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Theoretical Physics

ABSORPTION COEFFICIENT OF HOT ELECTRONS IN SEMICONDUCTORS

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Expressions are derived for the absorption coefficient of light by hot electrons in nondegenerate semiconductors. Scattering of hot electrons by ionized impuritities and optical phonons have been taken into account. The results have been compared with earlier theoretical results in the last section.

Keywords : Absorption Coefficient; Hot Electrons; Semiconductors; Optical Phonons; Acoustic Phonons; Inelastic Scattering; Transport Equation

§INTRODUCTION

THE presence of free electric carriers which characterize a conducting medium, alter the behaviour of light propagation. The absorption coefficient of light by hot electrons has been calculated for two cases (Sharnouby, 1971; and Bonch-Bruivich & Sharnouby, 1972) :—

Case 1 : The scattering of electrons by acoustic phonons is predominant.

Case 2 : The inelastic scattering of electrons by optical phonons is predominant and the distribution function of electrons is anisotropic, i.e., represented in terms of δ-function.

At low temperatures and high concentration of impurities in semiconductors, the scattering of electrons by acoustic phonons and impurities are predominant. For high static electric fields, $T_{\varphi\varphi} \simeq \bar{\epsilon}_{\mathbf{p}} > w_0 (w_0$ — energy of the emitted optical phonon; $\bar{\epsilon}_{\mathbf{p}} = \dfrac{\bar{\mathbf{p}}^2}{2m}$ — average energy of the carrier with momentum $\mathbf{p}$), the emission of optical phonons by hot electrons is predominant. Under these conditions, the absorption coefficient of hot electrons has been calculated in this paper. In (§ II) of this paper, we shall first discuss the Boltzmann equation relevant to hot electrons and its solution for the case of acoustic phonons and impurity scattering. Expressions for the absorption coefficient are given in (§ III). In (§IV), we obtain an expression for the absorption coefficient in the presence of optical phonons scattering. In the last section (V) some numerical results have been given for n-Ge and comparison is made with previous theoretical results.

(§II) TRANSPORT EQUATION

When the scattering of electrons is approximately elastic and isotropic, the distribution function of hot electrons can be written in the form

$$f(\mathbf{p}, t) = f_s(\mathbf{p}, t) + f_a(\mathbf{p}, t); f_a(\mathbf{p}, t) \ll f_s(\mathbf{p}, t)$$

where $f_s(\mathbf{p}, t) = f_s(-\mathbf{p}, t)$ — is the symmetric part of the distribution function; $f_a(\mathbf{p}, t) = -f_a(-\mathbf{p}, t)$ is the antisymmetric part. The Boltzmann kinetic equation for the distribution function $f(\mathbf{p}, t)$ has the form

$$\frac{\partial f_s(\mathbf{p}, t)}{\partial t} + e\mathbf{E} \cdot \vec{\nabla}_p f_a(\mathbf{p}, t) = I[f_s(\mathbf{p}, t)] \quad \ldots(1)$$

$$\frac{\partial f_a(\mathbf{p}, t)}{\partial t} + e\mathbf{E} \cdot \vec{\nabla}_p f_s(\mathbf{p}, t) = I[f_a(\mathbf{p}, t)] \quad \ldots(2)$$

here $\vec{\nabla}_p$ — denotes the gradient in momentum space; $e\mathbf{E}$ — force which act upon the particle of charge e under the effect of electric field of strength E; I — the collision integral.

In the presence of D. C. field $\mathbf{E}_0$ and a parallel small A.C. field $\mathbf{E}_1 e^{-iwt}$, the solution of (1), (2) can be put in the form

$$f_s(\mathbf{p}, t) = f_s^o(\epsilon_\mathbf{p}) + f_s^l(\epsilon_\mathbf{p}) e^{-iwt}; \; f_s^l(\epsilon_\mathbf{p}) \ll f_s^o(\epsilon_\mathbf{p}) \quad \ldots(3)$$

$$f_a(\mathbf{p}, t) = f_a^o(\mathbf{p}) + f_a^l(\mathbf{p}) e^{-iwt}; \; f_a^l(\mathbf{p}) \ll f_a^o(\mathbf{p}) \quad \ldots(4)$$

Considering (3) and (4), we have a system of equations

$$e\mathbf{E}_0 \cdot \vec{\nabla}_p f_a^o(\mathbf{p}) = I[f_s^o(\epsilon_\mathbf{p})] \quad \ldots(5)$$

$$e\mathbf{E}_0 \cdot \vec{\nabla}_p f_s^o(\epsilon_\mathbf{p}) = -\frac{f_a(\mathbf{p})}{\tau(\epsilon_\mathbf{p})} \quad \ldots(6)$$

$$-iw f_s^1(\epsilon_\mathbf{p}) + e\mathbf{E}_0 \cdot \vec{\nabla} p f_a^l(\mathbf{p}) + e\mathbf{E}_1 \cdot \vec{\nabla}_p f_a^o(\mathbf{p}) = I[f_s^l(\epsilon_\mathbf{p})] \quad \ldots(7)$$

$$\left(-iw + \frac{1}{\tau(\epsilon_\mathbf{p})}\right) f_a^l(\mathbf{p}) + e\mathbf{E}_0 \cdot \vec{\nabla}_p f_s^l(\epsilon_\mathbf{p}) + e\mathbf{E}_1 \cdot \vec{\nabla}_p f_s^o(\epsilon_\mathbf{p}) = 0 \quad \ldots(8)$$

The solutions of (5) and (6) are well known (Conwell, 1967) for acoustic phonons, optical phonons, and zero point lattice vibrations scattering. For n-Ge at 30 °K n_I (concentration of impurities) $\sim 10^{14}$ cm^{-3}, the scattering of electrons by impurities and acoustic phonons are predominant, i.e., we consider

$$\frac{1}{\tau(\epsilon_\mathbf{p})} = \frac{1}{\tau_{ak}(\epsilon_\mathbf{p})} + \frac{1}{\tau_I(\epsilon_\mathbf{p})} \text{ or } \tau(x) = \frac{l_a m^{1/2} x^{3/2}}{(2kT)^{1/2}(x^2 + q^2)},$$

$$If_s(\epsilon_\mathbf{p}) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{mp^3 KT}{l_a M}\frac{\partial}{\partial p} f_s(\epsilon_\mathbf{p}) + \frac{p^4}{l a M} f_s(\epsilon_\mathbf{p})\right]$$

where $x = \frac{\epsilon \mathbf{p}}{KT}$; $q^2 = \frac{6\mu_{ao}}{\mu_{lo}}$ (μ_{ao}, μ_{lo} (— are the low field mobility of acoustic phonons and impurity scattering);

$$M = \frac{KT}{v_s^2}; \text{ (v_s — sound velocity).}$$

l_a — mean free path of the electron which does not depend on **p**.

Considering the above conditions and averaging (5), (6) over the angle between **p** and **E**, the symmetric part of the distribution function of the carriers will take the form

$$f_s^0(x) = A \exp\left[-x + \frac{\xi}{2} \ln (x^2 + \xi X + q^2) - \frac{\xi^2}{\sqrt{4q^2 - \xi}} \cdot \tan^{-1}\left(\frac{2x + \xi}{\sqrt{4q^2 - \xi}} \right) \right], \quad \ldots(9)$$

where

$$\xi = \frac{KT}{6m}\left(\frac{eE_0\, l_a}{KT} \right)^2, \quad \ldots(10)$$

A is a constant which is determined from the normalization condition

$$n = 4\pi \int_0^\infty dpp^2\, f_s^0\, (\xi\, \mathbf{p}) \quad \ldots(11)$$

(§III) Absorption Coefficient

In the case of $e^2E_0^2\ l_a/wkT\ \sqrt{2mkT} \ll 1$, then

$$e\mathbf{E}_0 \cdot \vec{\nabla}_p f_s^1\ (\epsilon_\mathbf{p}) \ll e\mathbf{E}_l \cdot \vec{\nabla}_p f_s^0\ (\epsilon_\mathbf{p}), \text{ and from (8)}$$

we have

$$\text{Re } f_a^1\ (\mathbf{p}) = -\frac{e\tau(\epsilon_\mathbf{p})}{(1 + w^2\tau(\epsilon_\mathbf{p}))} \cdot \mathbf{E}_l \cdot \mathbf{p} \frac{\partial}{\partial x}\ f_s^0\ (x) \quad \ldots(12)$$

The current density of the electron is given by the relation

$$\mathbf{j} = \frac{e}{m} \int d\mathbf{p} \cdot \mathbf{p}\, f_a^1\ (\mathbf{p})\ e^{-iwt} = \sigma_d \mathbf{E}_l\ e^{-iwt} \quad \ldots(13)$$

The absorption coefficient of light is expressed in terms of the real part of the differential conductivity σ_d as

$$\alpha = \frac{4\pi}{C_0\sqrt{\epsilon_0}}\ \text{Re}\ \sigma_d \quad \ldots(14)$$

where C_0 — velocity of light in vaccum, ϵ_0 — dielectric constant.

For high frequency of light $w\tau \gg 1$ and large concentration of impurities $q \gg \xi$, then from (9-14), the absorption coefficient will take the form

$$\alpha = \frac{128}{3C_0} \frac{e^2 n\sqrt{\pi}}{mw^2\tau_0\sqrt{\epsilon_0}} \left[\frac{q^2(q^2+2)+3\xi}{8q^2+15\xi} \right] \qquad ...(15)$$

which shows that the effect of impurity scattering increases the absorption of light in the warm electron region.

For low frequency of light, we have

$$\alpha = \frac{128\, e^2 n\tau_0}{C_0\, mq^2} \left(\frac{\pi}{\epsilon_0} \right)^{\frac{1}{2}} \qquad ...(16)$$

which show that for low frequencies of the incident light the absorption of light decreases in the case of impurity scattering.

(§IV) Absorption Coefficient in Case of Optical Phonons Scattering

In the case of $T_{\varphi\varphi} \ll w_0$, the scattering of electrons by acoustic phonons is predominant and the symmetric part of the distribution function has the form (Conwell, 1967)

$$f_0(\epsilon) = A \exp\left[-\epsilon^2 \Big/ \frac{KT}{6mv_s^2} (eE_0' l_a)^2 \right] \qquad ...(17)$$

This formula is true till $T_{\varphi\varphi} = eE_0 l_a \left(\frac{KT}{mv_s^2} \right)^{1/2} \ll w_0$.

For $\epsilon_p > w_0$, the emission (not absorption since $T \ll w_0$) of optical phonons is effective and we can neglect the relaxation time of energy according to acoustic phonons i.e.,

$$I[f_s(\epsilon_p)] = \frac{-f_s(\epsilon_p)}{\tau_{op}(\epsilon_p)};$$

$$\frac{1}{\tau(\epsilon_p)} = \frac{1}{\tau_{ak}(\epsilon_p)} + \frac{1}{\tau_{op}(\epsilon_p)}, \text{ where}$$

$\tau_{op}(\epsilon_p) = \frac{\pi\sqrt{2}\,\hbar^2 \rho\sqrt{w_o}}{D^2 m^{3/2}} \left(\frac{\epsilon_p - w_o}{w_0} \right)^{-1/2}$ is the relaxation time for emission the optical phonons (Conwell, 1967), ρ being the density of the crystal and D the deformation potential.

From the above conditions, (5), (6) can be solved approximately for the region of energy

$\left(\frac{mv_s^2}{KT}\right)^2 w_0 \ll \epsilon_p - w_0 \ll w_0$, and the symmetric part of the distribution function has the form (Bichkov & Dichne, 1965).

$$f_s(\vec{\epsilon_p}) - A_1 \exp\left[- a \frac{w_0}{(eE_0 l)} \left(\frac{\vec{\epsilon_p} - w_0}{w_0} \right)^{5/4} \right].$$

where a is constant of order 1, depends on the structure of the energy band.

For $\vec{\epsilon_p} = w_0$, we have $A_1 = A_0 \exp - w_0^2/b$, where

$b = \frac{KT}{6mv_s^2} (cE_0 l_a)^2$: A_0 — is determined from (11)

$$A_0 = \frac{n/4\pi\sqrt{2}\ m^{3/2}}{\left[\frac{2}{3} w_0^{3/2} + \Gamma(\frac{9}{5})\ w_0^{3/2} (\exp - w_0^2/b) \Big/ \alpha_0^{4/5} \right]}$$

where $\alpha_0 = \frac{aw_0}{eE_0 l} \lesssim a \sqrt{\frac{KT}{mv_s^2}} \gg 1.$

Using (12), (13), (14), the absorption coefficient of light by hot electrons for high frequency A.C. field will take the form

$$\alpha = \frac{(4\pi)^2 e^2 b A_0}{3 w^2 l C_0 \sqrt{\epsilon_0}} \left[1 - \exp(-w_0^2/b) \left(1 - 2 \frac{\Gamma(9/5)\ w_0^2}{b \alpha_0^{4/5}} \right] \right) \qquad \ldots(18)$$

(§V) Numerical Results and Discussion

In order to illustrate the results obtained, we have performed some numerical evaluations taking the following values of characteristic paramers for n-Ge at 77 °K: $m = 0.3m_0$ (m_0 free electron mass), $\epsilon_0 = 16$, $v_s = 5.10^5$ cm/sec, $w_0 = 6.10^{13}\hbar$ erg, $l = 10^{-5}$ cm, $\mu_{a_0} = (3.8)10^3 \left(\frac{300}{T}\right)^{3/2}$ cm^{2}sec^{-1} V^{-1}, and $\mu_{I_0} = 10^6 \left(\frac{T}{300}\right)^{3/2} \frac{10^{15}}{n_I}$ cm^2 sec^{-1} V^{-1}. At $E_0 = 22$V/cm ($\xi \sim 1$), the absorption coefficient in case of acoustic phonons scattering α_a for low and high frequency of A.C. electric field has the form (Sharnouby, 1971)

$$\alpha_a = \frac{16\pi n e^2 \tau_0}{15 C_0 \sqrt{\epsilon_0} m} \frac{\Gamma_{(3)}}{\Gamma^{(3/2)}}; \quad \alpha_a = \frac{16\pi n e^2}{15\sqrt{\epsilon_0} w^2 \tau_0} \cdot \frac{\Gamma_{(4)}}{\Gamma^{(3/2)}} \qquad \ldots(19)$$

where Γ is the gamma function. At large impurity concentrations ($n_I \simeq 4.10^{15}$/cm^3 or 3.10^{14}/cm^3 at 30 °K), the ionized impurity scattering is predominant, then from (15), (16) and (19) we have

$\frac{\alpha_a}{\alpha} = 37$ per cent (at low frequency); $\frac{\alpha_a}{\alpha} = 3.3$ per cent (at high frequency).

At large E_0, the optical phonons scattering is predominant, in this case the absorption coefficient (18) is always positive and does not change its sign as it does according to Bonch-Bruivich and Sharnouby (1972). In addition, for values of E_0 at 380V/cm: 600 V/cm and 1350V/cm, the corresponding values of absorption coefficient α are (4.15) 10^{-2}cm^{-1}, (4.22) 10^{-2}cm^{-1} and (4.4) 10^{-2}cm^{-1} respectively (taking $W = 10^{13}\ \text{sec}^{-1}$). From these evaluations, one obtains that α increases with increasing E_0.

References

Bichkov, U. A., and Dichne, A. M. (1965) *Zh. eksper. teor. Fiz.* **48**, 4.
Bonch-Bruivich, V. L., and El Sharnouby, M. A. (1972) *Vest. Mosk. Univ., Fit. Astron.*
Conwell, E.M. (1967) *High Field Transport in Semiconductors.* Academic Press, New York.
El Sharnouby, M. A. (1971) *Vest. Mosk, Univ, Fit. astron.*, 3, p. 262.