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The Multiple Facets of Correlation Functions

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I will review studies of fluctuations and correlations carried out over the last 20 years and hopefully provide some insights for new measurements.

Key Words : QGP; Correlation; Charge Fluctuation; Momentum Fluctuation

Introduction

Correlation observables are a key component of the toolset used by researchers in the study of the dynamics of nuclear collisions at high energy, and for measurements of the properties of the matter produced in these collisions. As such, correlation observables take a multitude of forms and provide access to a broad range of phenomena and nuclear matter properties (Fig. 1). While they seemingly take many different forms, correlation observables share a basic definition and properties. They can be broadly divided into integral and differential correlation functions, which may be averaged over all interactions measured for a given type of collisions, or studied as function of global event observables such as the total transverse energy, or the charged particle multiplicity measured in a specific kinematic range. Differential correlations can be studied as function of selected kinematical variables of two or more particles and integral correlations are well suited towards the study of fluctuations. In these proceedings, I review advances in measurements of several types of correlation and fluctuation observables towards the identification and study of the quark gluon plasma (QGP) produced in relativistic heavy ion collisions at the CERN SPS, RHIC, and the LHC. I first introduce the basic definition of correlation observables in §. I then proceed to review a number of measurements that were designed to identify and study the properties of the QGP, including net charge fluctuations in §, the charge balance function in §, higher moments of charge fluctuations in §, measurements of K/π yield

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fluctuations in §, and p_T fluctuations in §. I conclude with a brief assessment of the results and impact of these various measurements in §.

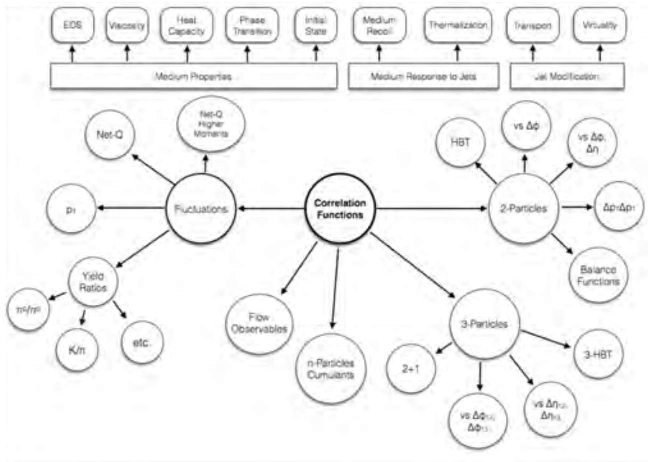

Fig. 1: The multiple facets of correlation functions: Differential and integral correlations have a common ground in terms of cumulants, and enable the definition of several types of correlation observables

Definitions

Measurements of correlations and fluctuations are rooted in the notion of joint probability distribution and multi-particle density. For instance, two random variables x and y can be said to be statistically independent if their joint probability distribution, $P(x, y)$, can be shown to factorize, i.e. if $P(x, y) = P(x)P(y)$ where $P(x)$ and $P(y)$ are the probabilities of observing x and y independently of each other. In the context of measurements of particle production in nuclear collisions, random variables of interest are the particle yield or particle density at different positions in momentum space. One is in particular interested in measuring whether the yield of particle production at a specific momentum (or range of coordinates) is correlated to the production at another momentum or range of coordinates. Loosely speaking, such a measurement can be accomplished by measuring the covariance of the two yields. But since particle densities are in fact functions of the momentum coordinates, one is led to the introduction of correlation functions which compare the two-particle density, $\rho_2(\vec{p}_1, \vec{p}_2)$, expressing the number of particle pairs at two specific momenta (or into two ranges) with the product of the single particle densities $\rho_1(\vec{p}_1)$ and $\rho_1(\vec{p}_2)$. Such correlation functions may be expressed as a difference or a ratio as follows:

$$\begin{aligned} C_{\alpha\beta}(\vec{p}_\alpha, \vec{p}_\beta) &= \rho_2(\vec{p}_\alpha, \vec{p}_\beta) - \rho_1(\vec{p}_\alpha) \times \rho_1(\vec{p}_\beta) \\ R_{\alpha\beta}(\vec{p}_\alpha, \vec{p}_\beta) &= \frac{C_{\alpha\beta}(\vec{p}_\alpha, \vec{p}_\beta)}{\rho_1(\vec{p}_\alpha) \times \rho_1(\vec{p}_\beta)} = \frac{\rho_2(\vec{p}_\alpha, \vec{p}_\beta)}{\rho_1(\vec{p}_\alpha) \times \rho_1(\vec{p}_\beta)} - 1. \end{aligned} \quad (1)$$

Generalization to three or more particles is readily achieved with the use of cumulants. Integration of single, two-, and n-particle densities over a finite momentum volume yield the average particle, $\langle N\rangle$, the average number of particle pairs, $\langle N(N-1)\rangle$, and factorial moments, $\langle N(N-1)\cdots(N-n)\rangle$ in that volume. Given the variance of the particle yield $\langle \Delta N^2\rangle = \langle N^2\rangle - \langle N\rangle^2$ may also be written $\langle \Delta N^2\rangle = \langle N(N-1)\rangle - \langle N\rangle^2 + \langle N\rangle$, one finds that integrals of the correlation functions C_2 or R_2 provide measures of particle fluctuations. Fluctuation observables based on R_2, in particular, offer the advantage that they are robust (i.e. independent) against particle loses associated with detection efficiencies, whereas $\langle \Delta N^2\rangle$ has a non trivial dependence on efficiencies.

Net Charge Fluctuations

In the late 1990s, early 2000s, Koch *et al.* (Jeon and Koch, 1999, 2000; Koch *et al.*, 2002; Bleicher *et al.*, 2000), Asakawa *et al.*, (Asakawa *et al.*, 2000), Heiselberg *et al.*, (Heiselberg, 2001; Baym and Heiselberg 1999), and several others (Shuryak, 1998; Shuryak and Stephanov, 2011; Aziz and Gavin, 2004; Zhang *et al.*, 2002; Bialas, 2002; Bopp and Ranft, 2001) elaborated on the notion that net charge fluctuations, i.e., fluctuations of the net charge of particles measured in a fixed momentum volume, might be severely suppressed if a quark gluon plasma phase is produced in the midst of heavy ion collisions, relative to fluctuations in proton-proton collisions, or a resonance gas. In particular, Koch *et al.*, (Jeon and Koch, 2000) predicted the observable $D \equiv \langle N_{ch}\rangle\langle \Delta R^2\rangle$ would span different ranges of values for a free gas of particles ($D \approx 4$), a resonance gas ($D \approx 3$), and a quark gluon plasma ($D \approx 1$). The observable D then emerged as an essential tool to determine whether a QGP is in fact produced in relativistic heavy ion collisions, and several experimental collaborations pursued its measurement at the CERN SPS, at RHIC, and eventually at the LHC. The problem initially arose that different groups chose to carry out measurements of net charge fluctuations using different and seemingly incompatible observables. For instance, the NA49 collaboration reported measurements using the Φ_q observable, while PHENIX and STAR used $\omega \equiv \langle \Delta Q^2\rangle/\langle N_{ch}\rangle$ and $\nu_{+-,dyn}$ respectively (Alt *et al.*, 2004; Adcox *et al.*, 2002a; Adams *et al.*, 2003a; Abelev *et al.*, 2009a). Fortunately, there exist a relatively simple one-to-one relationships between these observables (Pruneau *et al.*, 2003). The $\nu_{+-,dyn}$ observable involves the advantage of being defined in terms of ratios of two particle correlators to product of single particle yields, and is thus robust against particle losses due to detection efficiencies.

$$\begin{aligned} \nu_{+-,dyn} &= \frac{\langle N_+(N_+-1\rangle)}{\langle N_+\rangle^2} + \frac{\langle N_-(N_--1)\rangle}{\langle N_-\rangle^2} - 2\frac{\langle N_+N_-\rangle)}{\langle N_+\rangle\langle N_-\rangle} \\ &= R_{++} + R_{--} - 2R_{+-}. \end{aligned} \tag{2}$$

However, by construction, $\nu_{+-,dyn}$ is subject to a $1/m$ scaling in the presence of m independent particle sources. Measurements were thus reported both in terms of $\nu_{+-,dyn}$ and in terms of $N_{ch}\nu_{+-,dyn}$ where

N_{ch} is total number of charged particles measuring in the fiducial volume of experiments, corrected for detection efficiencies and instrumental effects. Fig. 2 presents measurements reported by STAR (Abelev *et al*., 2009a) for Au + Au and Cu + Cu collision systems at several beam energies as well as results by ALICE Abelev *et al*., 2013) for Pb + Pb and p + p collisions. One finds that $\nu_{+-,dyn}$ exhibits the anticipated $1/m$ scaling behavior for all collision systems and energies considered. However, the quantity $N_{ch}\nu_{+-,dyn}$ exhibits a clear dependence on collision centrality which indicate a change in the strength of the charged particle production correlation in central collisions. Both the STAR and ALICE measurements indicate a relative suppression of the fluctuations in central collisions. The effect is weaker at RHIC energies where the measured $N_{ch}\nu_{+-,dyn}$ observed in central collisions appears more or less consistent with a resonance gas, but quite a bit stronger at LHC energy hinting at a stronger suppression of net charge fluctuations. The interpretation of these data is however complicated by diffusion and radial flow effects (Shuryak, 1998; Aziz and Gavin, 2004).

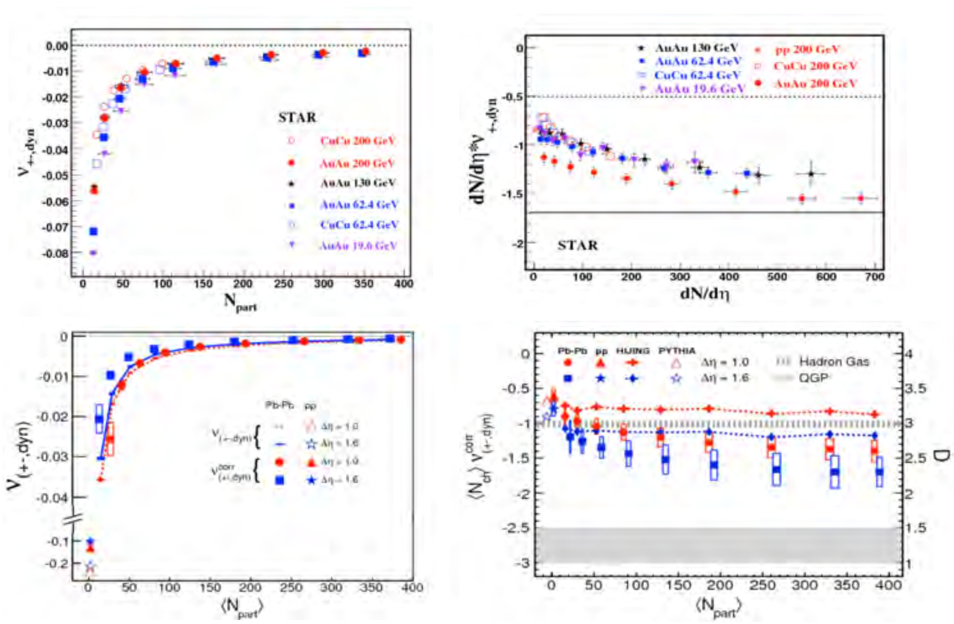

Fig. 2: Net charge fluctuations measurements by the STAR (Abelev ***et al*****., 2009a) and ALICE collaborations (Abelev** ***et al*****., 2013)**

Balance Function

The charge balance function is a differential correlation function closely related to measurements of net charge fluctuations. Pratt *et al.* proposed in (Bass *et al*., 2000; Jeon and Pratt, 2002) that a narrowing of the charge balance function, $B(\Delta\eta)$, measured as function of the pseudo rapidity acceptance, $\Delta\eta$, should signal delayed hadronization arising after the production of a "long-lived" QGP phase in relativistic heavy ion collisions.

$$B(\Delta\eta) = \frac{\langle N_- N_+ \rangle(\Delta\eta)}{\langle N_- \rangle} + \frac{\langle N_+ N_- \rangle(\Delta\eta)}{\langle N_+ \rangle} - \frac{\langle N_+(N_+ - 1)\rangle(\Delta\eta)}{\langle N_+ \rangle} - \frac{\langle N_-(N_- - 1)\rangle(\Delta\eta)}{\langle N_- \rangle} \quad (3)$$

At relativistic energies, $\langle N_- \rangle \approx \langle N_+ \rangle$, the balance function may then be expressed in term of integral correlators as follows:

$$B(\Delta\eta) = \langle N_{ch} \rangle R_{+-} - \langle N_+ \rangle R_{++} - \langle N_- \rangle R_{--} \quad (4)$$

STAR(Adams *et al.*, 2003b; Aggarwal *et al.*, 2010) has measured the balance function as function of $\Delta\eta$ in a variety of collision systems at several beam energies and for both inclusive and specific particle species (e.g. pions, kaons, etc). It reported a substantial narrowing of the balance function in central Au + Au collisions relative to widths measured in peripheral collisions, consistent with the notion of delayed hadronization. While radial flow may produce a slight narrowing of the balance function measured as function of $\Delta\eta$ (or $\Delta\phi$), the narrowing observed experimentally appears too strong to be explained solely on the basis of radial flow and is thus suggestive of the formation of a deconfined system with delayed particle production. ALICE (Abelev *et al.*, 2013) carried out similar measurements in the Pb + Pb system and obtained similar conclusions.

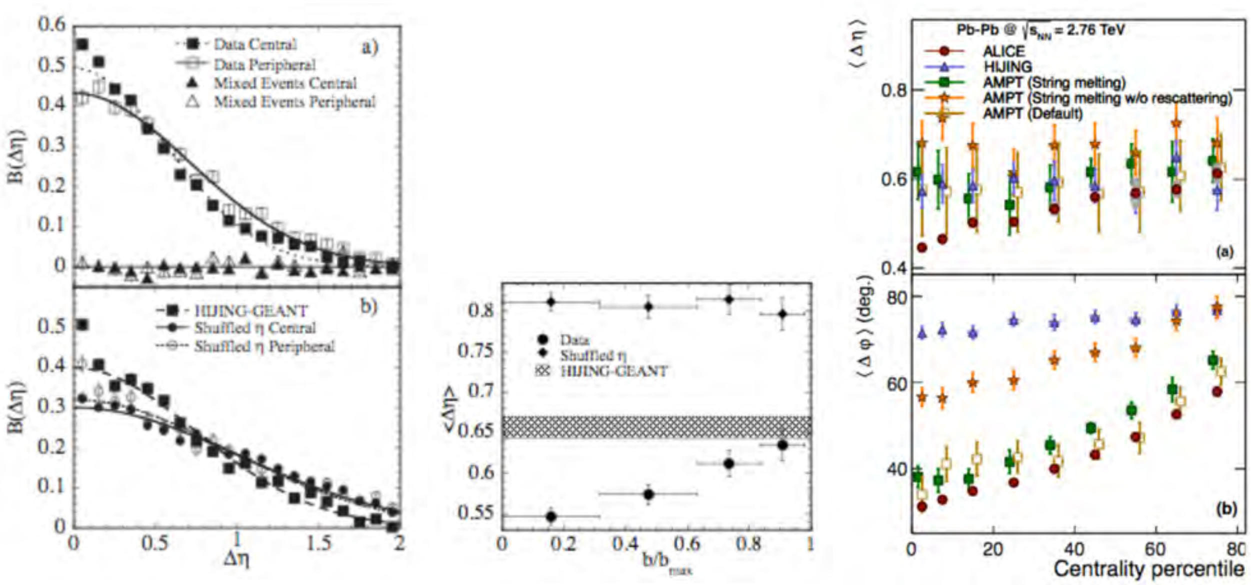

Fig. 3: Narrowing of the balance function observed by STAR (Adams *et al.*, 2003b) and ALICE (Abelev *et al.*, 2013)

Higher Moments of Net Charge Fluctuations

Theoretical studies suggest the phase diagram of nuclear matter may be rather complex. In particular, one expects that the transition from a hadron gas to a quark gluon plasma might be a cross over transition at very small net baryon density and a first order phase transition at large net baryon density. This thus suggests the existence of a critical point (CP) at the end of the first order phase boundary (Fodor and Katz, 2002; Stephanov, 2004, 2005; Fodor and Katz, 2004). The location of this CP is subject to much speculations and no theoretical consensus exists as to what its position might be. It might however be possible to locate the critical point experimentally based on the notion that the correlation length ξ is expected to diverge near CP in an infinite and static system. The systems formed in high energy heavy ion collisions are obviously not static or infinite, but various studies (Stephanov, 2009; Hatta and Stephanov, 2003; Karsch and Redlich, 2011; Rajagopal and Wilczek, 1993; Athanasiou *et al.*, 2010) indicate that it might nonetheless be possible to identify a noticeable increase in the correlation length through measurements of higher moments net charge fluctuations. The STAR collaboration has reported a measurement of the kurtosis of the net charge fluctuations produced in in Au + Au collisions at several beam energies (Aggarwal *et al.*, 2010; Luo *et al.*, 2011; Tarnowsky *et al.*, 2011). Measurements reported thus far indicate the kurtosis scaled by the variance is essentially independent of both collision centrality and beam energy. These measurements are unfortunately inconclusive because of limited statistics and issues involved in corrected for efficiency losses. It is also possible the RHIC beam energy scan has simply missed the relevant range of net baryon density. Better observational techniques and larger data samples over a wider span of beam energies are consequently needed before one decidedly concludes on the existence of a critical point through this line of investigation.

K to π Yield Ratio Fluctuations

The observation of a relatively sharp peak in the ratio of kaon to pion average yields, $\langle K^+\rangle/\langle\pi^+\rangle$, by the NA49 collaboration near $\sqrt{s_{NN}} = 7$ GeV generated a great deal of interest and theoretical studies. It was in particular suggested that this spike might signal the phase transition or perhaps even the presence of a critical point. The presence of a critical point should in principle engender abnormal fluctuations in the yield of kaons relative to the production of pions. Several measurements of fluctuations of ratio kaon to pion yields, $R = N_{K+}/N_{\pi+}$, were thus undertaken. NA49(Roland *et al.*, 2004; Alt *et al.*, 2009) reported results in terms of so called dynamical fluctuations consisting of the difference of the width of the R distribution measured in actual events relative to that obtained with mixed events. STAR(Abelev *et al.*, 2009b) based its analysis on the $\nu_{K,\pi,dyn}$ observable extended from measurements of net charge fluctuation measurements. Comparison of STAR and NA49 data, shown in Fig. 4, indicate the two data sets are difficult to reconcile, unfortunately. More robust data, with better kaon to pion separation, and much larger data samples are thus direly awaited.

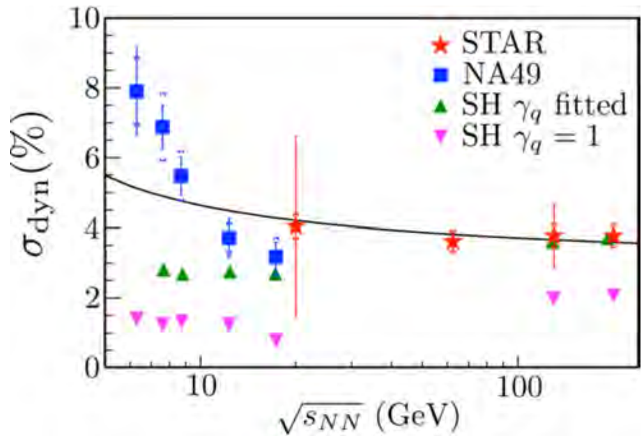

Fig. 4: Comparison of excess fluctuations of the kaon to pion fluctuations measured by the NA49 and STAR collaborations

Transverse Momentum Fluctuations

Transverse momentum fluctuations were suggested by Heiselberg (Heiselberg, 2001) and Stephanov *et al.* (Stephanov *et al.*, 1998) as a probe of phase instabilities near the QCD vs. hadron phase boundary. These speculations have motivated several measurements by the NA49(Appleshauser *et al.*, 1999). CERES (Adamova *et al.*, 2003), STAR (Pruneau, 2005; Adams *et al.*, 2005), PHENIX (Adcox *et al.*, 2002b), and ALICE Collaborations. As for net charge and K/π fluctuations, several distinct observables and methods can be used to measure transverse momentum fluctuations. Arguably, the simplest and most straightforward measurement method involves the $\langle \Delta p_T \Delta p_T \rangle$ observable first introduced by Voloshin (Voloshin *et al.*, 1999).

$$\langle \Delta p_T \Delta p_T \rangle = \frac{\int \rho_2(\vec{p}_{T,1}, \vec{p}_{T,2}) \Delta p_{T,1} \Delta p_{T,2} dp_{T,1} dp_{T,2}}{\int \rho_2(\vec{p}_{T,1}, \vec{p}_{T,2}) dp_{T,1} dp_{T,2}} \tag{5}$$

where $\Delta p_{T,i} \equiv p_{T,i} - \langle p_T \rangle$, and $\langle p_T \rangle$ is the inclusive transverse momentum average of measured particles. By construction, the correlation function $\langle \Delta p_T \Delta p_T \rangle$ scales inversely to the number of correlated particle sources, it is thus meaningful to examine the product $dN/d\eta \langle \Delta p_T \Delta p_T \rangle$ as reported by the STAR collaboration(Pruneau, 2005; Adams *et al.*, 2005). One finds that the scaled correlation strength has a very strong dependence on collision centrality thereby indicating that the systems formed in central collisions are considerably different than those produced in peripheral Au + Au collisions or in proton - proton collisions. The radial flow model formulated by Voloshin (Voloshin, 2006) indicate the collision centrality dependence observed by STAR is largely due to radial flow effects. Measurements by ALICE (Heckel, 2011) of $\langle \Delta p_T \Delta p_T \rangle$ scaled by p_T, shown in Fig. 5, however suggest the correlation function is in fact suppressed in most central collisions relative to most peripheral collisions. Considerable work remains to disentangle effects of collective radial flow and those potentially associated with the proximity of a phase boundary.

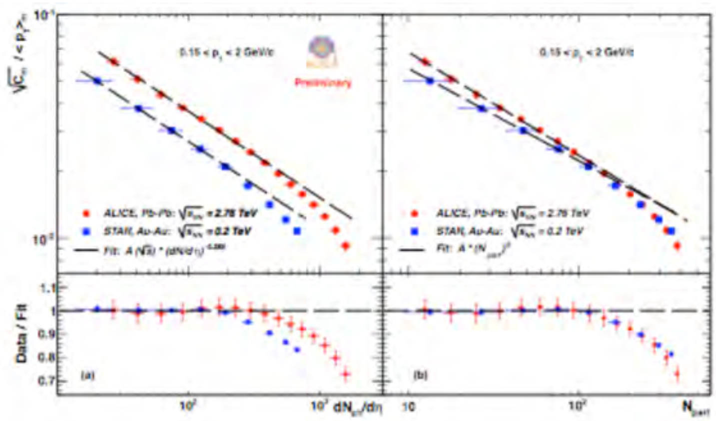

Fig. 5: Transverse Momentum Fluctuations Measured by the ALICE collaboration

Conclusions

Experiments at the CERN SPS, RHIC, and the LHC have conducted a wide range of correlation and fluctuation observables. While measurements of anisotropic flow coefficients lead to a rather unambiguous interpretation in terms of collective flow with low viscosity, considerably more work is yet required to reach a coherent picture based on measurements of fluctuation observables. Net charge fluctuation measured at RHIC and LHC show the systems produced in central collisions are clearly modified relative to those produced in peripheral or proton-proton collisions but the suppression of the fluctuations is not as large as that expected in the presence of a QGP phase. The suppression of the fluctuations is however larger at LHC possibly owing to the production of a more prominent QGP phase, smaller diffusion effects, or possibly more substantial radial flow effects. The identification of the QGP phase based on net charge fluctuation thus remains elusive. The observed narrowing of the balance function reported by both STAR and ALICE is, however, much more encouraging. Indeed, model calculations reported by Pratt indicate the observed narrowing cannot be explained on the basis of kinematic effects (radial flow) alone. The substantial narrowing observed is thus consistent with delayed hadronization, and the production of a deconfined QGP phase. Measurements of other fluctuations observables, such as higher moments of net charge, fluctuations of kaon vs pion yields, and transverse momentum fluctuations, though potentially promising, require further studies.

References

1. Abelev BI *et al.*, (2009a) Beam-Energy and System-Size Dependence of Dynamical Net Charge Fluctuations *Phys Rev* **C79** 024906

2. Abelev B *et al.*, (2013) Net-Charge Fluctuations in Pb-Pb collisions at $\sqrt{s_{NN}}$=2.76 TeV *Phys Rev Lett* **110** 152301
3. B Abelev *et al.*, (2013) Charge correlations using the balance function in Pb-Pb collisions at $\sqrt{s_{NN}}$= 2.76 TeV *Phys Lett* **B723** 267
4. Abelev BI *et al.*, (2009b) K/pi Fluctuations at Relativistic Energies *Phys Rev lett* **103** 092301
5. Adamova D *et al.*, (2003) Event by event fluctuations of the mean transverse momentum in 40, 80 and 158 A GeV / c Pb - Au collisions *Nucl Phys* **A727** 97
6. Adams J *et al.*, (2003) Multiplicity fluctuations in Au+Au collisions at $\sqrt{s_{NN}}$= 130 GeV *Phys Rev* **C68** 044905
7. Adams J *et al.*, (2003) Narrowing of the balance function with centrality in au + au collisions at $\sqrt{s_{NN}}$= 130 GeV *Phys Rev Lett* **90** 90
8. Adams J *et al.*, (2005) Event by event $< p(t) >$ fluctuations in Au - Au collisions at $\sqrt{s_{NN}}$ = 130 GeV *Phys Rev* **C71** 064906
9. Adcox K *et al.*, (2002a) Net charge fluctuations in Au+Au interactions at $\sqrt{s_{NN}}$ = 130 GeV *Phys Rev Lett* **89** 082301
10. Adcox A *et al.*, (2002b) Event-by-event fluctuations in mean p(T) and mean e(T) in $\sqrt{s_{NN}}$ = 130 GeV Au+Au collisions *Phys Rev* **C66** 024901
11. Alt C *et al.*, (2004) Electric charge fluctuations in central Pb+Pb collisions at 20 A GeV, 30 A GeV, 40 A GeV, 80 A GeV, and 158 A GeV *Phys Rev* **C70** 064903
12. Alt C *et al.*, (2009) Energy dependence of particle ratio fluctuations in central Pb + Pb collisions from s(NN)**(1/2) = 6.3 GeV to 17.3 GeV *Phys Rev* **C79** 044910
13. Aggarwal MM *et al.*, (2010) Balance Functions from Au+Au, d+Au, and p+p Collisions at $\sqrt{s_{NN}}$=200 GeV *Phys Rev* **C82** 024905
14. Aggarwal *et al.*, (2010) Higher Moments of Net-proton Multiplicity Distributions at RHIC *Phys Rev lett* **105** 022302
15. Appelshauser H *et al.*, (1999) Event-by-event fluctuations of average transverse momentum in central Pb + Pb collisions at 158 GeV per nucleon *Phys Lett* **B459** 679
16. Asakawa M, Heinz U and Muller B (2000) Fluctuation probes of quark deconfinement *Phys Rev Lett* **85** 2072
17. Athanasiou C, Rajagopal K and Stephanov M A (2010) Using Higher Moments of Fluctuations and their Ratios in the Search for the QCD Critical Point *Phys Rev* **D82** 074008
18. Aziz MA and Gavin S (2004) Causal diffusion and the survival of charge fluctuations in nuclear collisions *Phys Rev* **C70** 034905
19. Bass SA, Danielewicz P and Pratt S (2000) Clocking hadronization in relativistic heavy ion collisions with balance functions *Phys Rev Lett* **85** 2689
20. Baym G and Heiselberg H (1999) Event-by-event fluctuations in ultrarelativistic heavy ion collisions *Phys Lett* **B469** 7
21. Bialas A (2002) Charge fluctuations in a quark anti-quark system *Phys Lett* **B532** 249
22. Bleicher M, Jeon S and Koch V (2000) Event-by-event fluctuations of the charged particle ratio from nonequi-

librium transport theory *Phys Rev* **C62** 061902
23. Bopp FW and Ranft J (2001) Charged particle fluctuation as signal of the dynamics in heavy ion processes *Eur Phys J* **C22** 171
24. Fodor Z and Katz SD (2004) Critical point of QCD at finite T and μ, lattice results for physical quark masses *JHEP* **04** 050
25. Fodor Z and Katz SD (2002) Lattice determination of the critical point of QCD at finite T and mu *JHEP* **02** 014
26. Hatta Y and Stephanov MA (2003) Proton number fluctuation as a signal of the QCD critical endpoint *Phys Rev lett* **91** 102003
27. Heckel S (2011) Event-by-event mean p_T fluctuations in pp and Pb-Pb collisions measured by the ALICE experiment at the LHC *J Phys* **G38** 124095
28. Heiselberg H (2001) Event-by-event physics in relativistic heavy ion collisions *Phys Rept* **351** 161
29. Jeon S and Koch V (1999) Fluctuations of particle ratios and the abundance of hadronic resonances *Phys Rev Lett* **83** 5435
30. Jeon S and Koch V (2000) Charged particle ratio fluctuation as a signal for QGP *Phys Rev Lett* **85** 2076
31. Jeon S and Pratt S (2002) Balance functions, correlations, charge fluctuations and interferometry *Phys Rev* **C65** 044902
32. Karsch K and Redlich K (2011) Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations *Phys Lett* **B695** 136
33. Koch V, Bleicher M and Jeon S (2002) Event-by-event fluctuations and the QGP *Nucl Phys* **A698** 261
34. X Luo *et al.*, (2011) Probing the QCD Critical Point with Higher Moments of Net-proton Multiplicity Distributions *J Phys (Conference Series)* **316** 012003
35. Pratt S (2002) Balance functions: A signal of late-stage hadronization *Nucl Phys* **A698** 531
36. Pruneau CA (2005) Excitation function of $< p_t >$ and net charge fluctuations at RHIC *Acta Phys Hung* **A24** (2005) 85
37. Pruneau C, Gavin S and Voloshin S (2003) Net charge dynamic fluctuations *Nucl Phys* **A715** 661
38. Rajagopal K and Wilczek F (1993) Static and dynamic critical phenomena at a second order QCD phase transition *Nucl Phys* **B399** 395
39. Roland C *et al.*, (2004) Event by event fluctuations of particle ratios in central Pb+Pb collisions at 20 AGeV to 158 AGeV *J Phys* **G30** S1381
40. Shuryak EV (1998) Event per event analysis of heavy ion collisions and thermodynamical fluctuations *Phys Lett* **B423** 9
41. Shuryak EV and Stephanov MA (2001) When can long range charge fluctuations serve as a QGP signal? *Phys Rev* **C63** 064903
42. Stephanov MA (2004) QCD phase diagram and the critical point *Prog Theor Phys Suppl* **153** 139
43. Stephanov MA (2005) QCD phase diagram and the critical point *Int J Mod Phys* **A20** 4387
44. Stephanov MA (2009) Non-Gaussian fluctuations near the QCD critical point *Phys Rev lett* **102** 032301
45. Stephanov MA, Rajagopal K and Shuryak EV (1998) Signatures of the tricritical point in QCD *Phys Rev Lett*

81 4816

46. Tarnowsky TJ *et al*., (2011) Searching for the QCD Critical Point Using Particle Ratio Fluctuations and Higher Moments of Multiplicity Distributions *J Phys* **G38** 124054
47. Voloshin SA, Koch V and Ritter HG (1999) Event-by-event fluctuations in collective quantities *Phys Rev* **C60** 024901
48. Voloshin SA (2006) Transverse radial expansion in nuclear collisions and two particle correlations *Phys Lett* **B632** 490
49. Zhang QH, Pop VT, Jeon S and Gale C (2002) Charged particle ratio fluctuations and microscopic models of nuclear collisions *Phys Rev* **C66** 014909.