsilon_{0\pm}}{\epsilon_{zz}}\right]}, \quad \ldots(17)$$

and $f_{0\pm}$ is given by the equation

$$\frac{d^2 f_{0\pm}}{dz^2} = \frac{1}{4R_{d0\pm}^2 f_{0\pm}^3} - \frac{\epsilon_{2\pm}' (A_{00\pm}^2/f_{0\pm})\left(1 + \frac{\epsilon_{0\pm}}{\epsilon_{zz}}\right) A_{00\pm}^2}{2\epsilon_{0\pm} r_0^2 f_{0\pm}^2}, \quad \ldots(18)$$

where $R_{d0\pm}^2 = k_{0\pm}^2 r_0^4$, $k_{0\pm} = \frac{\omega_0}{c} \epsilon_{0\pm}^{1/2}$ and $\epsilon_{2\pm}'$ is the derivative of $\epsilon_{2\pm}$ with respect to its argument. The initial conditions on f_0 are $f_0 = 1$ and $\frac{df_0}{dz} = 0$ (plane wave front) at $z = 0$. The solution of eqn. (15*b*) can be written as

$$A_{2\pm} = E_{s00\pm} \exp[-i(k_{s\pm} z + S_{c\pm})] E_{s10\pm} \exp[-2i(k_{0\pm} z + S_{0\pm})]$$

where

$$k_{s\pm} = \frac{2\omega_0}{c} \epsilon_{\pm}^{1/2} (2\omega_0)$$

$$\epsilon_{\pm}(2\omega_0) = \left[1 - \frac{\omega_p^2}{4\omega_0^2\left(1 \mp \frac{\omega_c}{2\omega_0}\right)}\right], \qquad \text{...(19)}$$

wheie $S_{c\pm}$ and $S_{0\pm}$ are the eikonals introduced. Substituting (19) in (15*b*) and separating the real and imaginary parts, we get

$$\frac{1}{2}\left(1 + \frac{\epsilon_{\pm}}{\epsilon_{zz}}\right)\left[\frac{\partial^2 E_{s00\pm}}{\partial x^2} - \left(\frac{\partial S_{c\pm}}{\partial x}\right)^2 E_{s00\pm}\right] - 2k_{s\pm}E_{s00\pm}\frac{\partial S_{c\pm}}{\partial z} +$$

$$\frac{4\omega_0^2}{c^2}\cdot\frac{\omega_p^2}{4\omega_0^2}\,\frac{E_{s00\pm}}{\left(1 \mp \frac{\omega_c}{2\omega_0}\right)} - \frac{4\omega_0^2}{c^2}\cdot\frac{\omega_p^2}{4\omega_0^2}\cdot\frac{N_{0e}}{N_{00}}\,\frac{2\omega_0}{(2\omega_0 \pm \omega_c)}E_{s00\pm} = 0 \qquad \text{...(20a)}$$

and

$$\frac{1}{2}\left(1 + \frac{\epsilon_{\pm}}{\epsilon_{zz}}\right)\left[2\frac{\partial E_{s00\pm}}{\partial x}\cdot\frac{\partial S_{c\pm}}{\partial x} + E_{s00\pm}\frac{\partial^2 S_{c\pm}}{\partial x^2}\right] + 2k_{s\pm}\frac{\partial E_{s00\pm}}{\partial z} = 0, \qquad \text{...(20b)}$$

The solution of (20*a*) and (20*b*) are similar to self-focusing equation:

$$S_{c\pm} = \frac{x^2}{2}\beta_{s\pm}(z) + \phi_{s\pm}(z)$$

$$\beta_{s\pm} = k_{s\pm}\frac{1}{f_{s\pm}}\frac{df_{s\pm}}{dz}\frac{2}{\left[1 + \frac{\epsilon_{\pm}}{\epsilon_{zz}}\right]}, \qquad \text{...(21)}$$

$$E^2_{s00\pm} = \frac{B''^2}{f_{s\pm}}\exp\left[-x^2/b_0^2 f^2_{s\pm}\right]$$

$$\frac{d^2 f_{s\pm}}{dz^2} = \frac{1}{4}\left(1 + \frac{\epsilon_{\pm}}{\epsilon_{zz}}\right)^2\frac{1}{R^2_{ds\pm}f^3_{s\pm}} - \frac{f_{s\pm}\left(1 + \frac{\epsilon_{\pm}}{\epsilon_{zz}}\right)\omega_p^2\exp\left[-\alpha_{\pm}\frac{A^2_{00\pm}}{f_{0\pm}}\right]}{2f^3_{0\pm}r_0^2\,\epsilon_{\pm}(2\omega_0)\,2\omega_0^2\left(2 \mp \frac{\omega_c}{\omega_0}\right)}$$

where

$$R^2_{ds\pm} = k^2_{s\pm}r_0^4$$

B'' and b_0 are constants to be determined by the boundary condition viz. the second harmonic is zero at $z = 0$. Thus

$$B'' = -\frac{\frac{\omega_p^2}{c^2}\frac{N_1}{N_{00}}(z=0)A_{00\pm}}{[k^2_{s\pm} - 4k^2_{0\pm}]\left[1 \mp \frac{\omega_c}{\omega_0}\right]}, \qquad \text{...(22)}$$

and

$$r_0 = b_0. \qquad \text{...(23)}$$

Eqn. (15*b*) gives

$$E_{s10\pm} = \frac{\dfrac{\omega_p^2}{c^2}\dfrac{N_1}{N_{00}}\dfrac{\omega_0}{f_{0\pm}^{1/2}}\dfrac{A_{00\pm}}{(\omega_2 \mp \omega_c)}\exp\left[-x^2/2b_0^2 f_{0\pm}^2\right]}{[k_{s\pm}^2 - 4k_{0\pm}^2]}, \qquad \text{...(24)}$$

Therefore, E_2 can be obtained from eqn. (19) by making use of eqns. (21) and (24). The power carried by the second harmonic is given by

$$P = \frac{c}{8\pi}\epsilon_c^{1/2}(2\omega_0)\int_{-\infty}^{\infty} E_2 E_2^* \, dx$$

$$= \left[\frac{\omega_p^2}{c^2}\right]^2 A_{0\,0\pm}^2 \frac{c}{8\pi}\epsilon_b^{1/2}(2\omega_0)\left[\frac{H_1^2}{2}\frac{\pi^{1/2} r_0^3 f_{s\pm}^3}{[1 + f_{s\pm}^2]^3} + H_2^2 (\pi/2^5)^{1/2} r_0^3 f_{0\pm}^3\right.$$

$$\left. - \frac{H_1 H_2 (2^3\pi)^{1/2} r_0^3}{(p^2 + q^2)^{3/2}}\cos\left(\psi + \tan^{-1}\frac{r_0^2(\beta_{s\pm}^{-2}\beta_{0\pm})}{p}\right)\right]$$

where

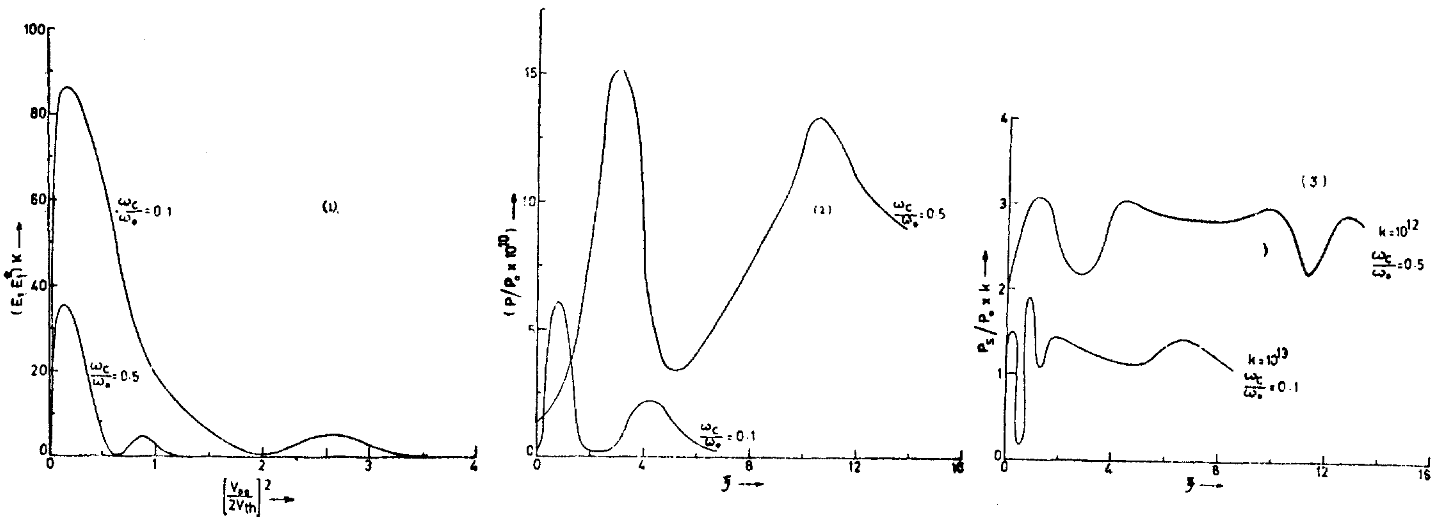
$$H_1 = \frac{N_{0e}(z=0)\, A_{00\pm} e v_{0\,0}^2\left(2 \mp \dfrac{\omega_c}{\omega_0}\right)\left[1 - \dfrac{\omega_c(\omega_c \pm \omega_0)}{(\omega_c^2 - \omega_0^2)}\right] M_{10}}{u_{th}^2\, 16 m r_0^2\left(1 \mp \dfrac{\omega_e}{\omega_0}\right)^3\left[\omega_0^2 - k_{0\pm}^2 u_{th}^2 - \omega_p^2\dfrac{N_{0e}}{N_{00}}(z=0)\right] N_{00}(k_{s\pm}^2 - 4k_{0\pm}^2)}$$

$$H_2 = -\frac{N_{0e}(z)\, A_{00\pm} e v_{0\,0}^2\left(2 \mp \dfrac{\omega_c}{\omega_0}\right)\left(1 - \dfrac{\omega_c(\omega_c \pm \omega_0)}{\omega_c^2 - \omega_0^2}\right)\left[1 - \dfrac{8u_{th}^2\left(1 \mp \dfrac{\omega_c}{\omega_0}\right)^2 f_{0\pm}}{v_{0\,0}^2\left(2 \mp \dfrac{\omega_c}{\omega_0}\right)}\right]}{16 r_0^2 f_{0\pm}^{7/2} m u_{th}^2\left(1 \mp \dfrac{\omega_c}{\omega_0}\right)^3 f_{s\pm}^{1/2} N_{00}\left(\omega_0^2 - k_{0\pm}^2 u_{th}^2 - \omega_p^2\dfrac{N_{0e}(z)}{N_{00}}\right)(k_{s\pm}^2 - 4k_{0\pm}^2)}$$

$$p = \left[\frac{1}{f_{s\pm}^2} + \frac{2}{f_{0\pm}^2} + 1\right]$$

$$\psi = (k_{s\pm} - 2k_{0\pm})\, z + \phi_{s\pm} - 2\phi_{0\pm}, \qquad \text{...(25)}$$

Numerical results for plasma wave and second harmonic powers have been calculated from eqns. (13) and (25) respectively. The results are presented in the form of graphs in Figs. 2 and 3.



FIGS. 1–3: (1) Variation of $E_1E_1^*$ with $[V_{00}/2\,v_{th}]^2$ for $\omega_c/\omega_0 = .1$ and $\omega_c/\omega_0 = .5$; (2) Variation of dimensionless plasma wave power, normalized by the initial power of the pump wave with $\xi(= (Z_c/\omega_0\, r_0^2)$ for $\omega_c/\omega_0 = .1$ and $\omega_c/\omega_0 = .5$; and (3) Variation of dimensionless power of the second harmonic, normalized by the initial power of the pump wave with $\xi(= (Z_c/\omega_0\, r_0^2)$ for $\omega_c/\omega_0 = .1$ and $\omega_c/\omega_0 = .5$.

Discussion

In the presence of an inhomogeneous beam, the electrons are redistributed from high field region to low field region on account of ponderomotive force. For a Gaussian beam considered in the present paper, the electrons get redistributed from axial region and a density gradient is established along the X-axis. This density gradient, as is obvious from eqn. (4), is determined by the strength of the magnetic field. Moreover, the effective electric field in the magneto-plasma on account of the pump wave is also determined by the strength of the magnetic field and it is given by

$$E^0_{eff} = \left[E_0 + \frac{1}{C}(V_0 \times B_0)\right], \qquad \ldots(26)$$

The plasma wave generation which depends on (*i*) Initial power of the main beam (*ii*) density gradient and (*iii*) intensity gradient of the E^0_{eff} must be significantly affected by the strength of the magnetic field. The density gradient along the X-axis is positive while the intensity gradient is negative and if the two are equal in magnitude the source term of eqn. (3) is zero and there will be no plasma wave generation as is evident from Fig. 1. Moreover, by changing the strength of the magnetic field, the magnitude of the plasma wave generation is significantly affected as is obvious from Figs. (1–3).

Fig. 1 depicts the variation of $E_1E_1^*$ with $[V_{00}/2\, v_{th}]^2$ for different values of the strength of the magnetic field at $z = 0$. This exhibits two maximum; one is of relatively lower magnitude than the second maximum and the magnetic field has significant effect on the magnitude of these maxima and their positions.

Fig. 2 displays the variation of the dimensionless power of the plasma wave (normalized by the initial power of the beam) against the distance of propagation for different strengths of the magnetic field. When the initial power of the main beam is more than the critical power for self-focusing, the beam propagates in the oscillatory wave guide and hence the plasma wave power, which depends on the intensity of the main beam, exhibits the corresponding variations. The effect of changing the strength of the magnetic field also affect the self-focusing of the main beam significantly and hence the plasma wave power exhibits the complicated behaviour.

Fig. 3 depicts the variation of the dimensionless second harmonic power (normalized by the pump wave power) with the distance of propagation — for different strengths of the magnetic field. The plasma wave power as discussed above is significantly affected with the distance of propagation and hence the power of the second harmonic is correspondingly modified. The effect of changing the strength of the magnetic field is quite significant on second harmonic generation.

In conclusion we have investigated that the plasma wave generated by a Gaussian beam at pump wave frequency is of considerable magnitude and the magnetic field has substantial effect on plasma wave generation. The second harmonic power as a result of this plasma wave is of significant magnitude and the magnetic field affects its magnitude further significantly. For the present set of parameters ($r_0 = 10\mu m$, $\omega_p/\omega_0 = .1$, $\omega_0 = 1.778 \times 10^{15}$ rad sec^{-1}) the maximum power of the plasma wave and second harmonic are $1.5\, P_0 \times 10^{-9}$ and $3.0\, P_0 \times 10^{-12}$ respectively.

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