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I. PHYSICS

Plasma Physics

INTERACTION OF AN ELECTROMAGNETIC WAVE WITH MOVING PLASMA SLAB

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Expressions for power reflection, transmission and absorption coefficients for *p*-polarized wave for a homogeneous, magnetized, collisional and moving plasma slab are derived. The collisional and magnetic effects are included through momentum transfer equation, ignoring the ion dynamics. The effects of plasma slab velocity ($\beta = v/c$), electron density (ω_p/ω) and cyclotron frequency $(\Omega/\omega_p)^2$ on reflection, transmission and absorption coefficients are investigated numerically.

Keywords : Plasma; EM Waves; Waves; Waves in Plasma; Moving Plasma

INTRODUCTION

THE problem of electromagnetic interaction with the moving plasma are of significant use in the fields of ionospheric studies, re-entry communication, meteorology etc., and have received considerable attention of many workers including Yeh (1966) Collier and Tai (1965), Jain *et al.* (1973), Tiwari and Tolpadi (1977), Phalswal and Varma (1978) and Phalswal *et al.* (1980). However, Phalswal and Varma (1978) and Phalswal *et al.* (1980) have studied the problem for moving plasma half-space and moving plasma slab within a waveguide in the absence of any external magnetic field.

Here, in the present paper, the power reflection (R), transmission (T) and absorption (A) coefficients are derived for *p*-polarized wave for a moving, homogeneous, collisional and magnetized plasma by solving the Maxwell's equations under necessary boundary conditions. The effects of plasma density and slab velocity are presented numerically in the presence of external magnetic field on R, T and A.

FORMULATION AND SOLUTION OF THE PROBLEM

Consider a collisional plasma slab with sharp boundaries at $z = o$ and $z = d_o$ which is embedded in a uniform static magnetic field $B_o \hat{y}$. The plasma slab can move with any uniform velocity v in the z-direction. Let us also consider two reference frames, the primed frame being fixed in the moving plasma slab and the unprimed frame is attached with the e.m.w. source. Thus the two reference frames are in relative motion along their z-axes.

Following Seshadri (1973) the permittivity of the collisional and magnetized plasma medium in the primed frame is given by

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$$\epsilon'_p = \epsilon'_o \left[1 - \frac{\omega_p'^2}{\omega'^2 - \Omega'^2} + \frac{i\nu'\omega_p'^2(\omega'^2 + \Omega'^2)}{\omega'(\omega'^2 - \Omega'^2)^2} \right] \quad ...(1)$$

where ϵ'_o, ω'_p, ω', ν' and Ω' are respectively, the permittivity of free space, plasma frequency, angular frequency, collisional frequency and cyclotron frequency of the the electrons in the primed system.

Also following Sommerfield (1952), the incident field equations for p-polarized wave in the unprimed system are

$$\left.\begin{aligned} B_y &= B_o \exp[-i(kz - \omega t)] \\ E_y &= 0 \end{aligned}\right\} \quad ...(2)$$

where B_o, k and ω are the amplitude of the incident magnetic field, wave vector in positive direction of z-axis and angular frequency of the wave respectively.

The field equations for incident, reflected and transmitted p-polarized wave in the primed frame at two interfaces of the plasma slab are given by (at $z = 0$ interface).

Incident fields

$$B'_y = B'_o \exp[-i(k'z' - \omega' t')]; E'_y = 0 \quad ...(3)$$

Reflected fields

$$B_y^{(r_1)'} = A'_{r_1} \exp[i(k'z' + \omega' t')]; E_y^{(r_1)'} = 0 \quad ...(4)$$

Transmitted fields

$$B_y^{(t_1)'} = G'_{t_1} \exp[-i(k'_s z' - \omega' t')]; E_y^{(t_1)'} = 0 \quad ...(5)$$

At $z = d' = d_o$ interface, the incident fields will be the same as given by equation (5) and reflected and transmitted fields are given by

$$B_y^{(r_2)'} = A'_{r_2} \exp[i(k'_s z' + \omega' t')]; E_y^{(r_2)'} = 0 \quad ...(6)$$

and

$$B_y^{(t_2)'} = G'_{t_2} \exp[-i(k'z' - \omega' t')]; E_y^{(t_2)'} = 0, \quad ...(7)$$

where B'_o, A'_{r_1}, G'_{t_1}, A'_{r_2} and G'_{t_2} are the arbitrary constants. ω' is the frequency of the wave in the primed frame and k' and k'_s are the wave vectors in 1ree space and plasma slab respectively.

Matching the boundary conditions at the two interfaces and following Heald and Wharton (1965), the average power reflection transmission and absorption coefficients are given as

$$\left.\begin{aligned} R' &= \frac{r_p'\left[1 + (1 + 2r_p')\exp\left(-\frac{4\omega'\chi'd'}{c}\right)\right]}{1 - r_p'^2 \exp\left(-\frac{2\omega'\chi'd'}{c}\right)} \\ T' &= \frac{(1 - r_p')^2 \exp\left(-\frac{2\omega'\chi'd')}{c}\right)}{1 - r_p'^2 \exp\left(-\frac{4\omega'}{c}\chi'd'\right)} \\ A' &= \frac{(1 - r_p')\, 1 - \exp\left(\frac{2\omega'\chi'd'}{c}\right)}{1 - r_p' \exp\left(-\frac{2\omega'\chi'd'}{c}\right)} \end{aligned}\right\} \quad \ldots(8)$$

If in the unprimed frame $k^{(r1)}$, $k_s^{(r1)}$, $k_s^{(t1)}$, $k_s^{(r2)}$, $k^{(t2)}$ and $\omega^{(r1)}$, $\omega^{(t1)}$, $\omega^{(r2)}$, $\omega^{(t2)}$ are the wave vectors and angular frequencies of the reflected and transmitted fields at the two interfaces of the slab respectively and A_{r1}, G_{t2}, A_{r2} and G_{t2} are the arbitrary constants as in equations (4–7), then making the use of phase invariance (Pauli, 1958); the variance of Maxwell's equations with respect to the Lorentz transformation (Sommerfeld, 1952); and satisfying the boundary conditions, we obtain the equations of transformations from primed to unprimed frame as

$$k^{(r1)} = \alpha\left(k' - \frac{\omega'\beta}{c}\right) = \alpha^2 k(1 - \beta)^2$$

$$k^{(r2)} = \alpha\left(k_s' - \frac{\omega'\beta}{c}\right) = \alpha^2 k(1 - \beta)(D - \beta) = k_s^{(t1)}$$

$$k^{(t2)} = \alpha\left(k' + \frac{\omega'\beta}{c}\right) = \alpha^2 k(1 - \beta^2)$$

$$\omega^{(r1)} = \alpha(\omega' - k'\beta c) = \alpha^2\omega(1 - \beta)^2$$

$$\omega^{(r2)} = \alpha(\omega' - k_s'\beta c) = \alpha^2\omega(1 - \beta)(1 - D\beta) = \omega^{(t1)}$$

$$\omega^{(t2)} = \alpha(\omega' + k'\beta c) = \alpha^2\omega(1 - \beta^2)$$

$$d = \frac{d'}{\alpha} = \frac{d_o}{\alpha}\,;\ \nu' = \alpha\nu$$

$$r_p = r_p'\left(\frac{1 - \frac{\nu k}{\omega}}{1 + \frac{\nu k^{(r)}}{\omega^{(r)}}}\right)^2 = \frac{(1 - k_r)^2 + k_i^2}{(1 + k_r)^2 + k_i^2}\left(\frac{1 - \beta}{1 + \beta}\right)^2, \quad \ldots(9)$$

where

$$\alpha = (1 - \beta^2)^{-(1+2)}\,;\ \beta = \frac{v}{c}\,;\ D = \left[1 - \frac{\omega_p'^2}{\omega'^2 - \Omega'^2} + \frac{i\nu'\omega_p'^2(\omega'^2 + \Omega^{2\prime})}{\omega'(\omega'^2 - \Omega'^2)^2}\right]^{1/2},$$

d_o is the proper thickness of the slab and

$$k_r = [\tfrac{1}{2}\{\sqrt{X^2 + Y^2} + X\}]^{1/2};\; k_i = [\tfrac{1}{2}\{\sqrt{X^2 + Y^2} - X\}]^{1/2},$$

$$X = 1 - \frac{\omega_p^2/\omega^2}{\alpha^2(1-\beta)^2 - \Omega^2/\omega^2}$$

$$Y = \frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2}\frac{[\alpha^2(1-\beta)^2 + \Omega^2/\omega^2]}{[\alpha^2(1-\beta)^2 - \Omega^2/\omega^2]^2}$$

Now following Phalswal and Varma (1978) and Phalswal *et al.* (1980), we obtain the reflection, transmission and absorption power coefficients in the unprimed frame as

$$\left.\begin{aligned} R &= \frac{r_p\left[1 + (1-2r_p)\exp\left(-\dfrac{4\omega\chi d_o}{c\alpha}\right)\right]}{1 - r_p^2\exp\left(-\dfrac{2\omega\chi d_o}{c\alpha}\right)} \\ T &= \frac{(1-r_p)^2\exp\left(-\dfrac{2\omega\chi d_o}{c\alpha}\right)}{1 - r_p^2\exp\left(-\dfrac{4\omega\chi d_o}{c\alpha}\right)} \\ A &= \frac{(1-r_p)\left[1 - \exp\left(-\dfrac{2\omega\chi d_o}{c\alpha}\right)\right]}{1 - r_p\exp\left(-\dfrac{2\omega\chi d_o}{c\alpha}\right)} \end{aligned}\right\} \quad ...(10)$$

where

$$\chi = [\tfrac{1}{2}\{\sqrt{X_1^2 + X_2^2} - X_1\}]^{1/2}$$

$$X_1 = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2};\; X_2 = \frac{\nu}{\omega}\frac{\omega_p^2(\omega^2 + \Omega^2)}{(\omega^2 - \Omega^2)^2}.$$

Results and Discussion

For $\omega_p = 0$, i.e., in the absence of plasma, the reflection and absorption coefficients reduce to zero. In case of collisional ($\nu \neq 0$) and unmagnetized plasma ($\Omega = 0$) for $d_o = \infty$, the reflection coefficient (R) in equations (10) reduces similar to the one obtained by Phalswal and Verma (1978). Taking $\nu/\omega_p = 0.01$, $\omega/c = d_o = 10$, the effects of slab velocity ($\beta = v/c$) and cyclotron frequency $(\Omega/\omega)^2$ are plotted in Figs. (1) and (2) for $\omega_p/\omega = 0.5$ and 1.5.

Fig. (1) shows the variation of reflection, absorption and transmission coefficients with slab velocity $\beta\,(= v/c)$, for $(\Omega/\omega_p)^2 = 5$. When the slab moves away from the e.m.w. source, then for $\omega_p/\omega = 1.5$, the reflection coefficient decreases with β, but the absorption coefficient increases, whereas for the case in which plasma slab moves towards the e.m.w. source, the reflection cofficient increases, but the

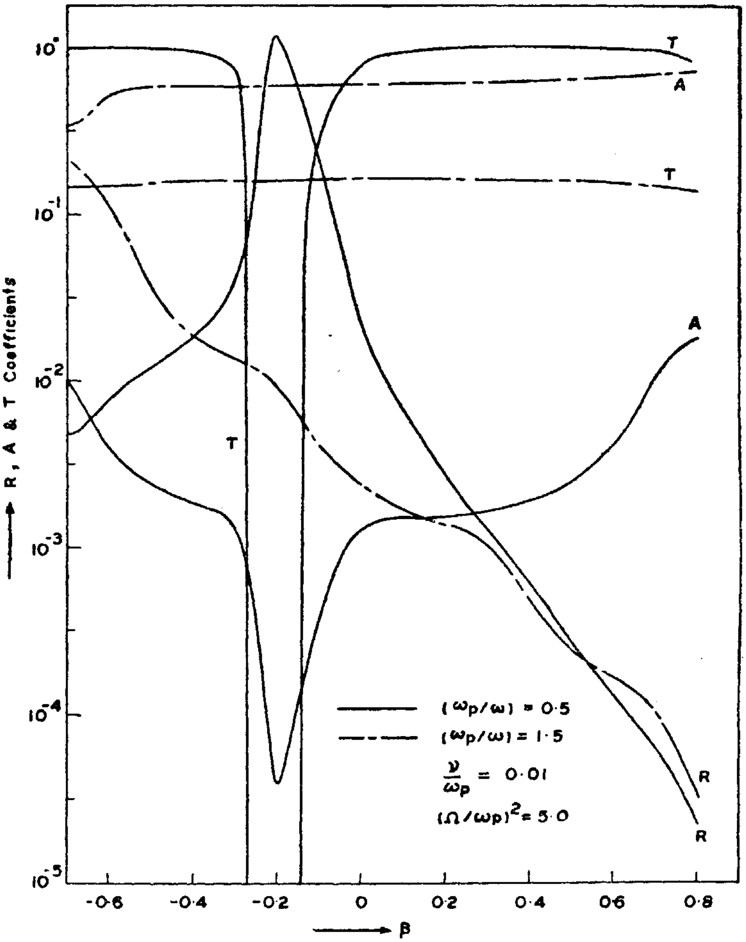

FIG. 1. Variation of R, T and A with plasma slab velocity ($\beta = v/c$).

absorption coefficient has the reverse trend. For plasma frequency greater than the wave frequency ($\omega_p/\omega = 1.5$), the variation of transmission coeffieient with β is negligible. For plasma frequency less than the wave frequency ($\omega_p/\omega = 0.5$), the absorption coefficient increases with β, but when the slab moves towards the e.m.w. source it decreases and minimum at $\beta = -0.2$, in the range $\beta < -0.2$, it again starts increasing. In the limits $-0.3 < \beta - 0.1$ the transmission coefficient is minimum, but outside the limits its variation is negligible. However, the reflection coefficient is maximum and greater than unity when $\beta = -0.2$ and it decreases in the limits $\beta \gtrsim -0.2$.

Fig. (2) shows the variation of R, A and T with $(\Omega/\omega_p)^2$ for $(\omega_p/\omega) = 1.5, 0.5$. It is observed that for $(\omega_p/\omega) = 1.5$ the transmission coefficient is maximum when cyclotron frequency is equal to the plasma frequency. As the cyclotron frequency exceeds the plasma frequency, R and T decreases, whereas the absorption coefficient increases. When $\Omega < \omega_p$, the transmission coefficient starts decreasing.

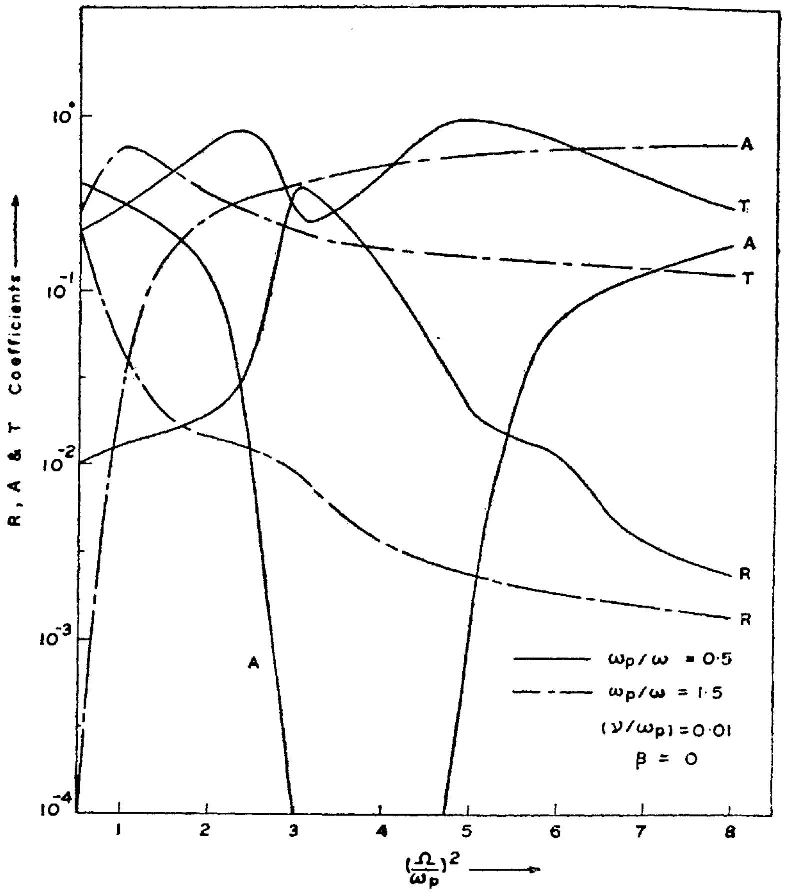

FIG. 2. Variation of R, T and A as a function of cyclotron frequency $(\Omega/\omega_p)^2$.

At lower plasma density ($\omega_p/\omega = 0.5$) in the limits $3 < \left(\frac{\Omega}{\omega_p}\right)^2 < 5$ the absorption coefficient is negligible, but in the range $5 < (\Omega/\omega_p)^2 < 3$ the absorption coefficient increases with $(\Omega/\omega_p)^2$. The reflection cofficient is maximum at $(\Omega/\omega_p)^2 = 3$ and it decreases with $(\Omega/\omega_p)^2$ in the range $(\Omega/\omega_p)^2 \gtrsim 3$. The transmission coefficient shows the oscillatory behaviour with plasma density as shown in Fig. (2).

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