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CLOUD DROPLET GROWTH IN A TROPICAL CUMULONIMBUS

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An adiabatic model is used to study the dependence of various parameters on the growth of a cloud droplet by condensation.

The results indicate that the growth rate depends directly on the composition of the droplet, as well as on the size of the cloud condensation nucleus.

Key Words: Droplet Growth; Supersaturation; Pressure; Temperature

Introduction

The formation and growth of cloud condensation nuclei is a problem of prime importance in cloud physics. The phenomenon is a complex one requiring knowledge of condensation, evaporation and the chemistry of the droplet composition. Numerous studies have been made on different aspects of droplet growth, but there are some aspects that are not yet well understood.

Adiabatic parcel models have been used in many studies such as the ones by Squires[1], Arnason and Brown[2] and Lee and Pruppacher[3]. Fukuta and Walter[4] derived a modified equation of growth for pure water and solution droplets having radii greater that 1 micron by including discrete vapour and temperature fields at the surfaces of growing droplets. Gilles Bergametti *et al.*[5] performed a continental aerosol sampling at a coastal location of N.W. Corsica and observed a seasonal pattern of aerosol composition. In a different approach of droplet growth, Srivastava[6] differentiated the supersaturation around a single drop from the bulk supersaturation over a population of drops. More recently, Srivastava and Coen[7] re-examined the assumptions underlying the traditional equations of hydrometers growth and evaporation and sought solutions to minimize the errors in results.

The present study utilizes an adiabatic model to understand the condensational growth of a droplet within a tropical cumulonimbus cloud. The influence of different parameters, such as, the ambient temperature, supersaturation, pressure and microphysical parameters, like the droplet size, composition on the growth of a droplet are studied. The droplet is assumed to be geometrically spherical, but electrical processes and radiation have not been considered at present.

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Mathematical Formulation

The adiabatic model considering the microphysics of the condensation process is adopted from Leaitch *et al.*[8] The model considers a parcel of air ascending adiabatically for a specified updraft velocity within a tropical cumulonimbus cloud. The microphysics of the growth of a droplet is divided into two parts as described below:

(1) For solution droplets having a salt mass concentration 1%, the rate of change of droplet growth is expressed by:

$$r\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{S+1-\exp\left(\frac{2M_w\sigma_{s/Q}}{R_g\rho_s T\cdot r}\right)\Big/\left[1+\frac{v\phi_s M_w m}{M_s(4/3\ \pi r^3\rho_s-m)}\right]^{\rho/\rho_s}}{\left[\left(\frac{L^2\rho_s M_w}{kR_g T^2}\right)-\left(\frac{L\rho_s}{KT}\right)+\left(\frac{\rho_s R_g T}{M_s De_s}\right)\right]} \qquad \ldots(1)$$

(2) For dilute solutions, the rate of change of droplet growth is[9],

$$r\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{S+1-\left(\frac{2M_w\sigma_{s/Q}}{R_g\rho Tr}\right)+\left(\frac{v\phi_s mM_w}{M_s(4\pi r^3\rho/3-m)}\right)}{\left[\left(\frac{L^2\rho M_w}{KR_g T^2}\right)-\left(\frac{L\rho}{KT}\right)+\left(\frac{\rho R_g T}{M_s De_s}\right)\right]} \qquad \ldots(2)$$

where

r = radius of droplet (m),
t = time (S),
S = supersaturation (%),
M_w = molecular weight of water,
M_s = molecular weight of nucleus material,
$\sigma_{s/s}$ = surface tension at solution/air interface (ntm^{-1}),
ρ_s = density of solution droplet ($\mathrm{kg/m^{-3}}$),
ρ = density of water ($\mathrm{kg/m^{-3}}$),
m = mass of dry nucleus (kg),
$\bar{v}$ = number of ions into which the salt dissolves,
ϕ_s = Osmotic coefficient of solution droplet,
T = temperature (°K),
R_g = universal gas constant ($\mathrm{J\ deg^{-1}, kg^{-1}}$),
L = latent heat of evaporation ($\mathrm{J\ kg^{-1}}$),
e_s = saturation vapour pressure at temperature T.(hPa)
K = thermal conductivity of air ($\mathrm{Jm^{-1}S^{-1}deg^{-1}}$)
D = diffusivity of water vapour in air ($\mathrm{m^2s^{-1}}$).

In the beginning it would be relevant to define some parameters. Saturation is defined as the equilibrium situation in which the rate of evaporation and condensation are equal.

The saturation ratio may be defined as the ratio of the vapour pressure to the saturation vapour pressure over the surface of a spherical droplet[9].

When the relative humidity exceeds 100%, the actual vapour pressure at that temperature is greater than the saturation vapour pressure, and then the air is said to be supersaturated. Supersaturation is expressed as a percentage.

The equation for supersaturation, S, in a parcel of ascending air rising with a velocity, V, is given by[10]

$$\frac{\mathrm{d}S}{\mathrm{d}t}=\left[\frac{Lq}{R_v T^2 C_p}-\frac{g}{R_a T}\right) V-\left[\frac{P}{\varepsilon e_s}+\frac{L^2}{R_v T^2 C_p}\right]\frac{\mathrm{d}q_L}{\mathrm{d}t} \qquad \ldots (3)$$

or

$$\frac{\mathrm{d}S}{\mathrm{d}t}=C_1 V-C_2\frac{\mathrm{d}q_i}{\mathrm{d}t}, \quad \text{say} \qquad \ldots (4)$$

where,

$$C_1=Lg/R_v T^2 C_p-g/R_a T$$

$$C_2=P/\varepsilon e_s+L^2 R_v T^2 C_p,$$

where

g = acceleration due to gravity (ms^{-2})
C_p = specific heat of air at constant pressure ($\mathrm{J\ kg^{-1}.K^{-1}}$)
R_v = specific gas constant for water vapour ($\mathrm{J\ kg^{-1}.K^{-1}}$)
P = pressure (h Pa)
ε = ratio of molecular weights of water and dry air
e_s = saturation vapour pressure (hPa)
q_L = liquid water mixing ratio (kg per kg)
R_a = gas constant for dry air ($\mathrm{J.K^{-1}, kg^{-1}}$)

The ratio of change of liquid water mixing ratio is then

$$\frac{\mathrm{d}q_L}{\mathrm{d}t}=\frac{4\pi S}{\rho_a}\left[\frac{R_v T}{e_s D}+\frac{JL^2}{KT^2 R_v}\right]^{-1}\Sigma n_i r_i, \qquad \ldots (5)$$

where

J = mechanical equivalent of heat
ρ_a = density of air ($\mathrm{kg\ m^{-3}}$)
n_i = number of droplets/unit volume with mass m and radii r_i.

or

$$\frac{\mathrm{d}q_L}{\mathrm{d}t}=\frac{4\pi S}{\rho_a}C_3\Sigma n_i r_i, \qquad \ldots (6)$$

where

$$C_3=\left[\frac{R_v T}{e_s D}+\frac{JL^2}{KT^2 R_v}\right]^{-1}$$

On substituting eq. (6) in (4) we find

$$\frac{\mathrm{d}S}{\mathrm{d}t}+\frac{4\pi S}{\rho_a}C_2 C_3 \Sigma n_i r_i = C_1 V$$

or

$$\frac{\mathrm{d}S}{\mathrm{d}t}+C_4(t)\,S=C_1 V, \qquad \ldots (7)$$

where

$$C_4=\frac{4\pi}{\rho_a}C_2 C_3 \Sigma n_i r_i \qquad \ldots (8)$$

Since C_1 and C_4 vary slowly with time these are assumed to be constants. Solving eq. (7) with the help of an initial condition,

at $t=0$, $S=S_0$,

where

S_0 = initial value of supersaturation,

we have

$$S=S_0\exp(-C_4 t)+\frac{C_1 V}{C_4}[1\text{-}\exp(-C_4 t)]. \qquad \ldots (9)$$

Results and Discussion

Studies were carried out with data taken from a premonsoon cumulonimbus thunderstorm cloud in the region of Calcutta. The thunderstorm occurred on the 2nd of April 1994.

Eq. (9) was used to find the value of supersaturation at different pressures within the cloud. The rates of growth of droplets were calculated by eq. (1) and (2).

The diffusitivity of water vapour in air and the thermal conductivity were obtained from the following expressions according to Pruppacher and Klett[9]

$$D = D'/[r/(r+\lambda) + (D/ra_c)(2\pi M_w/R'\,T_d)^{1/2}]$$

and

$$K = K'/[r/(r+\lambda) + (K/ra_T\rho C_p)(2\pi M_a/R'T_d)^{1/2}],$$

where

D' = Initial value of diffusivity (ms^{-1})
K' = Initial value of conductivity of air ($Jm^{-1}s^{-1}deg^{-1}$)
C_p = specific heat of air ($J\,deg^{-1}kg^{-1}$)
a_c = accommodation coefficient for the condensation of water vapour.
a_t = thermal accommodation coefficient
M_a = molecular weight of air
R' = universal gas constant (J/°K/k mol)
T_d = temperature at the surface of the droplet (°K)
λ = mean free path of air molecules (m).

The value of λ is taken to be 1.2×10^{-5} cm[8].

Computations were initiated with droplets containing solutions of sodium chloride and sulphuric acid. The radii of droplets were taken to be in the range of 0.1 to 60 microns. The mass of a nucleus was assumed to vary within the range 10^{-20} to 10^{-14} microns.

The variation of supersaturation at different levels within a cumulonimbus cloud is shown in Fig. 1. The base of the cloud is at 810 hPa. It is observed that the value of supersaturation is higher below the cloud base than at the base. The value of supersaturation again increases at higher pressures and attains a maximum in the mid-cloud zone (approximately 500 hPa). At higher upper pressures, supersaturation begins to decline again. Moreover, at any level, the value of supersaturation shows an initially rapid increase with time within a period of 0-70 secs. The value gradually settles at around 300 secs. This finding agrees with that of Twomey[11].

Fig. 2 shows this aspect in more detail. Here we can see the variation of supersaturation along the height of the cloud at different time intervals. We find that, at any time, supersaturation is at a maximum around 500 hPa. The supersaturation is at a minimum at the base of the cloud. This is expected because a change of phase occurs at this level bringing about a reduction in supersaturation just beyond the base of the cloud. This result is compared with the findings of Lee and Pruppacher[3] for a continental cumulus case.

The rate of growth of a droplet ($r = 10^{-6}$m) against time at different pressures is shown in Fig. 3. It is interesting to note that below the cloud base (810

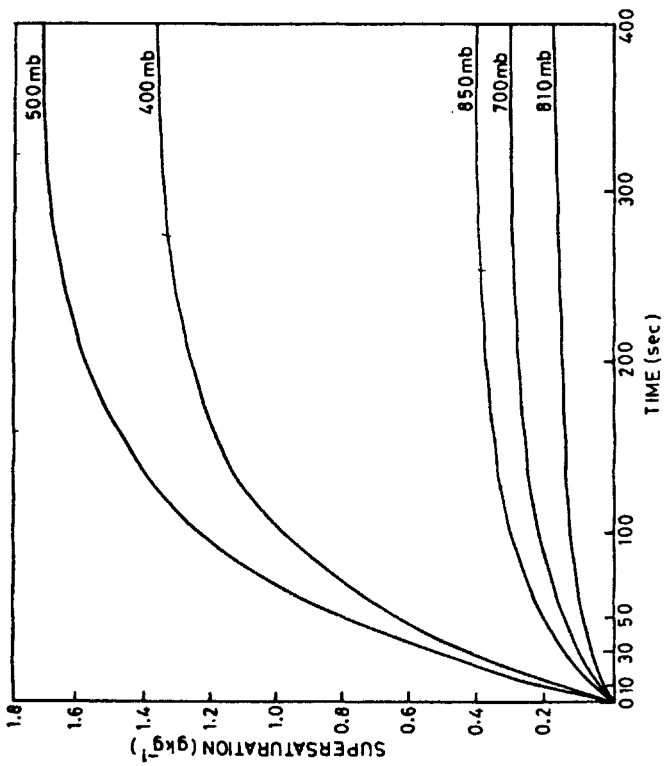

Fig 1 Supersaturation against time and different pressures within the cloud.

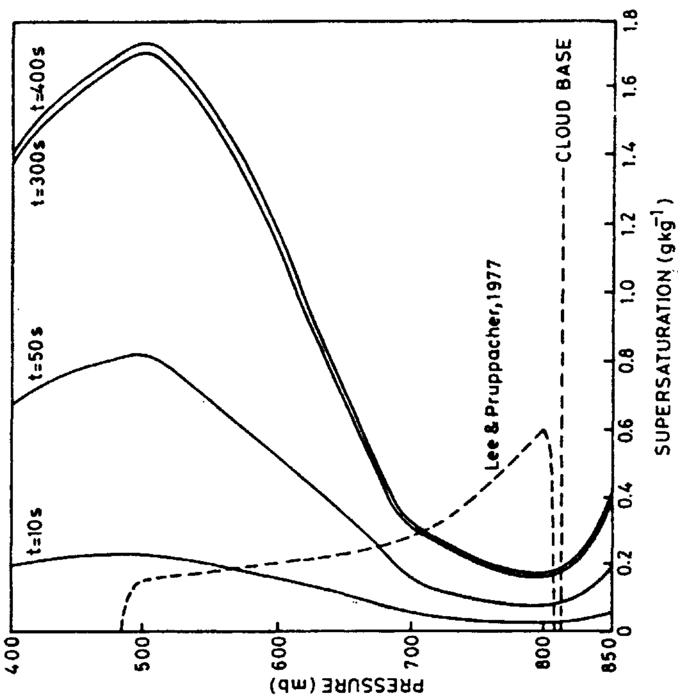

Fig 2 Variation of supersaturation within the cumulonimbus cloud at different time intervals.

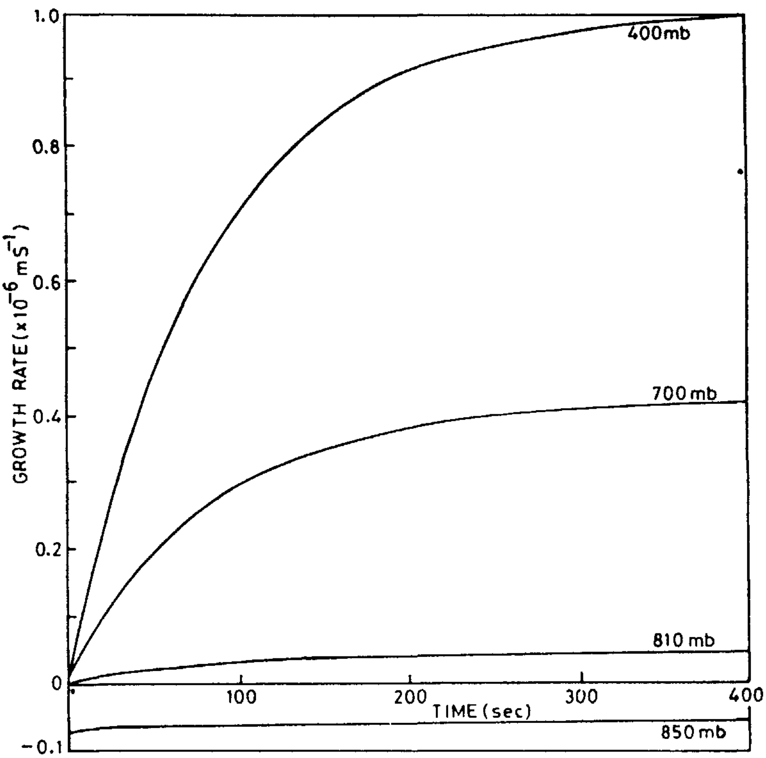

Fig 3 Growth rate against time at different pressures for droplet radius 10^{-6}m.

hPa) the rate of growth shows a negative trend. It indicates that the droplet has an evaporate tendency which slowly decreases with time. The rate of growth is observed to reach a maximum at 400 hPa, while at the base of the cloud it is very small. It is, approximately, of the order of 0.043×10^{-6} ms^{-1}. At all pressures, however, the initial growth rate is rapid which in later stages down to a steady pattern.

The rate of growth for droplets with different radii with respect to time is shown in Fig. 4. We find that smaller droplets have lower growth rates than the larger ones. But, there is a critical size above which the rate of growth begins to fall. This optimum radius is around 10^{-6}m. This is possibly due to the fact that the influence due to condensational growth becomes less dominant and collection growth takes over.

Fig. 5 shows the influence of the growth rate on the ambient temperature. We find that at fixed time intervals, the rate of droplet growth is inversely proportional to the temperature of the surrounding cloudy air. The decrease in the rates of growth is steeper for greater time intervals. An interesting feature is that, below the cloud base, the reduction in the growth rates is almost identical at any time interval.

Studies with droplets containing different solutions are shown in Fig. 6. Here, droplets containing salt solutions, sulphuric acid solutions and very dilute

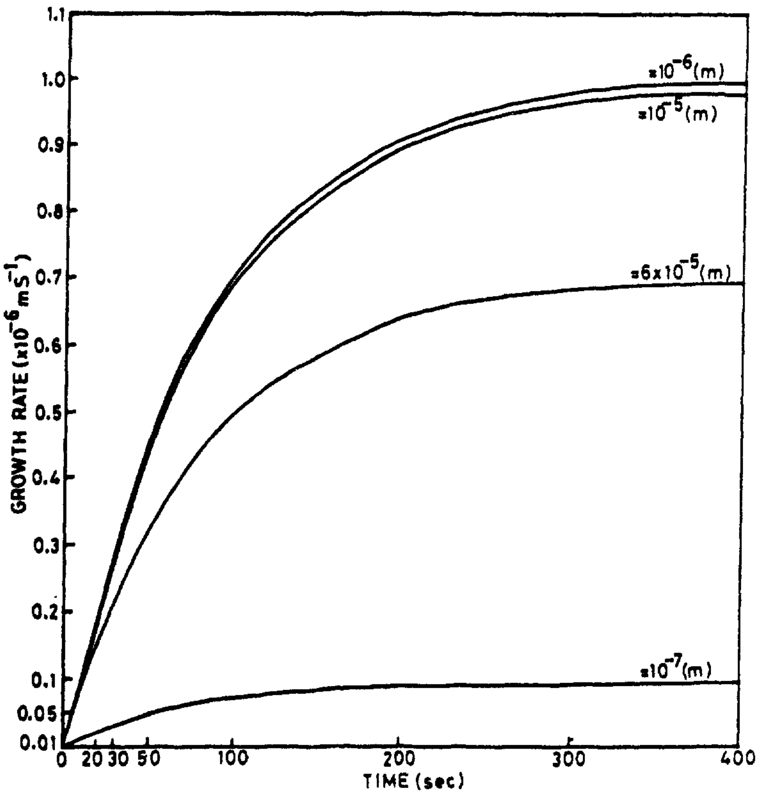

Fig 4 Growth rate against time for different sizes droplets at 400 hPa.

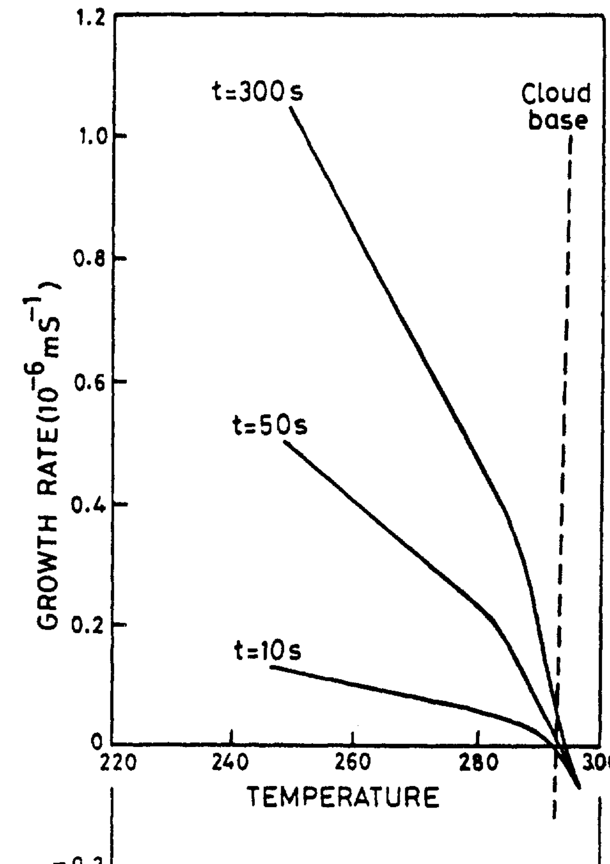

Fig 5 Growth rate of a droplet having radius 10^{-6}m against temperature at different time intervals.

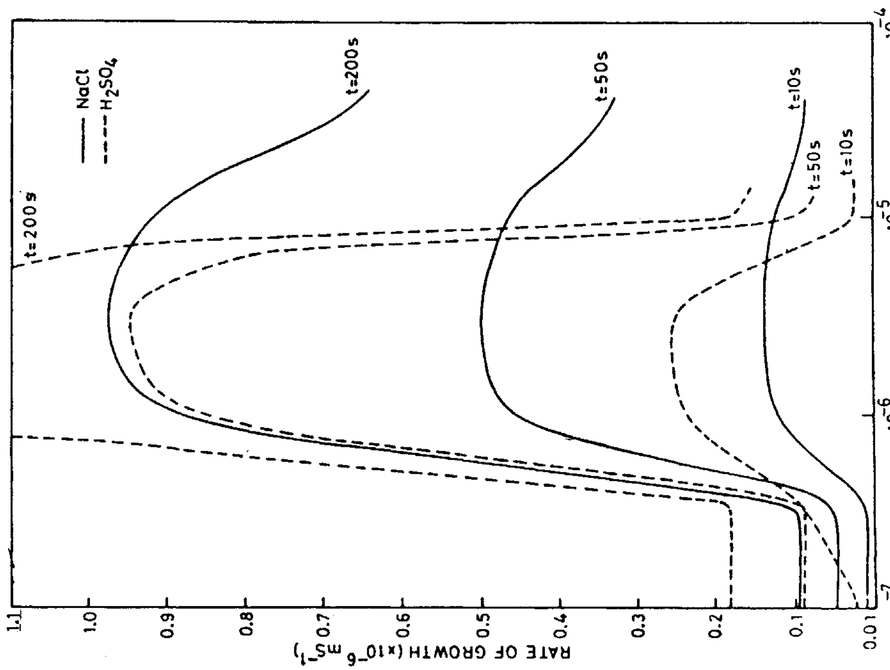

Fig 7 Rate of growth different radius and concentration at 400 mb.

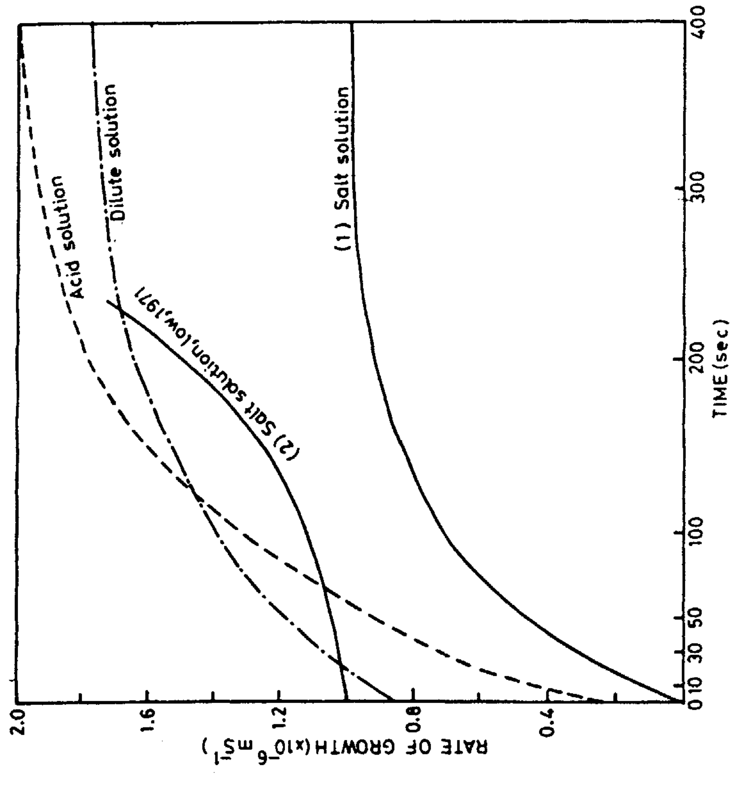

Fig 6 Rate of growths with time for different solution droplets of radii 10^{-6}m.

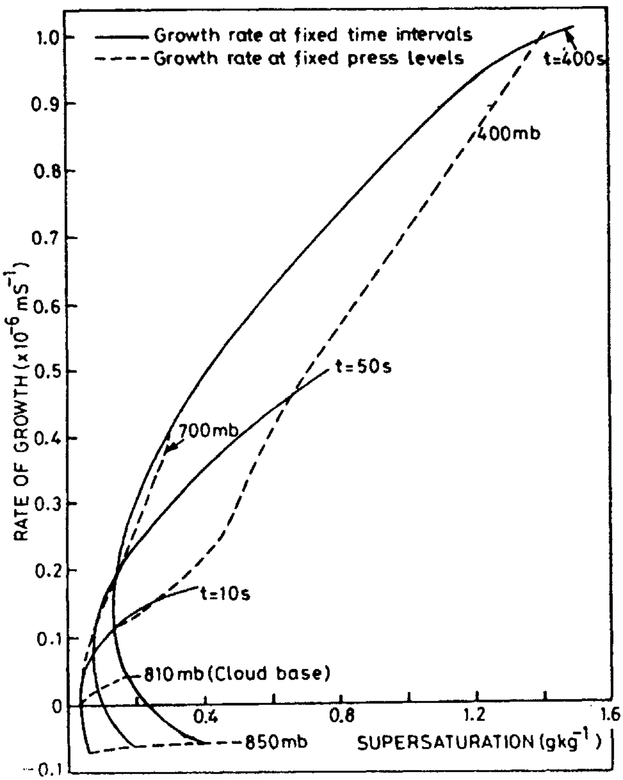

Fig 8 Variation of the growth rate of a droplet (radius 10^{-6}m) with respect to the ambient supersaturation.

solutions are considered. It is observed that droplets with salt solutions have the least growth rates, while sulphuric acid solutions have the steepest rate of growth. The droplet growth seems to be higher in industrial regions than in coastal regions.

Fig. 7 further emphasizes this factor. We made a comparative study with sodium chloride and sulphuric acid solutions. The growth rates with respect to droplet radii are shown for different fixed time intervals. Here again we note that droplets containing concentrated acid solutions have higher growth rates than those containing salt solutions. The growth rates of droplets having radii in the range of 10^{-6}-10^{-5}m are high, while smaller or larger droples have much lower rates of growth. This indicates that the growth rates of large cloud drops and small condensation nuclei are low. The growth rate is the highest among typical condensation nuclei and average sized cloud droplets. This indicates that condensational growth is highest for small sized droplets whereas it becomes less important as the droplet grows larger.

Fig. 8 shows the variation of droplet growth rate with the variation in supersaturation at different pressures. We observe that the growth rate of a typical cloud condensation nucleus indicates evaporate tendency below the base of the cloud. The droplet tends to evaporate below the cloud. This tendency diminishes as the droplet ascends towards the base of the cloud. The growth rate vanishes just below the base. At any particular time, the growth rate is exhibited by a parabolic curve.

Conclusion

This case study has revealed some interesting results. It is observed that the droplet growth rate depends directly on the composition of the droplet. It also depends on the size of the cloud condensation nuclei and the variation of ambient supersaturation. Meteorological variables, such as, pressure and temperature also influence the rate of growth of the droplet. Droplets below the cloud base show a tendency to evaporate. Droplets containing solutions of sulphuric acid have higher growth rates than those containing salt solutions.

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