

# PARTIAL AND COMPLETE SYNCHRONIZATION IN QUASIPERIODICALLY FORCED COUPLED MAPS

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We study different coupling schemes in a system of three coupled nonlinear maps in which the parameters are subject to quasiperiodic modulation. This gives rise to a phase diagram that has coexisting regimes of synchronized and desynchronized dynamics on torus and chaotic attractors, as well as on strange nonchaotic attractors (SNAs). We discuss the influence of coupling topology on the extent of partial synchronization; this suggests strategies for controlling the dynamics of assemblies of driven coupled map lattices.

**Key Words :** Coupled Map Lattice; Synchronization; Quasiperiodic Forcing; Strange Nonchaotic Attractor

## 1 Introduction

The circumstances under which coupled nonlinear systems display synchronization or desynchronization has been extensively studied in the last few years<sup>1</sup>. The subject has a long history, dating from the observation by Huygens<sup>2</sup> of synchronization between coupled periodic oscillators (pendula), to more recent work on the synchronization between coupled chaotic oscillators<sup>3</sup>. Pecora and Carroll<sup>4</sup> showed that identical (or nearly identical) nonlinear systems can be synchronized if coupled by a common drive signal even when the dynamics is chaotic, namely showing sensitivity to initial conditions. The motion of the coupled system, after transient unsynchronized dynamics, takes place on an attractor in the “synchronization manifold”<sup>5</sup> which is an invariant symmetric subspace (ISS) of the phase space. The Lyapunov exponent in directions transverse to the synchronization manifold (termed transverse Lyapunov exponents<sup>3</sup>) provide a measure of the average stability of synchronous motion. A number of studies of the dynamics of coupled chaotic systems have found different interesting cases of complete and partial<sup>6,7</sup>, generalized<sup>8</sup>, lag<sup>9</sup>, and phase synchronization<sup>10</sup>, along with riddled basins of attraction<sup>11</sup>, attractor bubbling<sup>12</sup>, and on-off intermittency<sup>13</sup>.

Similarly, quasiperiodic modulation has also been examined in fair detail in the past few years for a variety of reasons, including the fact that with quasiperiodic driving, the dynamics can be both aperiodic and stable,

on so-called strange nonchaotic attractors (SNAs)<sup>14</sup>. These attractors have all Lyapunov exponents (LEs) nonpositive (and are hence nonchaotic) and typically are geometrically fractal (or strange).

In the present work we study a system of three coupled quasiperiodically driven nonlinear maps. Quasiperiodic driving makes it possible to achieve synchronization<sup>15</sup> under at least three separate scenarios since the synchronous attractors can be chaotic and strange, or nonchaotic and either strange or nonstrange. Previous studies<sup>16–18</sup> have been restricted to two similarly coupled and driven maps and therefore here we explore the complexities introduced by different possible coupling schemes. This system of coupled logistic maps, which individually follow the period-doubling route to chaos<sup>19</sup>, demonstrate the universal scenario of the development of periodic and chaotic coexisting regimes. Quasiperiodic forcing transforms periodic attractors to quasiperiodic ones, and often these are SNAs<sup>20</sup>.

The enhanced stability that results from strange nonchaotic dynamics is the principal motivation for study of quasiperiodically forced coupled maps. The large number of possible parameters makes exploration of such systems difficult, and as a result we choose very specific (and restricted) parameters for coupling and forcing. It should also be pointed out that here we are interested in the detailed behaviour of small numbers of coupled maps, as opposed to studies that examine large coupled map lattices<sup>21</sup>. The behaviour of two or three such coupled maps have been extensively studied<sup>11,22–26</sup>. The effect of quasiperiodic forcing on

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