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AN APPROXIMATION METHOD FOR COMPUTATION OF HEAT TRANSFER RATE IN LIQUID METALS

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A mathematical model to predict heat transfer rate in liquid metals has been developed. The velocity distribution field is obtained by solving the momentum integral equation. The results are employed to solve the energy integral equation to get the temperature distribution and is coupled with the velocity gradients. An expression for Nusselt and Reynold and Prandtl numbers is derived and the results are compared with that of those investigations which attack the problem in a different way.

Key Words: Heat Transfer Rate; Liquid Metals; Velocity Distribution Field; Momentum Integral Equation

Introduction

Study of heat transfer in liquid metals is gaining importance because of their wide use in power generation system. As they possess high heat conductivity and realise low Prandtl numbers ranging 0.005 to .05, are used as a powerful means in ablation cooling. In order to gain insights into thermal problems associated with liquid metals, this paper makes an attempt to predict heat transfer rate through a mathematical model.

Sparrow and Gregg[1] obtained exact solution for the problem of forced-convection laminar boundary layer on flat plate with the help of computing machine for $0.006 \leqslant \Pr \leqslant 0.03$. Eckert[2] obtained an approximate solution of this problem on the assumption that the entire plate is heated. Grosh and Cess[3] tackled this problem and solved the same numerically. Evans[4] and Merk[5] suggested an approximate numerical method for its solution. The function representing a velocity profile is substituted in the momentum integral equation and thus got it in nonlinear form with one unknown $\delta(x)$. Using approximate method the value of $\delta(x)$ is obtained. This is used to solve the energy integral equation to obtain the temperature distribution. Using these results an expression relating Nu_x and Re_x and Pr is obtained to predict the heat transfer rate.

Problem Statement and Assumptions

It has been considered that liquid metal flows with constant velocity U_∞ and the temperature T_∞ along a thin semi infinite plate at constant temperature T_w. The origin of coordinates be at a leading edge of the plate, the x-axis along the plate and y-axis normal to it. The developed boundary layers flows are irrotational and two-dimensional nature and nonlinear in character and uncoupled.

Such flow when generated under thermal condition will further help in broadening our knowledge.

In deriving the governing equations the following assumptions are made:

1. There are no inner heat sources in the field.
2. The thermal and boundary layers develop along the surface of the body are not affected by the development of boundary layers on any adjacent surfaces.
3. In the absence of all body sources, the fluid is forced over the body.
4. The motion in this case is laminar steady and incompressible.

Governing Equations

In usual notations the equations governing the velocity and temperature distribution in a boundary layer flow past a flat plate in forced convection are, given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad \ldots (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad \ldots (2)$$

and

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \qquad \ldots (3)$$

where $a=\dfrac{K}{\rho c_p}$ (thermal diffusivity) with the boundary conditions

$$y=0: u=v=0;\ T=T_w \text{ or } \frac{\partial T}{\partial y}=0$$

$$y=\infty: u=U_\infty;\ T=T_\infty \qquad \ldots (4)$$

Eqs (1-2) after integration over $y=0$, $\delta(x)$, $\delta(x)$ being the boundary layer thickness, we get (2) in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^\delta\left(1-\frac{u}{U_\infty}\right)\frac{u}{U_\infty}\,\mathrm{d}y\right]=\frac{\nu}{U_\infty}\left[\frac{\partial}{\partial y}\left(\frac{u}{U_\infty}\right)\right]_{y=0}$$

Expressing this in term of η where $\eta=y/\delta$, we get

$$\delta\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^1 f(1-f)\,\mathrm{d}\eta\right]=\frac{\nu}{U_\infty}f'(0). \qquad \ldots (5)$$

Integrating eq. (3) with respect to y between the limits $y=0$ to $y=\delta_t(x)$, δ_t being the thermal boundary layer thickness and using assumption (4) in the result, we get

$$\frac{d}{dx}\int_0^{\delta_t}\frac{u}{U_\infty}(T-T_\infty)\,dy=-\frac{a}{U_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Introducing the dimensionless temperature θ_1 in it, we get

$$\frac{d}{dx}\left[\int_0^{\delta_t}\left(\theta_1\frac{u}{U_\infty}\right)dy\right]=-\frac{a}{U_\infty}\left(\frac{\partial\theta_1}{dy}\right)_{y=0} \qquad \ldots (6)$$

with the boundary conditions

$y=0$: $u=v=0$, $\theta_1=1$

$y=\delta$: $u=U_\infty$

$y=\delta_t$: $\theta_1=0$

Solution for Boundary Layer Thickness

To solve the momentum integral equation (5), the function in η satisfying all the required boundary and compatibility conditions is chosen in the form

$$\frac{u}{U_\infty}=f(\eta)=2.0667\eta-6.334\eta^4+9.0015\eta^5-4.4012\eta^6+0.667\eta^7 \quad 0\leqslant\eta\leqslant 1 \quad \ldots (7)$$

and $u=U_\infty$, $\eta>1$

For this profile δ_2, momentum thickness is given by

$$\delta_2=\int_0^\infty\frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy=\delta\int_0^1 f(1-f)\,d\eta=0.1074 \qquad \ldots (8)$$

and $f'(0)=2.0667$. ... (9)

Substituting (8) and (9) in (5) and after performing the required operations, we ultimately get

$$\delta=6.2037\sqrt{\frac{\nu_x}{U_\infty}} \qquad \ldots (10)$$

Solution for Enthalpy Thickness $H(\Delta)$ and Ratio of Thermal Boundary Layer Thickness to Boundary Layer Thickness Δ

The energy integral eq. (6) in terms of $H(\Delta)$ and Δ can be written as

$$\frac{d}{dx}\left[\Delta\delta H(\Delta)\right]=\frac{2.0667}{U_\infty\Delta\delta}, \qquad \ldots (11)$$

where

$$\Delta=\frac{\delta_t}{\delta}$$

$$H(\Delta)=\frac{1}{\Delta}\int_0^{\Delta} f(\eta)\,\theta_1 \mathrm{d}\eta \qquad \ldots (12)$$

and

$$\theta_1=1-2.0667\left(\frac{\eta}{\Delta}\right)+6.334\left(\frac{\eta}{\Delta}\right)^4-9.0015\left(\frac{\eta}{\Delta}\right)^5+4.4012\left(\frac{\eta}{\Delta}\right)^6-0.667\left(\frac{\eta}{\Delta}\right)^7 \qquad \ldots (13)$$

For $\Delta > 1$ i.e., $\delta_t > \delta$, (12) can be written as

$$H(\Delta)=\frac{1}{\Delta}\int_0^{1} f(\eta)\,\theta_1 \mathrm{d}\eta+\frac{1}{\Delta}\int_1^{\Delta}\theta_1 \mathrm{d}\eta. \{\because f(\eta)=1\,\eta>1\} \qquad \ldots (14)$$

Substituting values of $f(\eta)$ and θ_1 obtained from eq. (7) and (13) in (14) and performing the integration, we get

$$H(\Delta)=0.2784-\frac{0.2784}{\Delta}+\frac{0.1174}{\Delta^2}-\frac{0.0238}{\Delta^5}+\frac{0.0167}{\Delta^6}-\frac{0.0051}{\Delta^7}. \qquad \ldots (15)$$

Integration of (11) gives

$$\Delta^2 H^2(\Delta)=\frac{2\times 2.0667}{U_\infty \delta^2}\int_0^{x} H(\Delta)\,\mathrm{d}x \qquad \ldots (16)$$

Taking the value of Δ as Δ = constant and integrating (16) we get

$$\Delta^2 H(\Delta)=\frac{4.1334}{U_\infty \delta^2}\cdot\frac{1}{x}$$

Substituting value of δ obtained from (10) in it, we get

$$\Delta^2 H(\Delta)=\frac{0.1074}{Pr}.$$

Restricting only to three terms of the expansion series in (15) and substituting the same in it, and after doing some simplification, we get in the form

$$\Delta^2-\Delta-\frac{0.3857}{Pr}-0.4217=0$$

Solving this we get

$$\Delta=0.5\pm\sqrt{\frac{0.3858}{Pr}-0.1717} \qquad \ldots (17)$$

Results

The most important result to be derived in problems of the present type is an expression for heat transfer which is commonly represented by the introduction of the local Nusselt Number

$$Nu_x = -\Delta \left(\frac{\partial \theta_1}{\partial \eta}\right)_{\eta=0} \frac{x}{\Delta\delta} \qquad \ldots (18)$$

From (13), we get

$$\left(\frac{\partial \theta_1}{\partial \eta}\right)_{\eta=0} = -2.0667/\Delta \qquad \ldots (19)$$

Using values of $\left(\frac{\partial \theta_1}{\delta \eta}\right)_{\eta=0}$ from (19), Δ from (17) and δ from (10) in (18), we get

$$Nu_x = \frac{\sqrt{Re_x\, Pr}}{1.5009\sqrt{Pr} + \sqrt{3.4764 - 1.5472\, Pr}}. \qquad \ldots (20)$$

The results of earlier studies which will be used for comparison are the works of:

1. Sparrow and Gregg[1] obtained exact solution for this case and plotted the results and suggested a good approximation formula to the resulting curve in the form:

$$\frac{Nu_x}{\sqrt{Re_x\, Pr}} = \frac{0.564}{1 + 0.9\sqrt{Pr}} \qquad \ldots (21)$$

2. Eckert[2] obtained an approximate solution and represented his result in the form of equation

$$Nu_x = \frac{\sqrt{Re_x\, Pr}}{1.55\sqrt{Pr} + 3.09\sqrt{0.372 - 0.15\, Pr}} \qquad \ldots (22)$$

3. Evans[4] and Merk[5] who presented the solution in the form

$$\frac{Nu_x}{\sqrt{Re_x\, Pr}} = \frac{1}{\sqrt{\pi}}\left[1 - 1.7207\left(\frac{Pr}{\pi}\right)^{1/2} + \ldots\right] \qquad \ldots (23)$$

The formulae (20) to (23) have been used to calculate the heat transfer rate in the liquid metal range of Prandtl Number $0.005 \leqslant Pr \leqslant 0.05$ and for comparison are shown graphically in Fig. 1.

Using eqs. (17) and (13) various temperature profiles for different Prandtl Numbers have been calculated and are shown graphically in Fig. 2.

Concluding Remarks

1. Study of graphs in Fig. 1 shows that with rise in values of Prandtl Numbers the heat transfer rate decreases.
2. Observation of graphs drawn in Fig. 1 shows that the results obtained with the use of eq. 2 developed by the present method are more close to the results obtained with the use of approximation formula (21) suggested by Sparrow and Gregg.

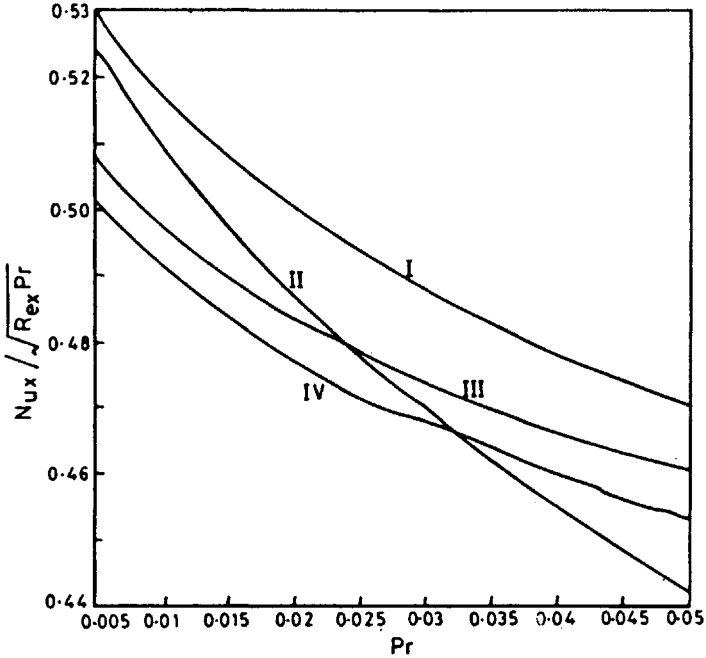

Fig. 1 I-Curve drawn with the use of equation (21); II-Curve drawn with the use of equation (23); III-Curve drawn with the use of equation (20); IV-Curve drawn with the use of equation (22).

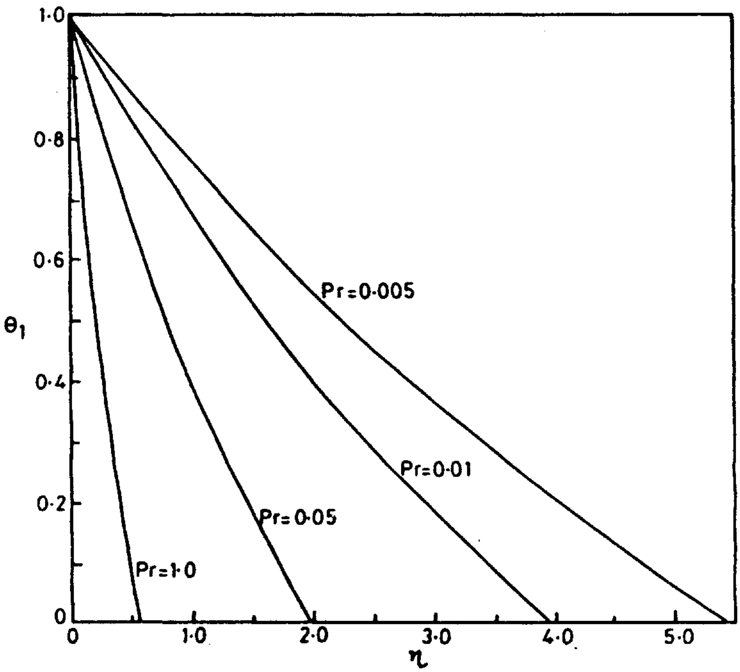

Fig. 2 Temperature distribution for different Pr numbers

3. From the graphs of Fig. (1), it is found that formula (20) is more suitable than that of (22) suggested by Eckert[2] as the former gives better results than the latter.
4. From the graphs of Fig. (1), it is found that formula (23) suggested by Evans[4] and Mark[5] gives better results than (20) for $0.005 \leqslant Pr \leqslant 0.024$ but for $0.025 \leqslant Pr \leqslant 0.05$ the results obtained by (23) are inferior than (20).
5. Observation of curves in Fig. (2) shows that the temperature distribution field becomes thicker with decrease in values of Prandtl Number.
6. Study of Eq. (20) shows that in the liquid metal range, its denominator is a weak function of *Pr*, hence the Nusselt number mainly depends on the product *Re Pr*.
7. Values obtained with the use of eq. (20) agree well, within ±5% for $0.005 \leqslant Pr \leqslant 0.05$ with the exact solutions.
8. From the eq. 17, it is found that in the liquid metal range, the ratio of flow boundary layer thickness to the thermal boundary layer thickness is proportional to the square root of the Prandtl number.
9. Study of curves of Fig. 2, proves that the thermal boundary is much thicker than the flow boundary layer.
10. From the curves of Fig. 2, it is seen that in the boundary layers of liquid metals the fluid attains the free stream velocity just after a thin zone of the plate and covers a wide range with the stream velocity. Thus fluid is viscous in the thin zone and becomes inviscid in the major area.

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