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FROM HADRON SPECTRA TO QUARK-LOOP DYNAMICS: AN INTEGRATED VIEW

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An overview is presented of the investigations carried out in the field of Quark Physics during the period 1989-94 under the INSA-Research Professorship. For a proper perspective, the account is integrated with a short background on a comprehensive programme of quark dynamics initiated by the author in an earlier phase (1981), characterized by a two-tier philosophy linking the (low energy) hadron spectroscopy to the high energy manifestations of QCD. In the present phase the basic thrust has been one of giving a concrete shape to this (two-tier) philosophy by (*i*) strengthening the foundations of this dynamical framework and (*ii*) (expending its scope to span a wider range of phenomena from mass spectra and transition amplitudes to more specialized topics like the evaluation of vacuum condensates via quark loops. The framework which is based on a QCD-motivated 4-fermion effective Lagrangian (with massless quarks connected by a non-perturbative gluon propagator) with built-in properties of Lorentz, gauge and chiral invariance, now provides an exact interconnection between the 3D and 4D forms of the Bethe-Salpeter wave function and thence a non-perturbative formula for the quark mass function. Simultaneously, a complex formulation of the relativistic 3-quark problem within a common dynamical framework for both qqbar and *qqq* systems has led to a sharper S_3-symmetric classification of baryonic states and the identification of several distinct SO(2,1) algebras of two-step ladder operators.

The resulting framework suffices to account for most hadron spectra as well as transition amplitudes (including vacuum condensates) with only two inputs (a reduced spring/string constant and a zero-point energy) without further assumptions.

Key Words: Hadron Spectra; Quark-Loop Dynamics Vacuum Condensates; Bethe-Salpeter Wave Function; Quantum Chromo Dynamics (QCD)

Introduction and Background

It was a great privilege for me to have been offered the Einstein Research Professorship by the Academy for 1989-94 to carry out a research programme of

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my choice during the said period, and I express my deepest gratitude to the Academy for this honour. At the same time there had been an uneasy consciousness of the responsibilities of such '*carta blanca*'. The biggest problem was the choice of a substantial theme with wide enough ramifications which should on the one hand keep one busy for the entire 5-year period, without resorting to a sequence of 'trivialities' adding up to a random collection of papers, and on the other hand without risking to tread on a totally unfamiliar ground "in the pursuit of excellence" for fear of drawing a total blank. In the end a middle course was adopted to ensure that the "end" would not be entirely out of sight. The chosen theme was in keeping with a long-term ambition of the author to generate in a practicable enough fashion a "doable" programme for simulating the non-perturbative features of Quantum-Chromo-Dynamics (QCD for short) that is now generally regarded as the leading candidate for the Theory of Strong Interactions, so that its applicational base may be significantly extended beyond routine processes. In such a quest he has admittedly been in the company of many other authors each vying with one another for the construction of a reasonable approximate framework for the description of diverse strong interaction phenomena. On the other hand, the generally 'piecemeal' nature of most of these efforts may be traceable to the fact that an exact solution of QCD in a viable enough form ready for diverse applications (somewhat on QED lines) had not been forthcoming for more than two decades since the original QCD proposal[1], despite valiant efforts (including 'lattice QCD') over a corresponding period. The only consensus on the "successes" of QCD has been on its perturbative[2] features which can be addressed only through its high energy ramifications but which, though impressively successful, are nevertheless not a proper testing ground for its genuinely non-perturbative aspects which are hidden in the low-energy regime of confinement. It is the latter region that has been of primary concern to the author's activities in the field which has remained largely open because of a formal lack of consensus on what aspects of QCD should constitute a minimal set of ingredients as a basis for an approximate implementation of its non-perturbative manifestations in hadronic phenomena. A fairly complete review of the early activities of the Delhi Group until the mid-eighties vis-a-vis the state of the art in the field may be found[3].

A Two-Tier (Spectroscopy-Linked) BSE Program

The basic strategy in the program (summarised below) has been to recognize three distinct features already present in QCD, all common to any "good" field theory, viz., Lorentz-invariance, Gauge-invariance and Chiral-invariance, and to formulate a viable theoretical framework in which all these three ingredients find their place within the limits of mutual compatibility. Since the main emphasis in the formulation is on a good handle on the non-perturbative domain characterized by (low-energy) spectroscopy and allied hadronic amplitudes, the starting point has had to be a corresponding dynamical framework which addresses this concern adequately. To that end the central ingredient was sought to be the 'wave function' or equivalently the (hydron-quark) vertex function which obeys an appropriate dynamical equation derivable from an input Lagrangian in principle (but can otherwise be motivated directly as the starting point as well). An obvious advantage of a dynamical-equation-based approach whose prototypes are the Bethe-Salpeter Equation (BSE) for the hadron-quark vertex function or the Schwinger-Dyson Equation (SDE) for the quark mass function, lies in its "micro-causality"[4] which in turn governs the entire evolution of the system under study, according to its own logic, once the three basic principles of Lorentz-Gauge-, and Chiral-invariance have been incorporated within its mathematical structure. Such a framework allows a fair degree of parametrization, e.g., to accommodate the phenomenology of low-energy spectroscopy as a starting point, and yet facilitates an automatic extension to, say, the high energy regime[3] (by virtue of its Lorentz-invariance property), without any fear of distortion of microcausality. In this respect its premises may be contrasted to other alternative approaches, such as QCD-sum rules[5], in which the transition on the energy scale is sought to be effected in the opposite direction (from the high energy to the low-energy end) but with the element of microcausality probably sacrificed in the process of "matching" the two ends. The other properties of chiral and gauge invariance can also be suitably incorporated in this spectroscopy-oriented description with corresponding predictive powers[3].

In the early eighties, a formulation on these lines was proposed in terms of a BSE for a *qq*-bar system and a corresponding BSE for a *qqq* system[6] for a unified understanding of meson and baryon spectroscopy within a common dynamical framework as it is generally believed that the confinement of both systems has a common underlying origin. To that end the kernel of the BSE was taken to be of the vector-type (one-gluon-exchange-like) which would simultaneously ensure confinement in both systems through a common sign for the respective, kernels in the colour-singlet (qqbar) and colour-antitriplet (*qq*) states. [This contrasts sharply to the input ansatz of a colour-dependent "scalar" kernel which gives opposite signs for the corresponding states]. The other benefits of a vector-type kernel, viz., gauge and chiral invariance (which were already present in the same structure) were to be exploited later (see below).

The main problem in the task of confronting this formalism with hadron spectroscopy for its initial calibration was on the issue of reconciling the observed O(3)-like spectra with the 'normal' 4D-structure of the BSE which would predict O(4)-like spectra without further precautions. On the other hand the 4D BSE structure is essential for giving a formal Lorentz-invariant meaning to the hadron-quark vertex function as a prerequisite for its appearance as an element in a typical Feynman diagram involving quark loops, so as to facilitate the evaluation of different types of transition amplitudes. Indeed without this facility of a twin predictability the BSE formalism itself would hardly qualify as a serious tool for any worthwhile hadronic investigation. On the other hand the problem of reconciliation of the conflicting demands of 3D-spectroscopy with the need for 4D-treatment for the evaluation of transition amplitudes via quark loops had compelled the abandonment of the program in the seventies[7].

The revival of the program by the Delhi-group in the eighties had to contend with this problem in the following manner: the 3D-form of the BSE in the so-called 'instantaneous approximation' was first employed to make contact with hadron spectra[6]. Then a reconstruction of the 4D BSE in terms of these

3D ingredients paved the way for the identification of the hadron-quark vertex function as the basic element for the calculation of transition amplitudes via quark loops[8]. In this way a two-tier approach was developed and found to be fairly successful in addressing the twin problems of spectroscopy of both *qq*-bar and *qqq* types, as well as transition amplitudes of several kinds[8], in a unified fashion. The principal defect of the approach on the other hand was a lack of formal covariance which dogged the program for several years without any immediate solution in sight.

The first step towards a covariant treatment consisted in the refinement to the so-called "null-plane" formulation wherein the above 3D reduction and subsequent reconstruction were effected in these standard linear combinations of the time-like and the longitudinal space-like momenta, so as to extend the energy range of validity of the predictions of this "two-tier" formalism[9]. It certainly improved several predictions of the theory, especially those on amplitudes corresponding to truly high energy transitions (such as structure functions[3] and fragmentation functions[3] which were beyond the reach of the instantaneous approximation, while the predictions on the spectral front were the result of several "phenomenological" improvements in the input structure of the BS-kernel, so as to bring it more in line with the standard QCD language. One important item in this regard was an ansatz of proportionality of the confining part of the (vector-like) kernel to the strong coupling constant of QCD, viz., $\alpha_s(Q^2)$[10], thus making it resemble more closely the infrared part of the gluon propagator, in company with the usual one-gluon-exchange (o.g.e.) term (which is proportional to this factor). Among other things this last ansatz resulted[10] in a marked improvement in the spectral predictions of both heavy and light flavoured mesons in a very natural manner without other adhoc assumptions on the way. On the other hand a fully Lorentz-covariant description of this "two-tier" formalism was still a long way off. This is where matters stood at the end of 1988, and were the subject of *S N Bose* (1986) *Lecture*[6] on the one hand, as well as the theme of a review article on the other[3].

Summary of Work During 1989-94

The subsequent work on this formalism was carried out during the tenure of the INSA Professorship, and may be summarised under the following broad categories:

(*i*) A manifestly Lorentz-covariant formulation, termed "Covariant Instaneity Ansatz" (CIA for short) which achieves an exact interconnection between the 3D and 4D forms of the BSE, and thus gives a precise meaning[11,12] to the "two-tier" philosophy of the BSE program[6,9].

(*ii*) Reinterpretation of the vector-form of the BS kernel as a chiral-invariant input 4-fermion effective QCD Lagrangian with current *ud*-masses, so as to facilitate the determination of their "constituent" values, *via* **dynamical breaking of chiral symmetry** (D_x SB for short)[3].

(*iii*) Derivation of an explicit non-perturbative formula for the momentum dependence of the quark mass function $m(p)$ to serve as a focal point for several, hitherto inaccessible, applications[13].

Applications of the mass function $m(p)$ include:

(*iv*) A new way to evaluate the total widths of light hadrons in an inclusive manner[14].

(*v*) A very general method to determine quark condensates of all types (both direct and induced), directly from their formal definitions as 4D quark loop integrals in a gauge-invariant way[15].

(*iv*) A general BSE formulation of a qqq system with unequal mass kinematics under Lorentz-covariant CIA, and the same input dynamics as for qq-bar mesons, with extensive checks from data and prediction of several new [56, odd] states[16].

(*vii*) Formulation of the qqq problem in a complex h.o. basis for better S_3 symmetry, leading to several distinct SO(2,1) algebras of two-step ladder operators, and a sharper classification of the qqq baryon states, in terms of a new quantum (N_a)[7].

(*viii*) A general formulation for the "Three-Hadron Form Factor" via quark triangle loops under the most general conditions of unequal mass kinematics, with wide applicational potential[18].

(*ix*) Effect of off-shell rho-omega mixing on "Charge Symmetry Breaking" in the N—N interaction[19].

The last two items concern the interface between particle (quark) and nuclear (hydronic) physics, and despite their considerable topical interest, will not be discussed further below. Also to be omitted from discussion are some miscellaneous applications of the same BSE formalism to other problems of topical interest: (1) Leptonic Decay Constants of Pseudoscalar Mesons[20]; (2) Amplitude Anomalies in Semi-leptonic $P \rightarrow V$ Decays[2].

In the following a brief is given in the "core" items, (*i*)-(*v*) which have resulted in a strengthening of the conceptual foundations of the original programme, as well as its extension to newer areas of applications. As part of item (*i*), are given some aspects of items (*vi*)-(*vii*) which (the author believes) represent a genuine progress in the understanding of the relativistic qqq problem (largely neglected in the literature) through the emphasis on the common framework that binds it to its (dual) qq-bar partner. Most of these results have also been reported in several international symposia during the said period.

A Lorentz Covariant Formulation of BSE

Item (*i*) closed an important theoretical gap in the two-tier philosophy underlying the entire BSE approach of the Delhi group and paved the way to the evaluation of several types of quark loop integrals in a manifestly Lorentz-invariant manner without losing the convergence property of the gaussian form factors associated with the vertex functions (since the time-like regions of the virtual 4-momenta were no longer an issue). On the other hand the CIA did not predict any differences from the earlier (already good) results on qq-bar spectra[22], since the structure of the 3D BSE[11] in the c.m. of the qq bar hadron is formally identical to the earlier (NPA) form[10] in which those results had been derived. However, the qqq system—see item (*vi*)-did need a considerable sharpening of the earlier (NPA) treatment (which had lacked formal Lorentz-covariance) since in the c.m. of a three-body system[9] the presence of the third quark induces off-shell effects on the c.m. momentum of the qq-subsystem. The Lorentz-covariance of CIA[12] had indeed produced results[16] which if anything are in somewhat better agreement with data than are the earlier NPA results[23]. While for the details of the methodology (which are on closely parallel lines to the earlier NPA treatment)[23] the interested reader is best referred to a recent publication[16], there is nevertheless an important aspect to the new formalism that probably merits some attention in view of its wider ramifications on the permutation symmetry S_3 of a three body (qqq) system, and this concerns the role of a complex formulation of the same problem.

A Complex H.O. Basis: A "New" Quantum (N_a)

It is well known that the quantum state of a three-body system in the real basis is specified by $|NJLS: \lambda>$ where λ characterizes the S_3-symmetry status of the system in terms of 'symmetric' (s), 'antisymmetric' (a) and 'mixed-symmetric' (m) all of which are expressible in terms of two independent (Jacobi) variables (ξ_i, η_i) forming a "mixed" representation of S_3-symmetry. However, a disadvantage of this "real" representation is that the above S_3-symmetry characterization (λ) is not quantitatively expressed in terms of some additional quantum numbers. This shortcoming is more acute in the case of a relativistic 3-body problem wherein a larger number of S_3-symmetry breaking terms are present in the interaction hamiltonian than for the corresponding non-relativistic system. This can be taken care of by the use of a more promising approach, viz., the complex basis defined in terms of the normalized coordinates ξ_i, η_i[24,25,26]:

$$\sqrt{2}\mathcal{Z}_i = \xi_i + i\eta_i; \qquad \sqrt{2}\mathcal{Z}_i^* = \xi_i - i\eta_i, \qquad \ldots (1)$$

A new quantum number[24,26] can be generated in a general way in terms of the quantity $\tan\gamma = 2\xi\cdot\eta/(\xi^2-\eta^2)$ and the related operator $-i\partial_\gamma$ whose eigenfunctions are $\exp(i\gamma\lambda/2)$ with eigenvalues $\lambda/2$ where $\lambda = 3n-1$, $3n$, $3n+1$ ($n=0$, 1, 2)[26]. The Harmonic representation, in addition[25,17], offers more specific information since the dynamics is now controlled by the two number operators

$$N_\xi = a^\dagger_{\xi i}\, a_{\xi i}; \qquad N_\eta = a^\dagger_{\eta i}\, a_{\eta i} \qquad \ldots (2)$$

where a, $a^\dagger$ are normalized ladder operators in ξ, η indices defined in the standard manner. In this ($\boldsymbol{\xi},\boldsymbol{\eta}$) basis, however, only the sum $N = N_\xi + N_\eta$ is diagonal but the difference $N_\xi - N_\eta$ is not. This shows that the real ($\boldsymbol{\xi},\boldsymbol{\eta}$) basis does not provide an h.o. representation with good S_3-symmetry properties. A more promising structure, i.e. one with better S_3-symmetry properties, would be obtained in terms of the following complex combinations

$$\sqrt{2}(a_1, a_i^*) = a_{\xi i} \pm i a_{\eta i}; \quad \sqrt{2}(a_i^\dagger, a_i^{*\dagger}) = a^\dagger_{\xi i} \pm i a^\dagger_{\eta i} \qquad \ldots (3)$$

satisfying the commutation relations

$$[a_i, a_j^\dagger] = [a_i^*, a_j^{*\dagger}] = \delta_{ij}; \; [a_i, a_j^*] = 0 \qquad \ldots (4)$$

In the new complex basis, the number operators

$$N_c = a_i^\dagger a_i;\ N_c^* = a_i^{*\dagger} a_i^*;\ N_m = a_i^\dagger a_i^*;\quad N_m^* = N_m^\dagger = a_i^* a_i^\dagger \qquad \ldots(5)$$

exhibit better symmetry properties *via* the separate S_3-singlets

$$N = N_c + N_c^* = N_\xi + N_\eta;\ N_a = N_c - N_c^* \neq N_\eta - N_\eta \qquad \ldots(6)$$

N_a is thus a new quantum number having no counterpart in the (ξ, η) basis. Its eigenvalues, modulo 3, are a measure of departure, of a given state with total quantum number N from a fully symmetric (antisymmetric) state. The situation is analogus to the diagonality of the charge operator for a scalar field when expressed in a complex basis (ϕ, ϕ^*), *versus* a real basis[17], while the energy remains diagonal in both. Thus the "charge" for the complex scalar field seems to play a role analogus to N_a in the three-body problem when viewed in the complex h.o. basis, a feature which clearly brings out the superiority of the complex basis over the real one[17].

SO(2.1) Algebras of Two-Step Ladders

To consider the SO(2,1) algebras of two-step bilinear operators made out of the set (3), it is convenient to distinguish them in accordence with their S_3-symmetry properties as follows[17]:

Symmetric:

$$A = 2a_1 a_i^*;\quad A^\dagger = 3a_i^\dagger a_i^{*\dagger} \qquad \ldots(7)$$

Mixed-Symmetric:

$$C = a_i a_i;\quad C^* = a_i^* a_i^*;\ C^\dagger = a_i^\dagger a_i^\dagger;\ C^{*\dagger} = a_i^{*\dagger} a_i^{*\dagger} \qquad \ldots(8)$$

We can identify three distinct sets of commuting SO(2,1) algebras whose spectra are bounded from below. Of these, the S_3-symmetric set (A, $A^\dagger$, $N+3$) satisfy the commutation relations[23]

$$[A,N] = 2A,\ [A^\dagger, N] = -2A^\dagger,\ [A, A^\dagger] = 4(N+3) \qquad \ldots(9)$$

so that the three normalized components are

$$Q_+ = A^\dagger/2;\ Q_- = -A/2;\ Q_3 = (N+3)/2$$

and the corresponding Casimir is[23]

$$u(u+1) = -(AA^\dagger + A^\dagger A)/8 + (N+3)^2/4\,. \qquad \ldots(10)$$

Since this spectrum is bounded from below[23], the eigenvalues of Q_3 are $-u+k$, $k=0, 1, 2..$ on the one hand, and $(N+3)/2$ on the other. Thus

$$u(u+1) = 3/4\ (\text{even } N);\ +2\ (\text{odd } N). \qquad \ldots(11)$$

Similarly, for the mixed symmetric set C, $C^\dagger$, N_c satisfying[17]

$$[C, N_c] = 2C;\ [C^\dagger, N_c] = -2C^\dagger,\ [C, C^\dagger] = 4(N_c + 3/2), \qquad \ldots(12)$$

the corresponding Casimir is[17]

$$u_c(u_c+1) \equiv -(CC^\dagger + C^\dagger C)/8 + (N_c + 3/2)^2/4; \qquad \ldots (13)$$

$$u_c(u_c+1) = -3/16\,(\text{even } N_c);\ 5/16\,(\text{odd } N_c). \qquad \ldots (14)$$

Identical results hold for starred counterpart (C^*, $C^{*\dagger}$, N_c^*).

Finally, the antisymmetric set ($N_m, N_m^\dagger, N_a$) satisfies a normal SO(3) algebra[17];

$$[N_m, N_a] = 2N_m, [N_m^\dagger, N_a] = -2N_m^\dagger; [N_m, N_m^\dagger] = -N_a \qquad \ldots (15)$$

with the Casimir

$$s(s+1) = (N_m N_m^\dagger + N_m^\dagger N_m)/2 + N_a^2/4 \qquad \ldots (16)$$

so that spectra is bounded from both below and above, just like an ordinary (spin) angular momentum[17]:

$$-N \leq N_a \leq N;\ s = N/2 \qquad \ldots (17)$$

It is of interest to see how compactly a qqq wave function can be expressed in the complex basis. Thus, after taking out the antisymmetric colour part ($\varepsilon_{\alpha\beta\gamma}/\sqrt{6}$), the reduced wave function which is symmetric in the orbital (ψ), spin (χ) and isospin (ϕ) degrees of freedom, may be subscripted by indices like (ψ_c; ψ_c^*) and (ψ_s; ψ_a) in a obvious S_3-notation (and similarly for χ, ϕ), may be expressed as

$$|56>^q = \psi^s\chi^s\phi^s; \qquad |56>^d = \psi^s(\chi_c\phi_c^* + \chi_c^*\phi_c)/\sqrt{2} \qquad \ldots (18)$$

$$|70>^q = \chi^s(\psi_c\phi_c^* + \psi_c^*\phi_c)/\sqrt{2}; \qquad |70>^d = (\psi_c\chi_c\phi_c + \psi_c^*\chi_c^*\phi_c^*)/\sqrt{2} \qquad \ldots (19)$$

$$|20>^q = \psi^a\chi^s\phi^a; \qquad |20>^d = \psi^a(\chi_c\phi_c^* - \chi_c^*\phi_c)/\sqrt{2} \qquad \ldots (20)$$

Applications to baryonic spectra are given elsewhere[16]. In this regard, the new quantum number N_a[17] has an important role as it affords a more complete classification of the baryonic states than hitherto available in terms of the standard "real" h.o. basis by allowing in a more transparent way the identification of the possible candidates for "mixing" under the different perturbative o.g.e. terms for calculating the energy shifts of the affected states. The complex basis also facilitates the identification and solution of several distinct SO(2,1) algebras[17] which in turn helped diagonalize the (highly non-linear) energy operator of the qqq-baryon directly in terms of the Casimirs[11,12] of these SO(2,1) operators. The evaluation of the various matrix elements is also found to be greatly compactified through the use of the complex representation[16].

Dynamical Breaking of Chiral Symmetry: Mass Function m(p)

Items (*ii*) and (*iii*) which may be taken together, represent an important dynamical consequence of chiral symmetry breaking through the solution of the SDE, using as input as spatially extended form of the effective 4-fermion interaction.

Though known in the literature[27] to provide a natural cut-off for the NJL mechanism[28] without needing a separate cut-off parameter, this feature of the extended 4-fermion interaction which also provides an effective confinement scale to serve ideally for quantitative hadron spectroscopy[13] has not been adequately appreciated in the literature[13]. More importantly, the same feature now provides, through the solution of the SDE, a concrete non-perturbative form for the momentum dependence of the quark mass function. In the present approach the mass function has been obtained[13] using another simple principle, viz., in the (chiral) limit of vanishing M_π, the mass function also satisfies the BSE for the pion[27,28]. While referring the interested reader to[13] for a fuller derivation of the mass function, the basic logic is as follows. One recalls that any hadron-qq-bar vertex function as a solution of the BSE under Cov. Inst. Ansatz (CIA)[11,13] is proportional to the product $D(\hat{q})\star\phi(\hat{q})$ where "D" is a 3D denominator function characteristic of CIA, "ϕ" the associated 3D wave function, and $\hat{q}_\mu = q_\mu - q{\cdot}PP_\mu/P^2$ is the covariant (3D) 4-momentum transverse to the hadron 4-momentum P_μ, viz., $\hat{q}{\cdot}P \equiv 0$. In the limit of a massless pion ($P_\mu = 0$), this quantity may be identified with the mass function $m(\tilde{p})$ which, under CIA, should read as $m(\hat{p})$ where $\hat{p}_\mu$ is the 4-momentum of either of the constituent quarks transverse to P_μ {since in the limit $P_\mu = 0$, $p_{1\mu} = -p_{2\mu}$}. Using a normalization which makes $m(0) = m_q$, the constituent mass ($= 265$ MeV)[22], the final form of the mass function becomes

$$m(\hat{p}) = \omega^3(p)m_q^{-2}\exp[-\hat{p}^2/2\beta_\pi^2]; \quad \omega^2(\hat{p}) = m_q^2 + \hat{p}^2, \qquad \ldots (21)$$

where β_π^2 ($=0.060$ GeV^2) is the "size parameter" of the pion obtained from the BSE-cum-SDE dynamics[11-14] linked to hadron spectroscopy. This is a non-perturbative form of the mass function which may be contrasted with the "perturbative" form[29]

$$m_{QCD}(-p^2) = (4\pi/3)\alpha_s(-p^2)\langle q\hat{q}\rangle_0(-p^2)^{-1} \qquad \ldots (22)$$

The mass function $m(\hat{p})$ represents a central ingredient of our formalism in as much as it has opened the way to several novel applications some of which have already been carried out[13-15], see under items (iv) & (v). In this respect, the CIA characteristic of the formalism shows up through the effectively 3D momentum dependence of the function $m(\hat{p})$ where $\hat{p}_\mu$ is the quark 4-momentum transverse to the (vanishing) 4-momentum P_μ of the "vacuum pion". This 3D dependence is in turn crucial for the convergence of the various loop-integrals involved in the evaluation of the various "vacuum condensates"[13,15], without having to introduce any ad hoc cut-off parameters. In what follows we indicate briefly first the logic of the 'condensate' derivation[13,15], followed by another novel application, viz., an inclusive derivation of hadronic total widths, item (v)[14].

Vacuum Condensates Directly From Definition

The vacuum condensates arise as coefficients of the 'higher twist' terms in the operator product expansion of Wilson[30], and they represent the long distance corrections to perturbative QCD. At the hands of the Russian School[5], this approach developed into a powerful technique termed QCD sum rules, in which these quantities played the role of key ingredients, albeit as free parameters. Although the method has proved very useful for many applications, it does not have the facility to generate new structures (e.g., induced condensates) without introducing fresh parameters. On the other hand an approach which proceeds to calculate vacuum condensates directly from their definitions does not require any parametric assumptions for the evaluation of any type of condensates (in particular, e.g., "induced condensates"), since the general definition takes care of them all! The formal definition for any condensate is[15]:

$$\langle \bar{q}\, O_i q\rangle \equiv -(2\pi)^{-4} \int d^4p\, Tr\{S_F^A(p) O_i\}, \qquad \ldots (23)$$

where O_i is the operator representing the nature of the condensate, the superscript A denotes the effect of a background field that may be present, and the quark propagator S_F has its perturbative part suitably subtracted beforehand. A simiiar definition holds for the gluon condensate[15]:

$$\langle g_s^2 GG\rangle = Tr\{\partial_\mu \partial_\nu - \delta_{\mu\nu}\partial'' \partial\} g_s^2 D_{\mu\nu}(0), \qquad \ldots (24)$$

where ∂_μ is a gauge covariant derivative and $D_{\mu\nu}(x)$ is the infrared part (nonperturbative) of the gluon propagator. In principle this quantity can be accessed from the qqbar potential, but some care is needed to ensure that the long range part that is extracted is flavour independent[15]. Examples of induced condensates are

$$O_i = \gamma_\mu \gamma_5; \quad \sigma_{\mu\nu}; \quad \lambda^a \sigma_{\mu\nu} G^a_{\mu\nu}/2; \quad \lambda^a G^a_{\mu\nu}/2, \qquad \ldots (25)$$

where the background fields are axial vector, electromagnetic, standard vacuum (gluonic, always present) and e.m. respectively. In the present BSE-cum-SDE model which is fully attuned to the spectroscopy of qqbar[22] and qqq[16] hadrons with only two free parameters ($C_0 = 0.27$ and $\omega_0 = 158$ MeV)[22,16], the structures of the respective propagators are:

$$-iS_F^{-1}(\Pi) = i\gamma \cdot \Pi + m(\hat{\Pi}^2); \quad \Pi = p - g_s \lambda^a G^a_\mu \qquad \ldots (26)$$

$$D^{ab}_{\mu\nu}(\hat{R}) = (3\omega_0^2/4) 2m_q \alpha_s(Q^2)[C_0/\omega_0^2 - \hat{R}^2/(1 + A_0 m_q^2 \hat{R}^2)^{1/2}] \qquad \ldots (27)$$

where R_μ is transverse to the "vacuum pion" direction, and[15]

$$2\pi/\alpha_s(Q^2) = [11 - 2n_f/3] \ln(Q/\Lambda_{QCD}); \quad \Lambda_{QCD} = 200 \text{ MeV}, \qquad \ldots (28)$$

The calculational details may be found in[14,15]; the results for the principal condensates *versus* the QCD-SR values[5,31] (in curly brackets) are[15]:

$$\langle q\bar{q}\rangle_0 = (266 \text{ MeV})^3 : 240\}; \quad \langle g_s^2 GG\rangle = 0.502 \text{GeV}^2 : \{0.47\} \qquad \ldots (29)$$

As to the induced condensates in the presence of an external, e.m. field, the results in the notation and normalization of ref.[31] for the quantities χ, x and ξ are as follows:

$$\chi = -3.52 \text{ GeV}^{-2} : \{-6 \pm 2\}; \; x = -0.11; \; \xi = +0.06 \text{ GeV}^{-2} \qquad \ldots (30)$$

Finally for the 'axial condensate' A_s defined by $\langle \bar{q} i\gamma_\mu \gamma_5 q\rangle_A \equiv A_s A_\mu$ the result which should be compared with f_π^2 is[32]

$$A_s = 0.021\ \text{GeV}^2 : vs\ f_\pi^2 = (0.133\ \text{GeV})^2 \qquad \ldots (31)$$

Hadronic Total Widths (Inclusive):

The use of the mass function has also proved crucial for the calculation of the total widths in an inclusive fashion[14], a "privilege" hitherto reserved for deep inelastic scattering (*via* e.m., kick to the hadron), or at most for the decays of heavy quarkonia (*via* gluon emission). The present venture, extending the idea to light hadrons, was considered too unorthodox and met severe "referee-resistance" through various types of objections for almost two years until all of them were finally resolved before the paper was accepted[14]. Only the main points will be outlined here, while the details be obtained from ref[14].

The essential idea is to recognize that, unlike in deep inelastic scattering where a hard 'e.m. kick' forces the hadron to dissociate into its 'partons' (current quarks), or the case of heavy quarkonia transitions to light species *via* the gluonic link, neither of these two mechanisms is available in the present situation of light quarkonia to light quarkonia transitions. An alternative mechanism to overcome the confinement barrier has been proposed[14], depending crucially on the existence of a short hiatus period—a sort of "breathing mode"-wherein the parent hadron (resonance) temporarily 'dissociates' into a pair of 'quasi-free' quarks before eventual hadronization. The latter event is of course envisaged with certainty, so that effectively the first stage dissociation itself, which is supposed to take place *via* the exchange of gluons (hard and soft), represents the entire process of hadronization. Further, the said mechanism is consistent with the general constraint of vanishing width (Γ) in the limit of the # of colours $N_c \Rightarrow \infty$[33]. In this mechanism, the main point of departure from that of the "DIS" process is that in the absence of the hard e.m. kick, the 'driving force' is the "urge" of the hadron (resonance) to dissociate, and the momentum dependence of the mass function $m(p)$ should ensure that the masses of the quasi-free quarks be somewhat intermediate between the "constituent" value $m(0)$ and the "current" value (≈ 0) for the non-strange (ud) quarks. The precise value of $m(p)$ is of course determined through the "on-shell" solution of eq. (21) and works out at $m(p) = 176$ MeV, c.f. $m(0) = 265$ MeV[22].

The mechanism of dissociation of a hadron into quasi-free quarks is shown schematically in Figs 1($a.b$), and the results are given in Tables I and II for non-strange, strange and hidden strange mesons. For the low excitations ($L \leq 1$), one should not expect good agreement because of "hadronic selection rules" inhibiting the available phase space (e.g., rho versus omega decays), but the agreement with data should progressively improve with higher excitations ($L > 1$). This is precisely what seems to be indicated from both tables. Despite the paucity of data for comparison for higher L-values, the general trend seems to support the suggested mechanism of (a temporary) quasi-free dissociation. Very recently, this 'quasi-free' mechanism was subjected to a more extensive test[34] in respect of both meson and baryon resonances within a general frame-

Table I

Total widths (in Mev) of non-strange hadrons with specified values of β^2 (in Gev^2)
(The widths Γ_{NP}, Γ_P, Γ_t are the contributions of Fig. (1a), (1b) and their resultant respectively)

Meson	$I^G J^{PC}$	β^2	N_H^{-2}	Γ_{NP}	Γ_P	Γ_t	Γ (expt.)
V_J mesons:							
ρ (775)	1^+1^{--}	0.0697	.0511	83	336	87	153 ± 3
f_2 (1270)	0^+2^{++}	0.0935	.1579	26	377	205	185 ± 20
a_2 (1320)	1^-2^{++}	0.0959	.1689	23	416	244	110 ± 5
ω_3 (1670)	0^-3^{--}	0.1130	.3642	12	336	221	166 ± 15
ρ_3 (1690)	1^+3^{--}	0.1140	.3727	12	344	230	215 ± 20
f_4 (2050)	0^+4^{++}	0.1313	.7270	6	315	232	204 ± 13
ρ_5 (2350)	1^+5^{--}	0.1456	1.2319	4	269	206	
f_6 (2510)	0^+6^{++}	0.1527	1.7479	4	213	158	
B^J mesons:							
h_1 (1170)	0^-1^{+-}	0.0886	.2001	36	346	159	335 ± 26
b_1 (1235)	1^+1^{+-}	0.0918	.2200	31	384	198	150 ± 10
π_2 (1670)	1^-2^{-+}	0.1130	.5780	13	357	235	250 ± 20
S_0 mesons:							
f_0 (975)	0^+0^{++}	0.0791	.1053	29	133	38	34
a_0 (980)	1^-0^{++}	0.0793	.1059	29	134	39	57 ± 11
A_1 mesons:							
a_1 (1260)	1^-1^{++}	0.0930	.1257	21	287	154	300-600
f_1 (1285)	1^+1^{++}	0.0942	.1306	19	295	164	24 ± 3

work independent of the above BSE-cum-SDE model, and found to work rather well, but this activity, though belonging to the very theme of the "Project", has nevertheless extended beyond its time limit (1989-94).

Summary and Conclusion: The Subjective Element

This report essentially marks the conclusion of a program started about 15 years ago for constructing a viable framework for the non-perturbative domain of QCD, incorporating its three pillars (Lorentz-, Gauge- and Chiral-invariance), one in which the main emphasis was to be on linking its low energy manifestations, i.e., spectroscopy, to numerous high energy aspects which reveal themselves in varying degrees through diverse types of "doable" transition amplitudes involving real and virtual processes. The natural language of these processes is quark loop dynamics whose crucial ingredient in turn is the "hadron-*qq*bar" vertex function for observable processes and the "vacuum-*qq*bar" vertex function for unobservable (virtual) ones. The latter shows up through the quark mass function $m(p)$ which is merely the pion-*qq*bar vertex function in the limit of vanishing pion mass, the so-called NJL[28] mechanism of DXSB[27]. An explicit construction of $m(p)$[13] within our non-perturbative BSE-cum-SDE formalism[11] has been a key ingredient of the "Project phase" of the program, leading to unorthodox applications like total widths and vacuum condensates. The entire structure rests of course on Item (*i*)—"Covariant Instantaneity Ansatz"[11] which

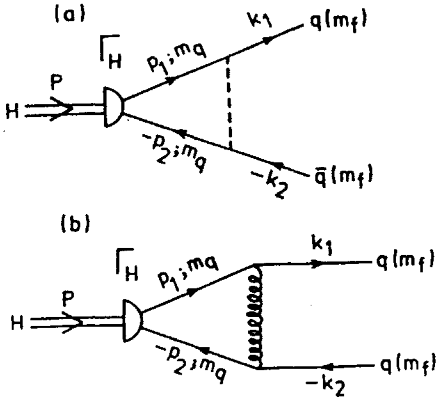

Fig. 1(a) & (b) Light flavour transitions via soft (dotted line) and hard (coiled line) gluon exchange mechanisms respectively

Table II

Total widths (in Mev) of strange and hidden-strange hadrons

(The entries Γ_{NP}, Γ_P, Γ_t are the contributions of Figs (1*a*), (1*b*) and their resultant respectively, as in Table I. The substractions m_c = 150-200 MeV from the kaon masses, and $2m_c$ = 300-400 MeV for the ϕ-like masses, have been made beforehand to bring their width calculations in line with those of non-strange mesons, Table I (see text for explanation). The indicated range of values for the different entries, corresponds to the range 150-200 MeV for m_c. The quantum numbers *G* and *C* are meant only for the ϕ-like hadrons)

Meson	I^GJ^{PC}	Γ_{NP}	Γ_P	Γ_t	Γ(expt.)
V_J mesons:					
K*(892)	$1/2\ 1^-$	90-105	267-236	49-30	50
ϕ (1020)	0^-1^{--}	95-114	252-207	40-16	4.43
K_2^*(1430)	$1/2\ 2^+$	25-29	383-338	211-169	109±5
f_2(1525)	0^+2^{++}	29-37	334-268	166-106	76
ϕ_3(1850)	0^-3^{--}	16-31	227-243	159-122	87±6
K_3^*(1780)	$1/2\ 3^-$	13.5-15	303-287	188-170	164±17
K_4^*(2045)	$1/2\ 4^+$	9-10.5	244-224	158-137	198±30
f_4(2300)	0^+4^{++}	7-9	284-246	201-160	150-200
B^J mesons:					
K_1(1270)	$1/2\ 1^+$	40-45	289-264	113-91	90±20
K_2(1270)	$1/2\ 2^-$	15-17	320-302	197-177	136±18
S_J mesons:					
f_0(1400)	0^+0^{++}	19-24	127-101	47-29	150-400
K_0^*(1430)	$1/2\ 0^+$	10-12	152-143	84-72	287±23
K*(1680)	$1/2\ 1^-$	67-73	105-85	59-41	323±110
A_1 (mesons):					
K_1(1400)	$1/2\ 1^+$	21-25	278-256	145-121	174±13
f_1(1510)	0^+1^{++}	24-33	260-228	125-87	35±15

gives a concrete shape to its two-tier philosophy[4] for a simultaneous handle on spectroscopy and quark-loop dynamics in a Lorentz-covariant fashion. And this in turn has facilitated a complex formulation of the relativistic *qqq* dynamics which has explicitly revealed[17] its (ortherwise hidden) S_3-symmetric structure.

Having thus "technically fulfilled" (?) my obligations to the "Project" to the best of my (limited) capacity, though I have no reason to expect an endorsement from you on my enthusiasm, I may be permitted at the end to share with you some "subjective" thoughts on this occasion, concerning my perceptions on the philosophy of science *vis-a-vis* these efforts. Admittedly, these thoughts took shape mainly during this 5-year period which is full of personal ups and downs of an author in his efforts to get his ideas across to the Community, in an area of physics where a lack of consensus on a common "paradigm" has been the order of the day, generating a strong conviction that Physics as a pursuit is, after all, "not all that objective", as it is dominated by a strong subjective element residing in the "powers that be". In so expressing myself I am no doubt vulnerable to your suspicions of a "sour grapes" (?) attitude,, yet a fond hope that the "thoughts" generated in the process may well be of interest to a wider community has led me to venture sharing some of them with you, before conclusion.

Along with these technical activities, I also got interested in some general aspects of the "Philosophy of Science", partly occasioned during this period by my involvement (as the Academy's nominee) with the Indian Council for Philosophical Research, but mainly for intrinsic reasons. In this respect I felt inspired enough by Thomas Kuhn's authoritative and intensely practical book[35] on the subject, and an equally thoughtful one by G Holton on the "Thematic component"[36] in scientific investigations, to express my own thoughts in the form of a short essay entitled[37]: "Profiles in Scientific Philosophy: Paradigms and Serendipity". Though by no means claimed as part of the "Project" work (!), the conclusions found therein[37], as well as in subsequent studies, may nevertheless serve for the conclusion of this report as well:

Despite all protestations to the contrary, it is perhaps fair to say that Science is not all as 'objective' as is often claimed on its behalf. Indeed its 'objective' aspect lies merely in the 'Contingency Plane'[36] of theory-experiment confrontation, but the driving force is still the 'Thematic'[36] component which originates in the human mind, and hence is 'subjective' in outlook. This subjectivity has two aspects: (*i*) a 'value judgement' on the 'themata' which operates in the abstract plane (and hence noble!); and (*ii*) a motive force which operates at the 'human' level (not always nable!) and is often involved in "paradigm conflicts", more reminiscent of socio-political conflicts, one in which the final arbiter is the "Assent of the Community"[35]. In this scenario, Science has a dual role in which the 'objective' part concerns only the impact of Nature and Logic, while the 'subjective' part has to deal with the "techniques" of persuasive argumentation, which is often dominant and gives a rather "chauvinistic" image to Science, upholding the "survival of the fittest" in a (Darwinian-type) "natural selection process"[35]. Thus Science seems to exhibit a dual personality[37]: (*i*) the objective (detached ?) aspect of laboratory investigations; and (*ii*) the subjective (ego ?) aspect of competing paradigms. In the words of Science (in the first person), this dual scenario may be captured in the following limerick[37]:

"I keep dancing in my ego phase; I am a particle, my ego a wave."

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