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$U_A(1)$ Breaking Effects and η' a t Finite Temperature

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In this talk proceeding, we will discuss the relation between chiral symmetry and the η' mass. We will also discuss whether the $U_A(1)$ breaking effect remains at high temperature.

Key Words : Chiral Symmetry; $U_{A(1)}$; Banks-Casher Formula

Introduction

The breaking and the possible restoration of a symmetry at finite temperature and/or density is a fascinating subject as the possible restoration in medium could be probed in relativistic heavy ion collision and/or in nuclear target experiments. The restoration of chiral symmetry has been linked to the vector meson spectral density and has been the subject of great theoretical and experimental interest up to this day (Hayano and Hatsuda, 2010; Leupold *et al*., 2010). It has also been linked to quenching of the pion decay constant (Jido *et al*., 2008) and possible observation of the sigma meson in nuclear matter through the $\pi - \pi$ correlation (Hatsuda and Kunihiro, 1985; Messchendorp *et al*., 2002). As for the $U_A(1)$ symmetry, its breaking is due to the anomaly, which induces an operator relation that remains broken above the QCD phase transition. However, the physical effect, such as the large η' mass, is intricately related to chiral symmetry breaking and the question of whether the mass will remain constant near the chiral symmetry restoration point is of particular interest as the partial quenching could be observed in nuclear target experiments (Nagahiro *et al*., 2009; Jido *et al*., 2012; Nagahiro *et al*., 2012, Nanova *et al*., 2012). In fact, a recent experimental observation by the CBELSA/TAPS collaboration of η' decaying into six photons inside a nuclear target finds that the effective optical potential is given as $V_0(\rho = \rho_0) = -(37 \pm (sta) \pm (syst))$ MeV (Nanova *et al*., 2013). Moreover, the two pion Bose-Einstein correlation observed at RHIC seems to suggest the quenching of the η' mass at high temperature (Csorgo *et al*., 2010; Vertesi *et al*., 2011, 2009).

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The talk is composed of two parts, we will first discuss the relation between chiral symmetry breaking effects and $U_A(1)$ breaking effects for a general number of color N_c in the correlation functions. This is accomplished through analysing the Banks-Casher formula (Banks and Cacher, 1980; Cohen 1996) and the contributions of the topologically non trivial configurations that depends on the number of flavors Lee, 1996; Evans *et al*., 1996). Next, we use the previous discussions together with the Wittten Veneziano (WV) formula (Witten, 1979; Veneziano, 1979) to obtain the relation between chiral symmetry restoration and the η' mass at finite temperature (Kwon *et al*., 2012; Lee and Cho, 2013).

Chiral Symmetry and U_A(1) Effects in Correlation Functions

Chiral Symmetry Breaking and Banks-Casher Formula

From a Euclidean path integral point of view, the origin of chiral symmetry breaking is the presence of zero eigen values for the Dirac equation in the presence of the gauge fields (Banks and Casher, 1980).

$$\begin{aligned} \langle \bar{q}q(0)\rangle &= -\langle \mathrm{Tr}[(0|\frac{1}{\not{D}+m}|0)]\rangle = -\pi\langle \mathrm{Tr}[J_{\lambda=0}(0,0)]\rangle \\ &= -\pi\langle \int \frac{d^4x}{V}\mathrm{Tr}[J_{\lambda=0}(x,x)]\rangle = -\pi\langle \rho(\lambda=0)\rangle, \end{aligned} \tag{1}$$

where we define a current density matrix of zero eigen values as follows:

$$J_{\lambda=0}(x,y) = \sum_{\lambda=0}\psi_\lambda(x)\psi_\lambda^\dagger(y). \tag{2}$$

Here, $i\not{D}\psi_\lambda = \lambda\psi_\lambda$, and the sum is over zero eigen values only. $\rho(\lambda)$ is the density of eigen value λ. Relation Eq. (1) shows that the essence of chiral symmetry breaking is the non-vanishing zero mode current density matrix $J_{\lambda=0}(x,y)$, which reduces to the scalar density of zero eigen values $\rho(\lambda=0)$ (Banks and Casher, 1980; Cohen, 1996) in Eq. (1).

One can subsequently show that the presence of non-vanishing density of zero eigen values is the unifying ingredient that dictates the breaking of chiral symmetry in these operators. Therefore, the same mechanism is at play when looking at other order parameters of chiral symmetry breaking. This is shown in the differences between meson two point functions of chiral partners.

$$\begin{aligned} \Delta^m_{S-P}(q) = &\int d^4x e^{iqx}\langle \mathcal{T}\Big(\bar{q}\tau^a q(x)\bar{q}\tau^a q(0) - \bar{q}\tau^a i\gamma^5 q(x)\bar{q}\tau^a i\gamma^5 q(0)\Big)\rangle \\ \xrightarrow{m\to 0} &\quad \int d^4x e^{iqx}\langle \mathrm{Tr}\Big[-\tau^2(0|\tfrac{1}{\not{D}+m}|x)\times J_0(x,0)\Big(2\pi\Big)\Big]\rangle, \end{aligned} \tag{3}$$

$$\begin{aligned}\Delta^m_{V-A}(q) = & \int d^4x e^{iqx} \langle \mathcal{T}\Big(\bar{q}\tau^a\gamma_\mu q(x)\bar{q}\tau^a\gamma_\mu q(0) - \bar{q}\tau^a\gamma^5\gamma_\mu q(x)\bar{q}\tau^a\gamma^5\gamma_\mu q(0)\Big)\rangle \\ \stackrel{m\to 0}{\longrightarrow} & \int d^4x e^{iqx} \langle \mathrm{Tr}\Big[-\tau^2\gamma_\mu(0|\tfrac{1}{\not{D}+m}|x)\gamma_\mu J_0(x,0)\Big(2\pi\Big)\Big]\rangle. \end{aligned} \tag{4}$$

Therefore, the difference vanishes when the current density matrix of zero modes J_0 vanishes as in Eq. (1). It should be noted that the argument of the wave function are identical in Eq. (1), while it is not so in Eq. (3), the difference in x will introduce a correlation length that is not of interest at this point.

If there is no flavor matrix in the scalar part of Eq. (3), such as in the SU(2) case, disconnected diagrams will contribute to the correlation function.

$$\begin{aligned}\Delta^m_{S-P}(q) &= \int d^4x e^{iqx} \langle \mathcal{T}\Big(\bar{q}q(x)\bar{q}q(0) - \bar{q}\tau^a i\gamma^5 q(x)\bar{q}\tau^a i\gamma^5 q(0)\Big)\rangle \\ &\stackrel{m\to 0}{\longrightarrow} \int d^4x e^{iqx} \langle \mathrm{Tr}\Big[\pi J_0(x,x)\Big]\mathrm{Tr}\Big[\pi J_0(0,0)\Big] + . \end{aligned} \tag{5}$$

Hence, the disconnected part also depends on J_0 (Cohen, 1996). Same goes for the difference in correlation functions involving baryons or mesons with one heavy quark (Lee and Cho, 2013).

$U_A(1)$ *Effect and Topological Configurations*

The integration over gauge fields can have nontrivial configurations characterized by $\nu = (g^2/32\pi^2) \int d^4x F\tilde{F} \neq 0$. The QCD partition function can be divided and summed over by different topological configurations.

$$Z = \sum_\nu Z_\nu. \tag{6}$$

Configuration with nontrivial topological charges have n_+ (n_-) number of right handed (left handed) zero mode solutions that satisfy the index theorem $\nu = n_+ - n_-$. For these configurations it is useful to explicitly write the zero mode contribution with asymmetric chirality as a separate quark determinant. For example, for configurations with $\nu = 1$, the partition function is given as follows:

$$Z_{\nu=1} = \int [d\mu]_{\nu=1} \mathrm{det}\Big(\int d^4x \bar{\psi}_0(x) m \psi_0(x)\Big). \tag{7}$$

Here, ψ_0 is the zero mode solution and the measure in the zero mode runs over all flavors. The prime measure is the positive definite measure after the zero mode has been taken out from the quark determinant. Therefore, the path integral for the difference in the correlation functions discussed in the previous section can be thought of the integration over gauge field configurations with $\nu = 0$.

However when calculating the n-point functions involving quark operators with asymmetric chirality containing all flavors, the topological configuration will give non vanishing contributions. This is so because in such $U_A(1)$ variant configurations, the integral over the chiral zero modes are saturated by the external operators and do not appear in the zero mode part of the quark determinant.

Let us consider the 2 flavor (u,d quarks) case. For the quark condensate,

$$\langle \bar{q}q(0) \rangle = \langle \bar{q}q(0) \rangle_{\nu=0} + \frac{-1}{Z} \int [d\mu]_{\nu=1} 2 \left(\bar{\psi}_0 \psi_0(0) \right) \left(\int d^4x \bar{\psi}_0(x) m \psi_0(x) \right). \tag{8}$$

As can be seen in the second term of the above equation, the topological configurations are proportional to the light quark mass and do not contribute to the quark condensate. However, when one looks at the two point function, the zero mode integral can be saturated by the external operators:

$$\begin{aligned} \langle \bar{q}q(x)\bar{q}q(0) \rangle &= \langle \bar{q}q(x)\bar{q}q(0) \rangle_{\nu=0} \\ &\quad + \frac{1}{Z} \int [d\mu]_{\nu=1} [du_R][d\bar{u}_L][dd_R][d\bar{d}_L] \left(\bar{q}_L q_R(x) \right) \left(\bar{q}_L q_R(0) \right) + \cdots \\ &= \cdots + \frac{1}{Z} \int [d\mu]_{\nu=1} 2 \left(\bar{\psi}_0 \psi_0(x) \right) \left(\bar{\psi}_0 \psi_0(0) \right) + \cdots . \end{aligned} \tag{9}$$

The topological configurations will therefore contribute when looking at the difference between correlation functions related by the $U_A(1)$ transformation. For example,

$$\begin{aligned} \langle \bar{q}q(x)\bar{q}q(0) \rangle &- \langle \bar{q}\tau^3 q(x) \bar{q}\tau^3 q(0) \rangle \\ &= \langle .. \rangle_{\nu=0} + \frac{1}{Z} \int [d\mu]_{\nu=1} 4 \left(\bar{\psi}_0 \psi_0(x) \right) \left(\bar{\psi}_0 \psi_0(0) \right) + \cdots . \end{aligned} \tag{10}$$

For the case of N_f flavors, $\nu = 1$ configuration will still be proportional to m^{N_f-2} and not contribute to the four quark operators. In this case, the lowest dimensional operators where the $\nu = 1$ configuration contribute are the $2N_f$ quark operators (Lee, 1996). Hence, in general, one can conclude that for meson correlation functions composed of N-meson currents, $U_A(1)$ breaking effect will appear with quark mass term proportional to m^{N_F-N}.

Chiral Symmetry Restoration and the η' Mass LET for Gluonic Operators

Let us first present the low energy theorems (LET) for gluonic correlations (Lee and Zahed, 2001). We define the scalar and pseudo-scalar gluonic two-point function,

$$\begin{aligned} S(q) &= i \int d^4x \, e^{iqx} \left\langle \mathcal{T} \frac{3\alpha_s}{4\pi} G^2(x) \frac{3\alpha_s}{4\pi} G^2(0) \right\rangle , \\ P(q) &= i \int d^4x \, e^{iqx} \left\langle \mathcal{T} \frac{3\alpha_s}{4\pi} G\tilde{G}(x) \frac{3\alpha_s}{4\pi} G\tilde{G}(0) \right\rangle . \end{aligned} \tag{11}$$

Here, $G^2 = G^a_{\mu\nu} G^a_{\mu\nu}$ and $G\tilde{G} = G^a_{\mu\nu} \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta}$. There is a well known LET that states

$$\frac{d}{d(-1/4g^2)} \langle O \rangle = i \int d^4x \langle \mathcal{T} O(x) g^2 G^2(0) \rangle, \tag{12}$$

where the bare coupling g^2 is related to the ultraviolet cutoff M_0 to the unique scale of QCD.

$$\Lambda = M_0 \exp\left(-\frac{8\pi^2}{bg_s^2}\right), \tag{13}$$

where $b = 11 - \frac{2}{3}N_f$ (Novikov *et al.*, 1981).

The LET depends on whether the correlation function is calculated in pure gauge theory or in the presence of light quarks. When there are light quarks present, the pseudo-scalar gluon current can be written as

$$\frac{N_f \alpha_s}{4\pi} G\tilde{G} = \sum_q \partial_\mu \bar{q} \gamma_\mu \gamma_5 q. \tag{14}$$

Therefore, for $N_f = 3$, $P(q)$ can be written in terms of the quark currents.

$$P(q) = \quad q^\mu q^\nu i \int d^4x \, e^{iq\cdot x} \langle \mathcal{T} \bar{q}\gamma_\mu\gamma_5 q(x) \, \bar{q}\gamma_\nu\gamma_5 q(0) \rangle. \tag{15}$$

Hence,

$$P(q=0) = 0 \tag{16}$$

However, when there are no light quarks, there are similar LET as in the scalar gluonic correlator.

$$\begin{aligned} S(q=0) &= \frac{18}{b} \langle \frac{\alpha_s}{\pi} G^2 \rangle \\ P_0(q=0) &= -\frac{8}{b} \langle \frac{\alpha_s}{\pi} G^2 \rangle, \end{aligned} \tag{17}$$

where the subscript 0 in the pseudo-scalar current represents calculation in pure gauge theory.

At finite temperature, the constant gets modified because there are extra scales in the system; temperature and/or density. Then the matrix element could depend directly on the extra scale. Hence, one has to make sure that the derivative in Eq. (12) do not act on these part. Such operations can be taken into account explicitly by noting that the matrix element can now be expressed as follows:

$$\langle O \rangle_{\mu,T} \quad = \quad \text{const} \times \Lambda^d f\left(\frac{T}{\Lambda}, \frac{\mu}{\Lambda}\right), \tag{18}$$

where the subscript μ, T denote that the expectation value is taken at finite chemical potential and/or temperature. Eq. (12) will then be modified as

$$\frac{d}{d(-1/4g^2)} \langle O \rangle_{\mu,T} \quad = \quad \frac{32\pi^2}{b} \left(d - T\frac{\partial}{\partial T} - \mu \frac{\partial}{\partial \mu} \right) \langle O \rangle, \tag{19}$$

and the LET will look as follows:

$$
\begin{aligned}
S(q=0) &= \frac{9}{2b}\left(d - T\frac{\partial}{\partial T} - \mu\frac{\partial}{\partial \mu}\right)\langle\frac{\alpha_s}{\pi}G^2\rangle \\
P_0(q=0) &= -\frac{2}{b}\left(d - T\frac{\partial}{\partial T} - \mu\frac{\partial}{\partial \mu}\right)\langle\frac{\alpha_s}{\pi}G^2\rangle
\end{aligned} \tag{20}
$$

Gluonic Correlator in Medium

Now let us go back to the pseudo scalar correlation function given in Eq. (11) and discuss their fate when chiral symmetry is restored. Here, we will again introduce light quarks with N_f flavors. Then using Eq. (14), the pseudoscalar gluonic current can be written in terms of the divergence of the axial current.

$$
P(q) = q^\mu q^\nu i \int d^4x\, e^{iq\cdot x}\left[\langle \mathcal{T}\bar{q}\gamma_\mu\gamma_5 q(x)\,\bar{q}\gamma_\nu\gamma_5 q(0)\rangle - \langle \mathcal{T}\bar{q}\gamma_\mu q(x)\,\bar{q}\gamma_\nu q(0)\rangle\right], \tag{21}
$$

where we have subtracted out the zero contribution from the conserved vector current. Using the previous representations,

$$
\begin{aligned}
P(q) &= q^\mu q^\nu \int d^4x e^{iq\cdot x}\Big[\langle \mathrm{Tr}[S(x,x)\gamma_\mu\gamma_5]\mathrm{Tr}[S(0,0)\gamma_\nu\gamma_5] \\
&- \mathrm{Tr}[S(x,x)\gamma_\mu]\mathrm{Tr}[S(0,0)\gamma_\nu]\rangle \\
&- \langle \mathrm{Tr}[S(0,x)\gamma_\mu\gamma_5 S(x,0)\gamma_\nu\gamma_5] + \mathrm{Tr}[S(0,x)\gamma_\mu S(x,0)\gamma_\nu]\rangle\Big].
\end{aligned} \tag{22}
$$

Now, when chiral symmetry is restored, the two terms in the last line of the above equation will cancel each other, as they are the same as the difference between flavored chiral partners; vector and axial vector currents. The remaining first line constitutes the disconnected contributions. However, as discussed before, the disconnected pieces all vanish in the chiral limit when chiral symmetry is restored. This is again because the disconnected contributions are proportional to the current density matrix of zero eigen values.

$$
\mathrm{Tr}[S_A(x,x)] \sim \mathrm{Tr}[S_A(x,x)\Gamma] = \mathrm{Tr}[J_{\lambda=0}(x,x)\Gamma] \to 0, \tag{23}
$$

where Γ is a Hermitian gamma matrix (Cohen, 1996). Therefore, when chiral symmetry is restored,

$$
P(q) \to 0, \tag{24}
$$

in the chiral limit for any finite external momenta q. It should be noted that Eq. (24) is true only when chiral symmetry is restored, while Eq. (16) is always true.

η' Mass in Medium

We have discussed the relationship between chiral symmetry and $U_A(1)$ effects in both the vacuum and in medium in the correlators. One still needs to establish the relation between the chiral symmetry and the η' mass. This can be established at the large N_c limit through the use of the WV formula together with the relation we obtained for the correlation functions (Kwon *et al*., 2012).

One can directly relate the pseudo-scalar current and the η' mass using the large N_c arguments (Witten, 1979; Veneziano, 1979). In terms of the physical states, the pseudo-scalar gluonic correlation function looks as follows:

$$\begin{aligned} P(q) &= -\sum_n \frac{|\langle 0|\frac{3\alpha}{4\pi} G\tilde{G}|\, n^{th} glueball\rangle|^2}{q^2 - M_n^2} - \sum_n \frac{|\langle 0|\frac{3\alpha}{4\pi} G\tilde{G}|\, n^{th} meson\rangle|^2}{q^2 - m_n^2} \\ &\equiv P_0(q) + P_1(q). \end{aligned} \tag{25}$$

The first term in the right hand side of the above equation indicates contributions from glueballs, while the second term shows those from the mesons composed of light quarks. As discussed before, $P(q=0) = 0$ while $P_0(q=0) \neq 0$ when we assume massless quark. This seems in contradiction to the large N_c argument because $P_0(q=0)$ that scales as N_c^2 is canceled by quark effects that scales as N_c. It was noted that this cancellation is possible by the existence of the η' whose mass scales as $1/N_c$ and cancels the gluonic effect in Eq. (25) only when $q = 0$; other meson masses have a smooth large N_c limit. This constraint directly relates the η' mass to the LET.

$$P_0(0) = -\frac{|\langle 0|\frac{3\alpha}{4\pi} G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2}. \tag{26}$$

By using the $U_A(1)$ anomaly $\langle 0|\frac{3\alpha}{4\pi} G\tilde{G}|\eta'\rangle = \langle 0|\partial_\mu J_5^\mu|\eta'\rangle = \sqrt{N_f}\, m_{\eta'}^2 f_\pi$, Eq. (26) becomes as follows:

$$P_0(0) = -m_{\eta'}^2 f_\pi^2 N_f, \tag{27}$$

where N_f is the number of light flavors and made use of $f_{\eta'} = f_\pi$ to lowest order in N_c. Eq. (27) is the celebrated WV formula. Then we find

$$m_{\eta'} = \sqrt{\frac{8}{33}} \frac{1}{f_\pi} \langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/2} \approx 464 \ \text{MeV}, \tag{28}$$

where we have used $f_\pi = 130\,\text{MeV}$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.35\text{GeV})^4$ and used $b = 11$ for pure glue theory. This is smaller than the vacuum value of the η' mass as expected; this is the part coming from the $\mathrm{U_A}(1)$ effect to the mass of η'.

To obtain the generalized formula at finite temperature, one notes that the thermal gluonic effects are of order N_c^2 while that of the quarks are of order N_c. If one is in the confined phase, the phase is composed of mesons, glueballs and baryons: The scattering of these states scale as order 1, 1 and N_c respectively. Hence as long as one assumes that the number of hadrons do not scale with N_c, hadronic effects can be neglected. Near the phase transition, the degeneracy of hadrons would increase and follow the scaling of gluons and quarks. Therefore, the leading order effect would come from the gluons. Taking into account the effect of the thermal gluons, the LET given in Eq. (20) will be modified only by the effect of finite temperature. Therefore, the generalization of the WV formula given in Eq. (26) would be the following.

$$m_{\eta'}^2 = \frac{|\langle 0|\frac{3\alpha_s}{4\pi} G\tilde{G}|\eta'\rangle|^2}{\frac{2}{b}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{T,puregauge}}. \tag{29}$$

In obtaining the result, we have assumed that the general structure given in Eq. (25) does not change.

First let us consider the denominator of Eq. (29). It has been known for a long time, that the gluon condensate has contribution from the perturbative and non-perturbative parts. Moreover, it was also known that at the critical temperature, the non-perturbative contribution changes abruptly, but does not vanish completely, and retains more than half of its non-perturbative value (Lee, 1989).

The effect of subtracting out the second term in the denominator of Eq. (29) is to get rid of the perturbative correction, or the seemingly scale breaking effect that is not related to scale breaking but due to the introduction of an external scale parameter T. The leading perturbative correction to the gluon condensate is proportional to $g_s^4 T^4$ (Kapusta, 1979; Boyd *et al.*, 1996). Therefore, assuming that the temperature dependence is $\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_T = G_0(T) + a g_s^4 T^4$, we find,

$$\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_T = \left(d - T\frac{\partial}{\partial T}\right) G_0(T), \tag{30}$$

if the temperature dependence of g_s is neglected. The only temperature dependence that survives is $G_0(T)$, whose scale dependence is coming from dimensional transmutation and not from the external temperature only. It is the non-perturbative part that dominates the behavior of the right hand side of Eq. (29). Moreover, as we discussed before, for the gluon condensate, one has to use the lattice result obtained in the pure-gauge theory calculation, where the critical temperature $T_{pure-gauge} \sim$ 260 MeV is known to be around 100 MeV larger than that from a full QCD calculation T_{QCD} (Boyd *et al.*, 1996). On the other hand, while the expected change of the gluon condensate is more abrupt in the pure-gauge calculation, the actual change in the condensate value itself at the critical temperature is found to be similar to the full calculation (Morita and Lee, 2008). This means that the change of the gluon condensate can be effectively neglected up to temperatures near T_{QCD}.

Finally, we use the fact that when chiral symmetry is restored, Eq. (29) is zero for large q as given by

Eq. (24), which is true for any N_c. Hence, when chiral symmetry is restored,

$$\langle 0|G\tilde{G}|\eta'\rangle \sim O(m_q). \tag{31}$$

Therefore, going back to Eq. (29) and making use of the previous discussions, we find that when chiral symmetry is restored,

$$m^2_{\eta'} \stackrel{\langle \bar{q}q\rangle\to 0}{\longrightarrow} 0, \tag{32}$$

in the chiral limit. One concludes that in the large N_c limit of QCD, η' mass will become degenerate with the other Goldstone bosons. A similar conclusion was obtained in Ref. (Benic *et al.*, 2011); that the anomalous $\mathrm{U_A}(1)$ η' mass squared vanishes at high T as the chiral quark condensate $\langle \bar{q}q\rangle$.

Summary

We have looked at how chiral and $U_A(1)$ symmetries and/or breaking are manifest in correlation functions and further studied the how the η' mass is related to the symmetries. The results can be summarized as follows:

1. Order parameter of symmetries can be constructed by taking the difference of correlation functions that are related by respective symmetry transformations (symmetry partners).

2. Chiral symmetry breaking occurs in these order parameters when the current density matrix of zero modes are non zero.

3. The $U_A(1)$ breaking effect comes about when the topological configuration contributes to the order parameter. These effects are due to the quark zero modes that are always present. However, since the zero modes consist of N_f left handed quarks and the same number of right handed quarks, their contributions will appear only when looking at correlation functions composed of at least N_F point functions. In another words,

   - For meson correlation functions composed of N-meson currents, $U_A(1)$ breaking effect will appear with quark mass term proportional to m^{N_F-N}.

   $$\langle J_1 J_2 .. J_N\rangle = \langle ..\rangle_{\nu=c0} + m^{N_F-N} \times (zero - mode)^N \tag{33}$$

4. Using the previous relations on correlation functions and the WV formula, we find that the η' mass will be quenched to the $U_A(1)$ symmetric value when chiral symmetry is restored.

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