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A STUDY OF VENTILATION FACTOR DURING DROPLET GROWTH

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The impact of the ventilation effect on a droplet during growth within cumulonimbus clouds in the tropics is investigated mathematically. The equation of growth is formed by taking into account the heat balance of a droplet.

The results reveal a more pronounced influence of the ventilation factor on the growth of large sized droplets than those of smaller size. It is observed that the importance of the ventilation factor is greater during the pre-monsoon days than in other seasons.

Key Words: Cloud Condensation Nuclei; Ventilation Effect; Droplet Growth

Introduction

The formation and growth of droplets in the atmosphere is an important meteorological process because it precedes cloud formation. A significant concentration of hygroscopic particles in the micron and submicron range act as condensation nuclei, which initiate the formation of droplets. The growth of droplets depends on different macrophysical and microphysical factors, such as, the collision efficiency, fall velocity, coalescence of drops and the cloud water content. It is also known that the fall velocity of a droplet relative to the vapour field introduces a "ventilation effect", which tends to increase the rate of droplet growth. This is taken in to account by including a ventilation factor (F), which is a function of the Reynolds and Prandtl numbers, in the growth equation of a droplet.

Studies on the ventilation effect date back to the early fifties when Dady[1] found that, in comparison with fall under gravity, eddies contribute very little to the ventilation of a cloud droplet. Squires[2] studied the growth of a droplet considering the ventilation factor, and showed that for growth of droplets upto a radius of about 50 microns, the neglect of a ventilation factor in the growth equation introduces an error of less than 28%. But, Fletcher[3] found that the ventilation effect was negligible for droplets with radii less than 10 microns. It becomes important with an increase in droplet size, until around 80 microns an error of 50% was estimated in the computed growth rate. As these studies were made on droplets suspended from rigid supports, computational errors were introduced by distortions in droplet shape, by the air flow and droplet oscillations. Pruppacher and Rasmussen[4] carried out wind tunnel experiments to estimate these errors. They found a more accurate expression for the ventilation factor. Srivastava[5] introduced his factor in his formulation of droplet growth by condensation.

The aim of the present paper is to analyse errors caused by neglecting the ventilation factor during the growth of cloud condensation nuclei, within cumulonimbus clouds, in premonsoon, post-monsoon and monsoon days. As the study is limited to the importance of errors due to the ventilation factor during droplet growth, the intricacies of different electrical and kinematical processes within clouds in different seasons have not been considered.

Mathematical Formulation

The growth of a droplet by condensation involves the transfer of heat and water vapour. The latent heat released during the change in phase accompanying growth is large in comparison with the thermal capacity of droplet. Assuming that a spherical droplet has a diameter large in comparison to the mean free path of the molecules, the ventilation factor may be expressed by the following equations[2]: (Squires, 1952),

$$F(R_e, \sigma) = 1 + 0.246\, R_e^{-1/2} \qquad \ldots (1)$$

and

$$F_1(R_{e'}, \sigma') = 1 + 0.232\, R_e^{-1/2} \qquad \ldots (2)$$

In eq. (1), $F(R_e, \sigma)$ is the ventilation factor in the transfer of heat, R_e is the Reynolds number ($2Vr/v$), with V and r being the velocity and radius of a droplet and ν is the kinematic viscosity of air. σ is the Prandtl number (ν/ν_c) where,

$$\nu = 1.72 \times 10^{4}\left(\frac{303}{T+120}\right)\left(\frac{T}{273}\right)(\mathrm{m}^2/\mathrm{s}) \qquad \ldots (3)$$

and ν_c is the thermal diffusivity of air (m^2/s).

In eq. (2), $F_1(R_e, \sigma')$ is the ventilation factor during the transfer of water vapour and σ' is expressed by

$$\sigma' = \nu/D\,, \qquad \ldots (4)$$

where D is the diffusion coefficients of water vapour in air (m^2/s). Using Clausius Clapeyron's equation, we may express the heat balance equation of a droplet of radius $(r)^2$ by

$$\frac{4}{3}\pi r^3 \rho c \frac{\mathrm{d}T_d}{\mathrm{d}t} = 4\frac{\pi D r L \varepsilon}{RT}(e_a - e_d)\,[F_1(R_e, \sigma')] - 4\pi K r[F(R_e, \sigma)](T_d - T), \qquad \ldots (5)$$

where

ρ = density of water ($\mathrm{kg/m^3}$)
c = specific heat of water (J/°K/kg)

t = time (s)
T = temperature of environmental air (°K)
T_d = temperature at surface of droplet (°K)
K = thermal conductivity of air ($Jm^{-1}s^{-1}k^{-1}$)
ε = ratio of molecular weights of water and dry air
L = latent heat of condensation (J/kg)
R = universal gas constant of dry air (J/°K/kg)
e_a = vapour pressure of air (hPa)
e_d = equilibrium vapour pressure of droplet (hPa)

Eq. (5) shows that the time rate of change of temperature at the surface of the droplet depends on the transfer of water vapour towards the droplet and the transfer of heat away from the ventilated spherical droplet.

The equilibrium vapour pressure over a droplet is

$$e_d = e(Td)\left[1 + \frac{2\zeta\varepsilon}{R\rho rT} - \frac{3MM_v}{4\pi\rho r^3}\right], \qquad \ldots (6)$$

where

ζ = surface tension of water (dynes cm^{-1})
M = Molar mass of condensation nuclei (kg)
M_v = molecular weight of water (kg)
Eq. (6) is introduced in the second term of eq. (5).

Thus,

$$\frac{4}{3}\pi r^3\rho c\frac{dT_d}{dt} + 4\pi Kr(T_d - T)[F(R_e, \sigma)] = \frac{4\pi Dr}{RT}L\varepsilon[F_1(R_e\sigma')]\left[e_a - e(T_d)\times\left\{1 + \frac{2\zeta\varepsilon}{R\rho rT} - \frac{3MM_v}{4\pi\rho r^3}\right\}\right] \qquad \ldots (7)$$

From Clausius Clapeyrons equation, a small vapour pressure difference of air at dew point, T_s, may be expressed by

$$\frac{e(T_s)}{e} = \frac{L\varepsilon}{RT^2}(T_s - T) \text{ and } \frac{e(T_d)}{e} = \frac{L\varepsilon}{RT^2}(T_d - T),$$

where

e = average vapour pressure, (hPa)
and
T_s = dew point temperature (°K)

Hence,

$$e(T_s) - e(T_d) = \frac{L\varepsilon Je}{RT^2}(T_s - T_d), \qquad \ldots (8)$$

where J is the mechanical equivalent of heat.

Since at dew point $e_a = e(T_s)$, eq. (7) becomes,

$$\frac{4}{3}\pi r^3 \rho c \frac{dT_d}{dt} + 4\pi K r F(R_e, \sigma)(T_d - T) = \frac{4\pi JDL}{R^2 T^3} \varepsilon^2 re\, F_1(R_e, \sigma')\left[T_s - T_d - \frac{2\zeta T}{JL\rho r} + \frac{3MM_v RT^2}{4\pi\rho\varepsilon r^3}\right] \quad \ldots (9)$$

or

$$\frac{dT_d}{dt} = \frac{3KF(R_e, \sigma)}{r^2\rho c} T - \frac{3KF(R_e, \sigma)}{r^2\rho c} T_d + \frac{3\varepsilon^2 L^2 DJe}{R^2 T^3 r^2 \rho c} F_1(R_e, \sigma')\left[T_s - \frac{2\zeta T}{JL\rho r} + \frac{3MM_v RT^2}{4\pi JL\varepsilon\rho r^3}\right] - \frac{3\varepsilon^2 L^2 DJeF(R_e, \sigma')}{R^3 T^3 r^2 \rho C} T_d$$

This may be expressed by,

$$\frac{dT_d}{dt} = aT - aT_d + bT_s - bT_d \quad \ldots (10)$$

$$a = 3KF(R_e, \sigma)/r^2\rho C,$$

$$b = \frac{3\varepsilon^2 L^2 DJe(F(R_e, \sigma)}{R^2 T^3 r^2 \rho C}$$

and

$$T_s = T_s - \frac{2\zeta T}{JL\rho r} + \frac{3MM_v RT^2}{4\pi JL\varepsilon\rho r^3}.$$

Omitting the first term in eq. (9) since it is very small, we find

$$T_d = \frac{aT + bT_s}{(a+b)}. \quad \ldots (11)$$

From eqs (10) and (8), we get the rate of growth of a droplet of mass m. This is

$$\frac{dm}{dt} = 4\pi A_1 r F_1(R_e, \sigma')\left[S - \frac{B}{r} + \frac{GM}{r^3}\right], \quad \ldots (12)$$

where

$$A_1 = \frac{\varepsilon^2 L^2 DJe}{R^2 T^3} \Bigg/ \left[1 + \frac{\varepsilon^2 L^2 DJe}{KR^2 T^3} \frac{F_1(R_e, \sigma')}{F(R_e, \sigma)}\right], \qquad \ldots (13)$$

$$S = T_s - T,$$

$$B = = 2\zeta T/JL$$

and

$$G = 3M_v RT^2/4\pi\rho JL\varepsilon.$$

Results and Discussion

The effect of ventilation on a growing droplet within a thundercloud is obtained from eq. (12) with the aid of data (Table I) collected from the Regional Meteorological Office, Calcutta. The present study concerns three thunderstorm events with different characteristics in the pre-monsoon, monsoon and post-monsoon period of 1969. The intention of the study was not to generalize the concept, but to see the nature of the ventilation effect on a growing droplet during the three cases. The range of radii of droplets under study was taken to vary from 5 microns to 100 microns. The coefficients of diffusion of water vapour and thermal conductivity were obtained from the following expressions by Leaitch *et al.*[6]:

$$D = D_0 \Bigg/ \left[\frac{r}{r+\lambda} + \frac{D_0}{r\,a_c}(2\pi M/R_g T)^{1/2}\right] \qquad \ldots (14)$$

Table I

Effect of ventilation on a growing droplet (data)

Pressure (hPa)	17 April 1969		17th July 1969		9 October 1969	
	Temp (°C)	Dew pt (°C)	Temp (°C)	Dew pt (°C)	Temp (°C)	Dew pt (°C)
1000	28	25	26.4	25.8	23	22
950	25	20	26	23	22	18
850	25	04	20	18	17	12
800	21	−03	19	14	13	10
700	11	−02	12	10	07	04
600	00	−06	06	03	00	00
500	−10	—	−02	−06	−06	−07
450	−10	—	−07	−11	−11	−13
400	−23	—	−13	−18	−16	−20
300	−35	—	−28	−31	−33	−39
250	−44	—	−38	−43	−44	—
200	−53	—	−52	—	−58	—

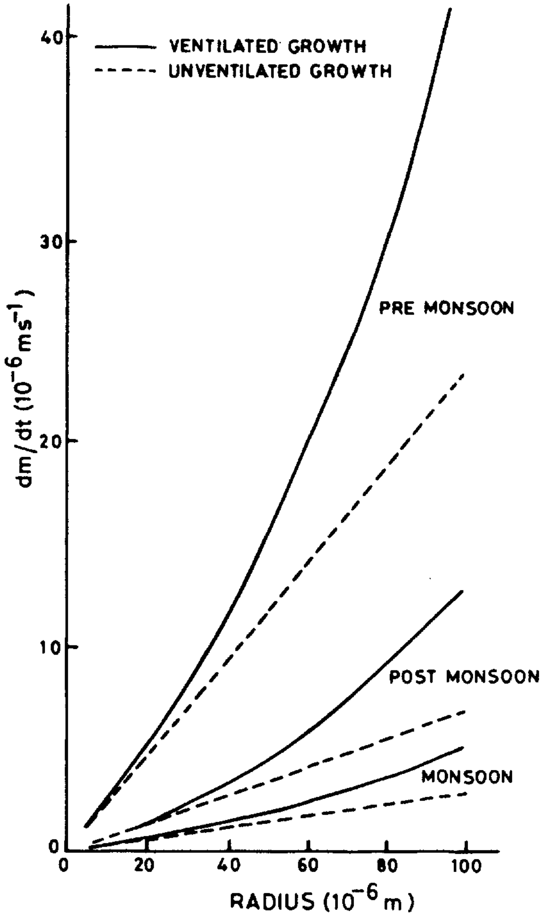

Fig 1 Variation of growth of droplets with respect to different radii for ventilated (unbroken) and unventilated (broken lines) cases

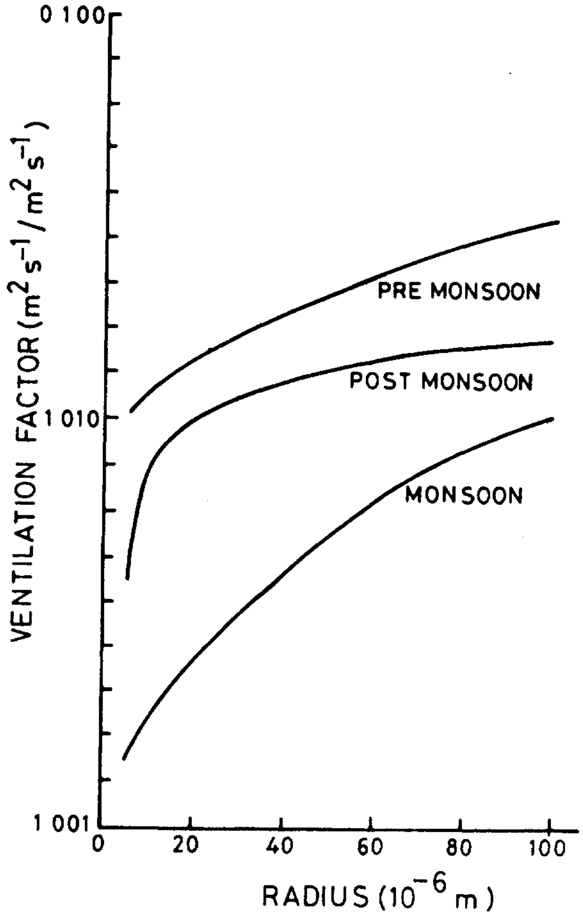

Fig 2 Variation of the ventilation factor with radius of the droplet during pre-monsoon, monsoon and post-monsoon days

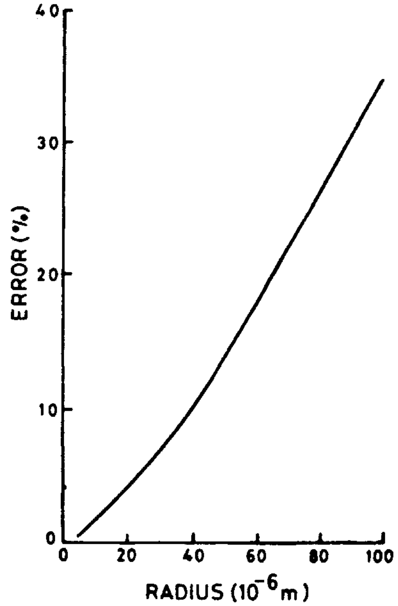

Fig 3 Variation of the percentage of errors with respect to droplet radius in the case of an unventilated droplet

and

$$K = K_0 \Bigg/ \left[\frac{r}{r+\lambda} + \frac{K_0}{r\alpha_t \rho c_p}(2\pi m_a / R_g T)^{1/2}\right], \qquad \ldots (15)$$

where

c_p = specific heat of air (J $\text{deg}^{-1}\text{kg}^{-1}$)
m_a = molecular weight of air,
R_g = universal gas constant,
α_c = condensation coefficient,
α_t = thermal accommodation coefficient
λ = mean free path of an air molecules (1.2×10^{-5} cm) as in Leaitch *et al.*[6],
D_0 = initial value of diffusivity (m^2s^{-1})

and

K_0 = initial value of thermal conductivity (m^2s^{-1}).

Fig. 1 shows the variation of errors due to the ventilation factor as a function of the droplet radius. We find that at the base of a thunder-cloud, for radii less than 10 microns, the effect of ventilation is nominal. But, it becomes significant for $r > 70$ microns. The impact of this factor is particularly effective during the pre-monsoon season. It is less during the post-monsoon season and the least during the monsoon.

In Fig. 2 we show the rate of droplet growth as a function of its radius in a ventilated and unventilated atmosphere during the three days considered by us.

Although the effect is not significant for radii less than 15 microns, it becomes important when the radius exceeds 60 microns. The omission of the ventilation effect in the latter case will introduce errors of approximately 40%. This is more explicitly revealed in Fig. 3.

Table II

A comparison of the percentage of errors at different droplet radii with those of Squires (1952)[2]

Radius (μ)		10	30	50	70	100
Percentage of error	Present paper	0.01	0.07	0.14	0.22	0.36
	Squires (*loc.cit*)	0.01	0.07	0.13	0.19	0.29

The percentage of error that enters in computations of different sized droplets is shown in Fig. 3. The error increases proportionately with radius. The percentage errors introduced by the ventilation factor for different droplet radii are compared with those of Squires (1952) in Table II. We find that for radii upto 30 microns, the errors are comparable with those of Squires. But for larger droplets, namely about 100 microns, the percentage error in the present study is about 35.8%. This is more than the values of Squires (*loc. cit*) in mid-latitudes.

Conclusion

The main features of the study are as follows:

(i) The influence of the ventilation factor increases with the growth rate of a droplet.

(ii) The ventilation factor is proportional to the droplet size.

(iii) For droplets of medium to large dimensions, the omission of this factor will introduce serious errors.

(iv) The importance of ventilation is larger in the pre-monsoon seasons. It is lesser during the post-monsoon and least during the monsoon.

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