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Research Paper

Suppression of Wall Turbulence by Stability and Turbulence Analysis Using a Compliant Surface

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This paper discusses the role of hydrodynamic stability theory in understanding wall turbulence and its possible suppression by using compliant surfaces. Our work reveals that, in wall turbulent flows, there are three important 'mode classes'; namely, the Tollmien-Schlichting (TS) mode class, the Static- Divergence (SD) mode class, and the high-speed highly damped (HSHD) mode class. All these modes scale with inner wall variables (with wave speed, C_r, close to 0.3) and so do the material properties of the compliant surface. The general thrust should be to replace TS modes by HSHD stable modes. Outer modes (with C_r, close to 1.0) were also investigated and found to be damped. Comparative analysis of the present work was established with the experimental results of Gad-el-Hak et al. (1985), Veeravalli, Sen & Joshi (2006) and theoretical analysis of Yeo (1988).

Key Words: Wall turbulence, instability, compliant wall, TS mode.

Introduction

Hydrodynamic stability theory and the results based on experiments have provided ample scope for research over the past several years. As a result of intensive research, we now have extensive and in-depth details on laminar-to-turbulent transition. The question regarding the connection between hydrodynamic stability and turbulence itself has often been asked. However, there has not been any definite answer to this question.

One significant aspect that emerges is that hydrodynamic stability theory plays an important role in understanding free turbulent shear flow. The prominent coherent structure in free turbulent shear flow can be obtained from an inviscid instability analysis of the turbulent mean-velocity profile, which is inflectional in the cases of free turbulent shear flows. Several authors working on the turbulent mixing layer have confirmed the above finding. Furthermore, a more recent overview by Roshko [1] on stability and turbulent shear flow also confirms the above findings. However, for wall turbulence, no such simple connection has been found.

Sen and Veeravalli [2&3] have re-examined the question of a connection between instability and wall turbulence, and have obtained an improved theoretical model for the problem, with the turbulence stress tensor, modeled similarly as in Pope [4]. This model gives further extensions of the Orr-Sommerfeld equation over what Reynolds and Hussain [5] obtained. The results show that a very strong unstable wall mode exists over a wide range of the spatial wave number α. The instability characteristics scale very well with the inner variables of the turbulent flow and are virtually independent of the outer conditions. Therefore, the results are quite universal for wall turbulence and depend very little on the specific geometry of the problem. Extensive numerical computation has been done both for turbulent boundary-layer flow and channel flow to confirm this. Sen and Veeravalli [2&3] have concluded that organized disturbances leading to hydrodynamic instability modes may have a very definite connection with wall turbulence. Their experiments also confirm the theoretical finding of a turbulence-generating mechanism, which they called "*a possible root cause mechanism*" of turbulence in wall-bounded shear flows. Assuming that the findings of Sen and Veeravalli do actually correspond to "*the root cause mechanism*" of turbulence, one of the possible methods of suppressing these instabilities, and therefore the "*root cause* mechanism", could be the use of compliant surfaces.

The use of compliant surfaces has been widely proposed in the past to delay laminar to turbulent transition. The present work discusses the problem of wall turbulence and its possible suppression by using compliant surfaces. We have used two types of wall models, in the lines of those used by Carpenter and Garrad [6] and Sen & Arora [7]. The inverse methods as discussed in Sen & Arora [7] was used both for selecting the parameters for stabilisation of the flow, and also, for investigating physical realisability of the predicted modes. The results obtained for the rigid wall case with our new simplified numerical scheme, are similar to the ones reported in the Sen and

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Veeravalli [2&3] papers. Numerical computation was also done for outer modes. However, they do not scale with inner variables and are invariably stable. Thereafter, material selection for the compliant surface was carried out with an illustrative practical problem.

Preliminary results of the present work have been presented in brief paper by Sen, Josan and Veeravalli [8].

Formulation of the Problem

In the present study the formulation is based on a combined fluid-solid problem. This requires the separate specification of the dynamics of the fluid side and that of the solid side, and matching the two at the interface.

Theory

We now discuss briefly the Sen and Veeravalli theory (Details may be seen in Sen and Veeravalli [2&3]). In the discussion to follow the instantaneous velocity vector u_i, and pressure p, obey the Navier-Stokes and continuity equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \qquad \ldots(1a)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad \ldots(1b)$$

The velocity and pressure fields are decomposed in turbulent flows by the well-known Reynolds decomposition; typically,

$$u_i = \overline{u}_i + u_i' \;;\; p = \overline{p} + p' \qquad \ldots(2)$$

Here $\overline{u}_i$, $\overline{p}$ are respectively the mean velocity and pressure, and u'_i, p' are the (random) turbulent fluctuations. If we now superpose an organised (solenoidal) disturbance $\tilde{u}_i$, $\tilde{p}$ (with zero mean), the instantaneous velocity and pressure are respectively given as follows:

$$u_i = \overline{u}_i + \tilde{u}_i + u_i' \;;\; p = \overline{p} + \tilde{p} + p' \qquad \ldots(3a)$$

The time averages of u_i, p are still respectively $\overline{u}_i$, $\overline{p}$, but, the ensemble (phase- locked) averages are respectively:

$$\langle u_i \rangle = \overline{u}_i + \tilde{u}_i \;;\; \langle p \rangle = \overline{p} + \tilde{p}, \qquad \ldots(3b)$$

and are different. Moreover, the organised disturbance is assumed to be small, or linear in the following sense:

$$\left|\langle \tilde{u}_i \tilde{u}_j \rangle\right| << \left|\langle u_i' u_j' \rangle\right| \qquad \ldots(4)$$

After some algebra, one is in a position to obtain the linearised dynamic equation for the organised disturbance,

$$\frac{\partial \tilde{u}_i}{\partial t} + \overline{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} + \tilde{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} + \frac{\partial \tilde{r}_{ij}}{\partial x_j}, \quad \ldots(5)$$

where $\tilde{r}_{ij}$ is the modulation in the Reynolds stress tensor, given by

$$\tilde{r}_{ij} = -\left(\langle u_i' u_j' \rangle - \overline{u_i' u_j'}\right) \qquad \ldots(6)$$

We now introduce (twice) the rate of strain tensor and (twice) the vorticity tensor, respectively as s_{ij} *and* ω_{ij} which are given below:

$$s_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}; \quad \omega_{ij} = \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \qquad \ldots(7a,b)$$

The mean and ensemble averages of the above quantities are $\overline{s}_{ij}$, $\overline{\omega}_{ij}$ and $\langle s_{ij} \rangle$, $\langle \omega_{ij} \rangle$ respectively.

The expression for $\tilde{r}_{ij}$, may be written as (see details in Sen & Veeravalli [2&3]):

$$\tilde{r}_{ij} = \varepsilon \tilde{s}_{ij} - \varepsilon \left(\frac{\lambda}{(\overline{u})'}\right) \left[\frac{1}{2}\left\{\overline{\omega}_{ik}\tilde{s}_{kj} + \tilde{\omega}_{ik}\overline{s}_{kj} - \overline{s}_{ik}\tilde{\omega}_{kj} - \tilde{s}_{ik}\overline{\omega}_{kj}\right\}\right] \qquad \ldots(8)$$

In equation 8 above, ε, is the eddy viscosity and, λ, is a parameter that characterizes the anisotropy of the turbulence near the wall.

We next look at the stability equation corresponding to organised disturbances. The mean flow is assumed to be parallel, or 'quasi-parallel', and two-dimensional. The disturbance equation (5) for $\tilde{u}_i$, with $\tilde{r}_{ij}$ given by (8), sets the framework for obtaining normal-mode solutions. As already mentioned, two-dimensional disturbances are considered here, which leads to an extended form of the Orr-Sommerfeld equation, to be discussed later.

To continue with the formulation, a stream function ψ is introduced for the organised disturbances, such that $\tilde{u} = \partial\psi/\partial y$ and $\tilde{v} = -\partial\psi/\partial x$. After assuming normal modes, ψ may be expressed in the form $\psi = \phi(y)e^{i\alpha(x-ct)}$, where α is the temporal wave number and $c = c_r + ic$ is the (complex) wave speed. For unstable modes $c_i > 0$, introducing ψ in the evolution equation (5) for $\tilde{u}_i$, and remembering the quasi-parallel approximation for the mean flow and the closure equations for $\tilde{r}_{ij}$, one arrives at extended forms of the Orr-Sommerfeld equation, given below:

$$i\alpha[(\overline{u} - c)(\phi'' - \alpha^2\phi) - \overline{u}''\phi]$$
$$-1/R[\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi]$$

$$-1/R[E\{\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi\} + 2E'\{\phi''' - \alpha^2\phi'\}$$

$$+E''\{\phi''+\alpha^2\phi\}]-\frac{\lambda E}{R}[-2i\alpha\phi''''+2i\alpha^3\phi']$$

$$-\frac{2i\alpha\phi'}{R}[\lambda E''+2\lambda' E'+\lambda'' E]=0 \qquad \ldots(9)$$

Here $(')$ denotes differentiation with respect to *y*. All quantities in eq. (9) have been non-dimensionalised by outer variables. However, in eq. (9) the eddy viscosity has been non-dimensionalised by molecular viscosity ν and is denoted by *E*. Expressions for λ and *E* may be seen in Sen and Veeravalli [2&3].

The unstable modes obtained are wall modes, which are found to scale nearly perfectly with inner scales. Therefore eq. (9) may also be written in terms of inner variable using the friction velocity u_* as the velocity scale and ν / u_* as the length scale. We denote quantities that are non-dimensionalised by the inner scales with the superscript (+). The scheme of non-dimensionalisation is as follows:

$$\lambda=\lambda^+ ; \quad E=E^+; \quad B\equiv\frac{u_*^2}{U_\infty^2};$$

$$y^+=y\big/R\sqrt{B} ;$$

$$u^+=\bar{u}/u_* ; \quad \alpha^+=\alpha\big/R\sqrt{B} ,$$

where *B* is the non-dimensional wall shear stress. Now, equation (9) becomes:

$$i\alpha^+[(\bar{u}^+-c^+)(\phi''-\alpha^{+2}\phi)-\bar{u}^{+''}\phi]-[\phi''''-2\alpha^{+2}\phi''+\alpha^{+4}\phi]$$

$$-[E^+\{\phi''''-2\alpha^{+2}\phi''+\alpha^{+4}\phi\}+2E^{+'}\ \{\phi'''-\alpha^{+2}\phi'\}$$

$$+E^{+''}\{\phi''+\alpha^{+2}\phi\}]-\lambda^+E^+[-2i\alpha^+\phi'''+2i\alpha^{+3}\phi']$$

$$-2i\alpha^+\phi'[\lambda^+E^{+''}+2\lambda^{+'}\ E^{+'}\ +\lambda^{+''}E^+]=0 \qquad \ldots(10)$$

Equation (10) is in a general form applicable to all wall modes, because the Reynolds number becomes unity in inner variables; only the eddy viscosity *E* has a very weak outer dependence particularly for the inner modes. It has been seen by our present numerical calculations that wall-modes are confined to the inner wall region.

Boundary Conditions

Having formulated the stability equation, we next look at the boundary conditions for channel-flow and boundary-layer flow.

(i) Rigid Wall

At the wall $y = 0$ $\quad \phi, \phi' = 0,$ …(11a)

At channel center line $y = 1$ $\quad \phi, \phi''' = 0,$ …(11b)

At boundary-layer edge $y \geq 1$ $\quad \phi \approx e^{-\alpha y}$ …(11c)

However, in the present work, for inner wall modes, we apply the outer boundary condition at $y \cong 0.3$, using a local solution of the Rayleigh equation. This is because the inner modes decay very rapidly for $y \geq 0.3$, and thus it is superfluous to apply the outer boundary conditions at *y* = 1. This is also the reason why inner modes, for both channel flow and boundary-layer flow are virtually identical, and so also for all cases of wall turbulence.

(ii) Compliant Wall

For the compliant wall the boundary conditions at *y* = 0, after linearisation, are $\phi(0)=ac$ and $\phi'(0) =-a\ u_w'$, where subscript *w* refers to the wall, and, *a* is a non-normalised form of the amplitude of the wall displacement. Substituting $a = \phi_w/c$ we get

$$c\phi_w'+u_w'\phi_w \ = \ 0 \ \text{at}\ y=0 \qquad \ldots(12)$$

The second boundary condition is obtained by equating the wall pressures, or a pressure-derived response coefficient, like admittance *Y*, calculated from the fluid side and from the solid side:

$$Y = -\ \frac{i\,\alpha\,\phi_w}{\hat{p}_w} \quad ;\ \text{at}\ y=0, \qquad \ldots(13)$$

where $\hat{p}_w$ is the amplitude of wall pressure defined as

$$\hat{p}_w=-\frac{i}{\alpha R}(\phi_w'''-\alpha^2\phi_w') \qquad \ldots(14)$$

We next consider the solid side. The plate - spring model of Carpenter and Garrad [6] is used for the dynamics of the compliant wall, which is, accordingly, governed by the following equation

$$m\frac{\partial^2\eta}{\partial t^2}+\bar{d}\,\frac{\partial\eta}{\partial t}-T\frac{\partial^2\eta}{\partial x^2}+B\frac{\partial^4\eta}{\partial x^4}+K\eta=\hat{p}_W\,, \qquad \ldots(15)$$

where the displacement η of the interface from its stationary or mean position is given as

$$\eta = ae^{i\alpha(x-ct)}, \qquad \ldots(16)$$

and $m,\ T, \bar{d}$ are respectively the mass per unit area, longitudinal tension per unit width and damping, in suitable non-dimensional form. Also *B* is the flexural-rigidity term, *K* is the spring stiffness term, also $d=\bar{d}\big/m$. The admittance from the solid side Y_0, is given as

$$Y_0=\frac{i\alpha}{m\alpha\left(\bar{c}_0^2-c^2-\dfrac{icd}{\alpha}\right)}, \qquad \ldots(17)$$

where $\bar{c}_0$ is the surface wave speed given as

$$\bar{c}_0^2=\frac{B\alpha^2}{m}+\frac{T}{m}+\frac{K}{m\alpha^2}. \qquad \ldots(18)$$

Thereafter the second boundary condition at the wall in the combined fluid-solid model is obtained by matching the admittance from the fluid side and that from the solid side, viz.

$$Y - Y_0 = 0 \quad \text{at} \quad y = 0. \qquad \ldots(19)$$

Results and Discussions

The instability analysis in turbulent flow using compliant surface was carried out both in channel-flow and boundary layer flow mainly for inner modes, and also for some outer modes; though, in the context of the present problem, the effect of outer modes is not relevant as these are all damped. The inverse method as discussed in Sen-Arora is used both for selecting the parameters for the stabilisation of flow, and also for investigating physical realisability of the predicted modes. The results show that there are two important inner 'mode classes', namely the Tollmien-Schlichting (TS) mode class and the Static Divergence (SD) or Kelvin Helmholtz (KH) mode class, present in the flow. It is the TS mode class, which is believed to be the so-called "root cause mechanism of turbulence", and, suppression of this mode class is desirable if turbulence is to be suppressed.

In the Sen-Arora method, the wall value of ϕ, viz. ϕ_w, is parameterized as $\bar{\phi}_w = |\bar{\phi}_w| e^{i\theta}$, where ϕ_w is the value of . ϕ at the wall subjected to some adopted normalization of ϕ in the outer region. In our present problem . ϕ is notionally normalized as . $\phi = 1+0i$ at channel centre line (i.e. at $y = 1$), or, at the outer edge of the boundary layer. Also $|\bar{\phi}_w|$ is called the "Kinematic Compliance parameter", and, ϕ is the phase. $|\bar{\phi}_w|$ is expressed herein as a percentage of the maximum value of . ϕ (obtained for the rigid wall) in the given range of y.

Extensive investigations have been carried out for both the modes. It is observed that for a compliant surface with kinematic compliance $|\bar{\phi}_w| > 70\%$, and with carefully selected material properties, the TS mode class ceases to exist, and bifurcates into a high-speed highly damped (HSHD) stable mode class. Further, for such a surface, the unstable SD modes also do not appear, if material properties are carefully selected.

Typical eigenfunctions for the inner modes, for four different Reynolds numbers (5000, 6000, 7000 and 10000) for channel-flow and 6250 for boundary layer flow (corresponding approximately to the channel-flow R =5000) were plotted. The near indenticality of the two sets of curves speaks strongly in favour of universality in inner variables (for details see Josan thesis [9]). Figure 1 shows the growth rate of the organised disturbance $\alpha^+ c_i^+$, for the inner mode, as a function of the wave number α^+, in the rigid wall case. Since the unstable modes are wall modes, we find that the curves, in inner variables, collapse into one curve for all the four Reynolds numbers, considered here. Hence, α^+, and not α, is a natural parameter of the problem, and, the *Reynolds number R is not a significant parameter of the problem*. The typical unstable values of α^+ for which extensive numerical computation have been carried out correspond to modes I to VII as shown in figure 1.

The curves of real and imaginary wave speed c_r^+, c_i^+; real and imaginary admittance Y_r^+, Y_i^+ surface wave speed c_0^{2+} and damping coefficient d^+, when plotted against phase angle θ, with inner variable scaling, for any particular mode, appear to be independent of the Reynolds number for a given set of values of α^+, m^+ and $|\bar{\phi}_w|$. These curves are applicable to both channel flow and boundary layer flow. A typical set of curves of c_r^+ and c_i^+ versus θ for the TS mode for α corresponding to mode class III is shown in figure 3.

It has been stated earlier that generally speaking, both TS modes and SD modes are present in the flow. When one varies the kinematic compliance from 1% to about 70%, for mode class I to mode class VI, the growth rate c_i^+ remains unstable (positive). A typical curve of c_i^+ for mode class III, for various values of $|\bar{\phi}_w|$ = 20%, 30%, 40% & 50%, is shown in figure 4. Also mode class VII ceases to exist for $|\bar{\phi}_w| \geq 50\%$; in the sense that the TS mode bifurcates into the high-speed highly damped (HSHD) stable mode. A typical curve of growth rate c_i^+ for the HSHD mode (mode VII) is shown in figure 5. We get negative values of c_i. The negative value of c_i thus indicates stability. Modes I – VI also show the same transition to the HSHD modes, for $|\bar{\phi}_w|$ ranging from 50% to 80%. As a general norm it may be stated that bifurcation to the high-speed highly damped (HSHD) stable mode class occurs for a kinematic compliance $|\bar{\phi}_w| > 70\%$. However, SD modes can still be present in the flow, for $|\bar{\phi}_w|$ approaching zero and also for large values of $|\bar{\phi}_w|$.

Carefully selected and optimised material properties from the HSHD stable mode class, will ensure exclusion of most of the TS type modes

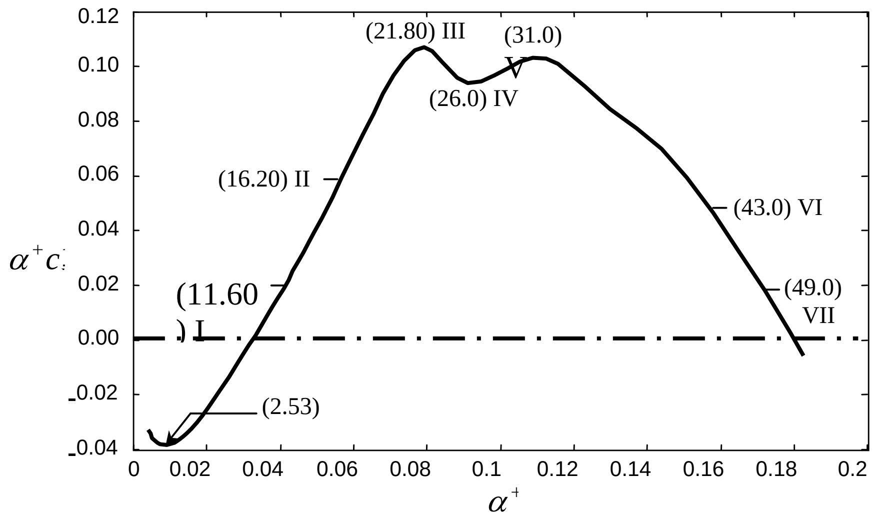

Fig. 1: *Growth curve + + i c a versus + a (inner variable scaling), R for channel flow is 5000. Figures in the bracket correspond to unstable wave numbers a for R=5000*

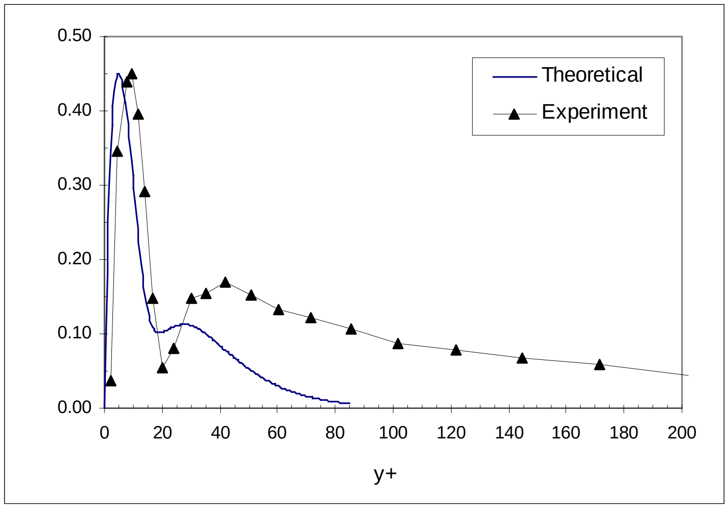

Fig. 2: *Graph of comparison of experimental and theoretical graphs of u~ versus y+ in channel flow. + a is nearly the same for both the curves.*

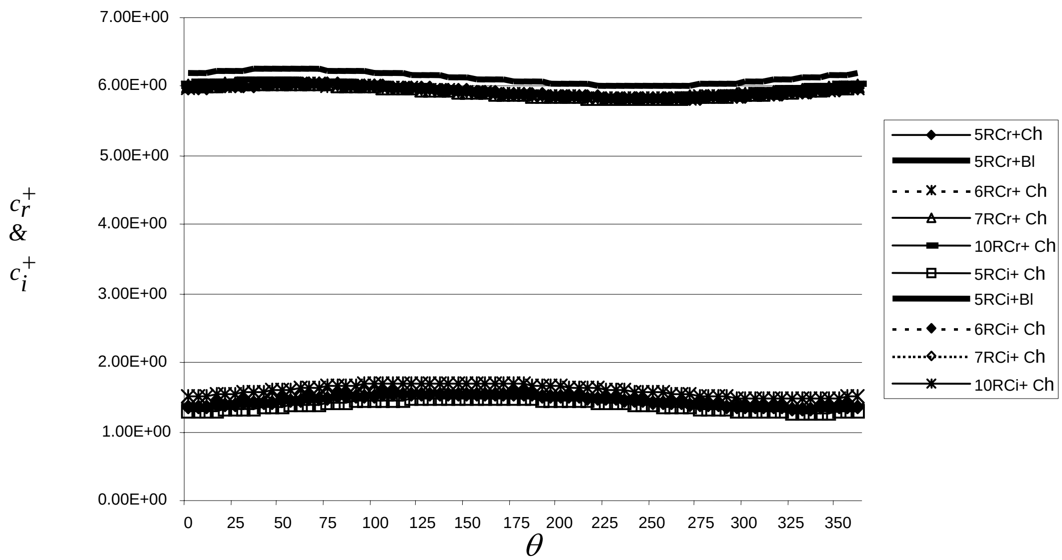

Fig. 3: *Graph of +r c and +i c versus q for channel flow and boundary layer flow for the TS mode, corresponding to mode class III with = q 10% 5RCr+Ch means R=5000, + r c channel flow*

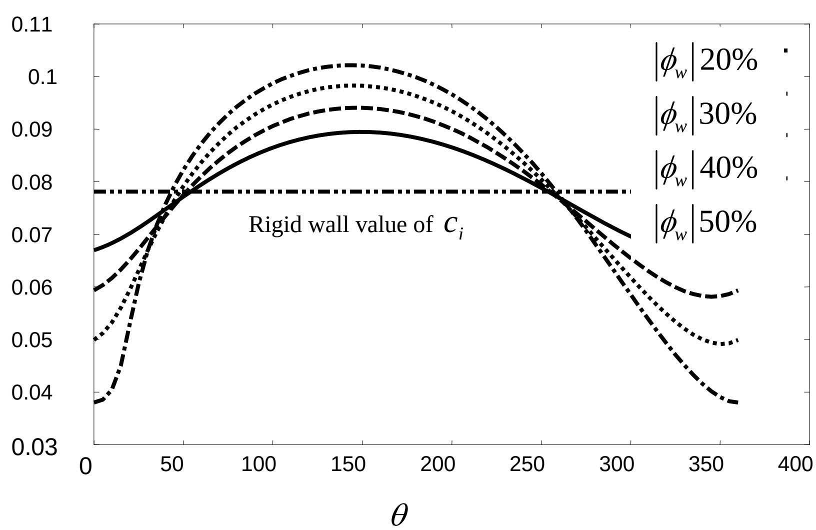

Fig. 4: *Graph for ci versus q for channel-flow for the TS mode, corresponding to mode class III for R=5000, and for w f = 20, 30%, 40% & 50%. Minimum value of ci = 3.8023 E-02 at w f = 50%*

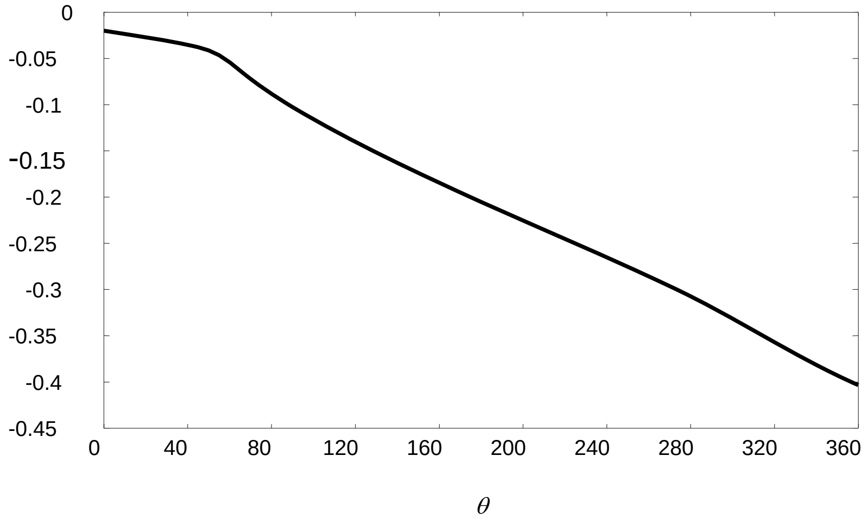

Fig. 5: *Graph for versus + i c for channel-flow in HSHD stable mode class, corresponding to mode class VII for R = 5000 and with w f = 50%. Minimum value of c_i = –0.4035E+00*

including suppression of many unstable or marginally stable or marginally damped oblique modes. Further, this would result in suppression of regeneration process in the near-wall region. Such a choice will also suppress or control the SD modes if material properties are carefully selected by ensuing that the SD modes are damped. Thus, it could result in substantial control of wall turbulence. With reference to figure 6 and 7, it is seen that K^+ and $\bar{d}^+$ may be selected corresponding to HSHD mode class, which is always stable. For example if from figure 6, we select K^+ = 0.2081E+03 at m^+ = 284.97, $|\bar{\phi}_w|$ = 70% and θ = 355^o. The corresponding value of $\bar{d}^+$ from figure 7 at the same m^+, and the same kinematic compliance $|\bar{\phi}_w|$ and θ, is 0.2797E+03. If we look at figure 6, and draw a horizontal line at K^+ = 0.2081E+03, it will cut a typical TS mode (and its neighbouring sets, which are not shown in the figure) having m^+ = 284.97 and $|\bar{\phi}_w|$ = 1% at about θ = 225^o. From figure 6, this typical TS mode, at 02 m^+ = 284.97 and $|\bar{\phi}_w|$ = 1% at about θ = 225^o has $\bar{d}^+$ = 0.800E+03. But, we have already selected $\bar{d}^+$ = 0.2797E+03 with a corresponding K^+= 0.2081E+03. We conclude from the above analysis that this typical TS mode (and its neighbouring sets, which are not shown in the figure) will not be present. Such a choice will also eliminate SD modes, which are well below the HSHD modes. Nevertheless these are not "carefully optimised values" with respect to the entire unstable range of α^+, although it shows suppression of mode class III.

Comparison with other Works

Gad-el-Hak et al. [10] have reported the existence of unstable static divergence waves experimentally, and

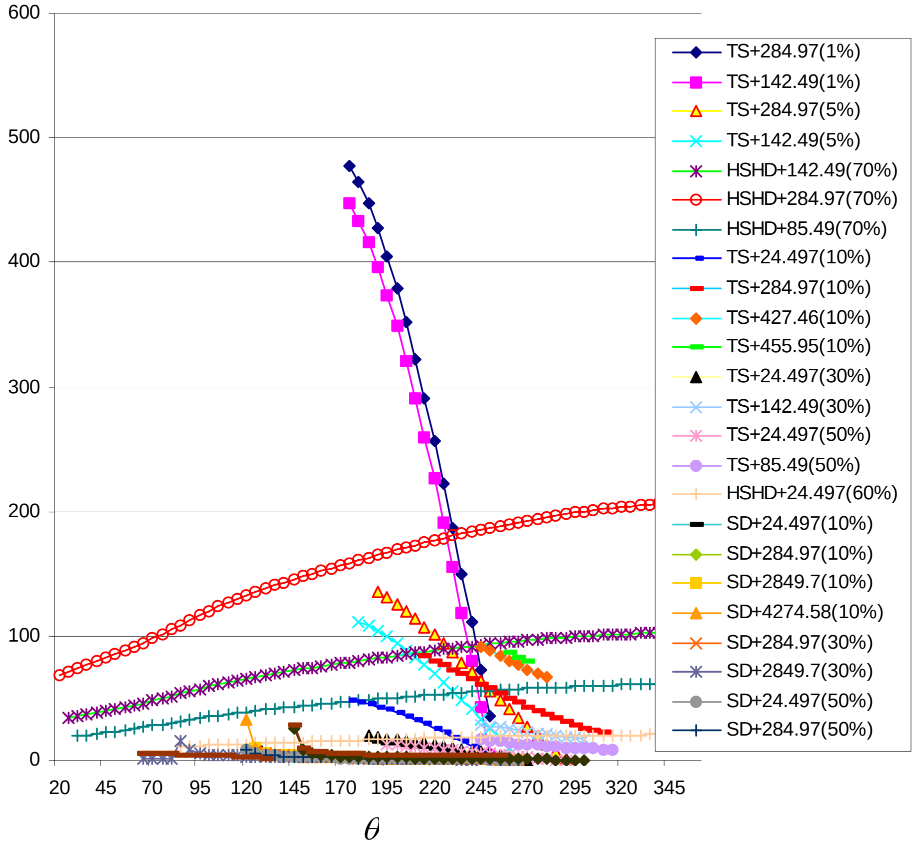

Fig. 6: *Graph of K_+ and versus q for the TS and SD modes, corresponding to mode class III with $_w f$ ranging from 1% to 70% and with m_+ ranging from 28.497 to 4274.58 (corresponding to m = 0.1 tp 15.0 at R = 5000). Key to legend: TS+105.44 (50%) means values of K_+ in TS mode, m_+=105.44 and $_w f$ = 50% or HSHD+284.97(70%) means values of K_+ in HSHD mode, m_+=284.97 and $_w f$ =10%*

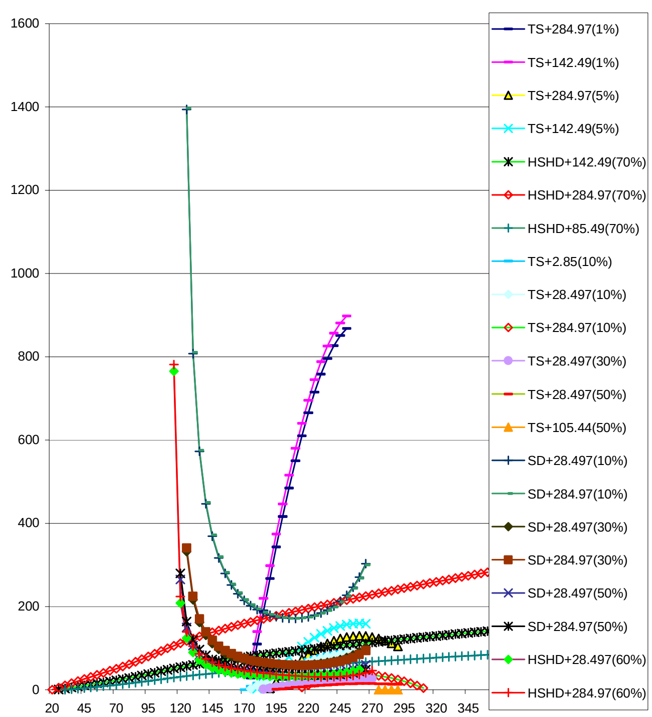

Fig. 7: *Graph of + d versus q the TS and SD modes, corresponding to mode class III with $_w f$ ranging from 1% to 70% and with m_+ ranging from 2.85 to 284.97 (corresponding to m = 0.01 to 1.0 at R=5000). Key to legend: SD+284.97 (50%) means values of + d in SD mode, m_+ =284.97 and $_w f$ = 50%, or HSHD+284.97(10%) means values of + d in HSHD Mode, m_+ = 284.97 and $_w f$ = 10%.*

these waves are also predicted in our theoretical investigation. The characteristics of unstable SD waves in the experimental model of [10] have been converted into inner variables (based on inferred or extrapolated) data given in Gad-el-Hak, and then compared with our non-dimensional wave number α^+ in inner variables. It was observed that the range obtained in the experiments is the same as that obtained in our theoretical analysis.

Further, there is no contradiction with respect to our theory, regarding the existence of SD modes in the turbulent boundary layer. Our theoretical results show that considerable overlap exists for our TS (so called "root cause mechanism of turbulence") and SD modes. So, SD modes can in general, definitely exist in turbulent flow over a compliant wall. However, in the presence of SD modes local energy transfer to the wall can be very high and in practice can damage the compliant wall. Thus the presence of SD modes, even if they are damped, is not desirable and our wall parameters have to be chosen accordingly.

Experiments by Veeravalli, Sen and Joshi [11], in a turbulent channel with rigid wall, confirm the existence of the theoretically predicted wall modes. The plots for $\tilde{u}$ versus $y^+$are in remarkably good agreement with the experiments, as shown in figure 2. The wave number, wave speed and growth rate measured in the experiments agree very well with the theoretical predictions. The experiments therefore provide good confirmation of the existence of wall modes, for organised disturbances introduced in turbulent flow.

The study presented herein suggests that eliminating TS modes altogether, by bifurcation into the so-called "high-speed highly damped (HSHD) stable mode class" (discussed in the next section), would succeed in the elimination of TS-like three-dimensional modes as well. This in turn will also succeed in controlling the transient growth processes as well. This would result in suppression of the regeneration processes in the near wall region. It would also result in substantial control of wall turbulence. The present work does not however look at the longitudinal coherent structure called Klebanoff modes.

Conclusions

All the parameters corresponding to the inner mode viz. equivalent spring constant, K^+ and the damping coefficient $\bar{d}^+$ etc. are independent of Reynolds number R, for a given set of mass m^+, wave number α^+, and kinematic compliance $\bar{\phi}_w$. Outer modes do not scale with inner wall parameters and material properties of compliant wall. However, they are always damped, and irrelevant to the present problem. The experiments of Veeravalli, Sen and Joshi [11] provide sufficient evidence for the existence of TS Mode in the case of the rigid wall. Good qualitative equivalences are also found with the experimental results of Gad-el-Hak et al. [10] and theoretical analysis of Yeo et al. [12].

We have shown that with a judicious choice of compliant wall parameters (which are physically realizable) the unstable TS waves can be eliminated completely and the flow can made to bifurcate to support only the benign HSHD modes. Further, it is anticipated that control of so-called two-dimensional TS mode would also result in the control of three-dimensional modes because these modes also belong to the oblique TS mode class defined by us. Thus this work has significant implications for turbulence control and drag reduction in open flows.

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