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# Contribution of $\pi$-$\eta$ Mixing to the Difference Between $p\,p$ and $nn$ Scattering Lengths

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We revisit the problem of charge symmetry violation (CSV) in nucleon-nucleon interactions due to $\pi$-$\eta$ mixing driven by the neutron-proton mass difference. We construct the CSV potential and estimate the contribution to the difference between $pp$ and $nn$ scattering lengths.

**Key Words : Charge Independence; Charge Symmetry; Mixing**

## Introduction

The study of symmetry violating phenomena of nucleon-nucleon ($NN$) interaction is an interesting area of research in nuclear physics. Such research might provide significant insight into the dynamics of $NN$ interaction.

In nature, $NN$ interaction violates both the symmetries - charge independence (CI) and charge symmetry (CS). CI means neutron-neutron ($nn$), proton-proton ($pp$) and neutron-proton ($np$) interactions are equal, while CS implies the equality between $nn$ and $pp$ interactions only. CI is violated if CS violates, but violation of CI may not lead to the violation of CS (Stevens, 1965; Henley *et al.*, 1972; Downs *et al.*, 1967).

In this paper we consider only CS violation (CSV) of $NN$ interactions which can be observed in various experiments (Cheung *et al.*, 1979) such as difference between $pp$ and $nn$ scattering lengths in the $^1S_0$ state (Miller *et al.*, 1990), difference of binding energy between mirror nuclei (Nolen *et al.*, 1973), decay of $\Psi'(3686) \rightarrow (J/\Psi)\pi^0$ etc (Miller *et al.*, 2006).

There are various mechanisms lead to the violation of CS in $NN$ interaction. For example, mixing of isoscalar-isovector mesons can generate CSV $NN$ interactions. At the fundamental level, neutral mesons with same spin and parity of different isospins can mix because of up-down ($u - d$) quark mass difference

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which causes neutron-proton mass splitting at hadronic level. Such mass splitting may also leads to the mixing like $\rho$-$\omega$ mixing (McNamee *et al*., 1975, Coon *et al*., 1977; Blunden *et al*., 1987; Machleidt *et al*., 2001) and $\pi$-$\eta$ mixing (Coon *et al*., 1982; Maltman, 1993; Piekarewica, 1993).

Here we estimate the contribution of $\pi$-$\eta$ mixing to the difference between $pp$ and $nn$ scattering lengths. The mixing amplitude is driven by the $n-p$ mass difference. This mixing amplitude is the essential part of the two body CSV $NN$ potential. Note that the space-like mixing amplitude is relevant for the construction of CSV potential.

The paper is organized as follows. The formalism is presented in Section II and we summarize our results in Section III.

## Formalism

To construct CSV two body $NN$ potential one should calculate the Feynman amplitude of the diagram shown in Fig. 1. In this figure, nucleons and mesons are presented by solid and dashed lines, respectively; and the mixing is indicated by the crossed circles. The relevant Feynman amplitude is given in Eq. (1) where, $u_N$ represents the Dirac spinors, $\Pi_{\pi\eta}(q^2)$ is the $\pi$-$\eta$ mixing amplitude, $\tau_3(1)$ and $\tau_3(2)$ are isospin operators at vertices '1' and '2'. The vertex factor and meson propagator are denoted by $\Gamma_j(q)$ $(j = \pi, \eta)$ and $\Delta_j(q)$, respectively.

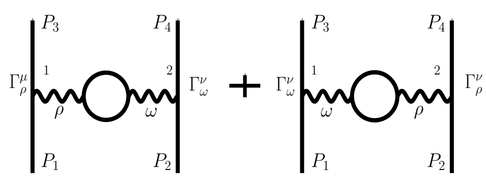


Figure 1: **Feynman diagram of CSV $NN$ potential.**

$$\begin{aligned}\mathcal{M}^{NN}_{\pi\eta}(q) &= [\bar{u}_N(p_3)\tau_3(1)\Gamma_\pi(q)u_N(p_1)]\Delta_\pi(q)\Pi_{\pi\eta}(q^2)\Delta_\eta(q)[\bar{u}_N(p_4)\Gamma_\eta(-q)u_N(p_2)] \\ &+ \ [\bar{u}_N(p_3)\Gamma_\eta(q)u_N(p_1)]\Delta_\eta(q)\Pi_{\pi\eta}(q^2)\Delta_\pi(q)[\bar{u}_N(p_4)\tau_3(2)\Gamma_\pi(-q)u_N(p_2)]. \qquad (1)\end{aligned}$$

One may obtain the momentum space two body $NN$ potential from Eq. (1) substituting $q_0 = 0$. In this work we assume that the mixing is generated by the $N\bar{N}$ loops and the difference between proton and neutron loops contribute to the mixing amplitude $\Pi_{\pi\eta}(q^2)$

$$\Pi_{\pi\eta}(q^2) = \Pi^{(p)}_{\pi\eta}(q^2) - \Pi^{(n)}_{\pi\eta}(q^2), \qquad (2)$$

where $\Pi^{(p)}_{\pi\eta}(q^2)$ and $\Pi^{(n)}_{\pi\eta}(q^2)$ are the $p$-loop and $n$-loop contribution to the $\pi$-$\eta$ mixing self-energy. Isovector meson $\pi$ and isoscalar meson $\eta$ couple to proton with the same sign but couple with opposite sign to neutron which brings a relative sign between proton and neutron loops. The one loop contribution to the mixing self-energy is given by

$$i\Pi^{(N)}_{\pi\eta}(q^2)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\Gamma_\pi(q)G_N(k)\Gamma_\eta(-q)G_N(k+q)]\,, \tag{3}$$

where $N = p$ or $n$ and nucleon propagator:

$$G_N(k) \;=\; \frac{\not{k} + M_N}{k^2 - M_N^2 + i\zeta}. \tag{4}$$

The vacuum contribution of $\pi$-$\eta$ mixing self-energy for pseudoscalar (PS) and pseudovector (PV) couplings are respectively

$$\Pi^{(N)}_{vac}(q^2) \;=\; 4i\mathrm{g}_\pi\mathrm{g}_\eta\int\frac{d^4k}{(2\pi)^4}\left[\frac{M_N^2-k\cdot(k+q)}{(k^2-M_N^2)((k+q)^2-M_N^2)}\right]. \tag{5}$$

$$\begin{aligned}\Pi^{(N)}_{vac}(q^2) \;&=\; 4i\left(\frac{\mathrm{g}_\pi}{2M_N}\right)\left(\frac{\mathrm{g}_\eta}{2M_N}\right)\int\frac{d^4k}{(2\pi)^4}\\ &\times\;\left[\frac{q^2(M_N^2-k\cdot(k+q))-2q\cdot(k+q)(k\cdot q)}{(k^2-M_N^2)((k+q)^2-M_N^2)}\right].\end{aligned} \tag{6}$$

From the dimensional counting it is found that both the integrals *i.e.* Eq. (5) and Eq. (6) are divergent. We adopt dimensional regularization to isolate the singularities of the above equations. Following the technique described in (Biswas *et al.*, 2010) one obtains the approximated $\pi$-$\eta$ mixing amplitude in vacuum:

$$\Pi^{PS}_{vac}(\mathbf{q}^2) = \Pi^{PV}_{vac}(\mathbf{q}^2) = -a_1\mathbf{q}^2, \tag{7}$$

where $a_1 = \frac{\mathrm{g}_\pi\mathrm{g}_\eta}{4\pi^2}\ln\left(\frac{M_p}{M_n}\right)$. In the limit $M_p = M_n$ mixing amplitude vanishes. Thus CSV $NN$ potential does not exist if $M_p = M_n$. This mixing amplitude is used to construct the CSV $NN$ potential which reads

$$V^{NN}_{CSV}(\mathbf{q}^2) \;=\; T_3^+\frac{\mathrm{g}_\pi\mathrm{g}_\eta}{4M_N^2}\frac{\Pi^{PS}_{\pi\eta}(\mathbf{q}^2)\,(\sigma_\mathbf{1}\cdot\mathbf{q})(\sigma_\mathbf{2}\cdot\mathbf{q})}{(\mathbf{q}^2+m_\pi^2)(\mathbf{q}^2+m_\eta^2)}\left[1-\frac{\mathbf{q}^2}{8M_N^2}-\frac{\mathbf{P}^2}{2M_N^2}\right], \tag{8}$$

where $T_3^+ = \tau_3(1)+\tau_3(2)$ and $\mathbf{P} = (\mathbf{p}_1+\mathbf{p}_3)/2 = (\mathbf{p}_2+\mathbf{p}_4)/2$. The coordinate space CSV $NN$ potential reduces to

$$V^{NN}_{vac}(r) \;=\; -T_3^+\frac{\mathrm{g}_\pi\mathrm{g}_\eta a_1}{48\pi M_N^2}\left[\frac{m_\pi^5U(x_\pi)-m_\eta^5U(x_\eta)}{m_\eta^2-m_\pi^2}\right]. \tag{9}$$

Here

$$U(x_i) \;=\; Y_0(x_i)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + S_{12}(\hat{\mathbf{r}})Y_2(x_i) \tag{10a}$$

$$Y_2(x_i) \;=\; \left(1+\frac{3}{x_i}+\frac{3}{x_i^2}\right)Y_0(x_i) \tag{10b}$$

$$S_{12}(\hat{\mathbf{r}}) \;=\; 3(\sigma_\mathbf{1}\cdot\hat{\mathbf{r}})(\sigma_\mathbf{2}\cdot\hat{\mathbf{r}}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) \tag{10c}$$

where $x_i = m_i r$, $i = \pi, \eta$ and $Y_0(x_i) = e^{-x_i}/x_i$.

Since hadrons have internal structures one needs to incorporate vertex corrections through phenomenological form factors:

$$F_i(\mathbf{q^2}) \;=\; \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + \mathbf{q}^2}\right), \;\; i = \pi, \eta. \tag{11}$$

Here $\Lambda_i$ is the cut-off parameter. With the inclusion of form factors Eq. (9) reduce to

$$\begin{aligned} V_{vac}^{NN}(r) \;=\; & -T_3^+ \frac{g_\pi g_\eta a_1}{48\pi M_N^2}\left[\left(\frac{a_\pi m_\pi^5 U(x_\pi) - a_\eta m_\eta^5 U(x_\eta)}{m_\eta^2 - m_\pi^2}\right)\right. \\ & \left. - \; \lambda\left(\frac{b_\pi m_\pi^5 U(X_\pi) - b_\eta m_\eta^5 U(X_\eta)}{m_\eta^2 - m_\pi^2}\right)\right], \end{aligned} \tag{12}$$

where $X_i = \Lambda_i r, \; i = \pi, \eta$ and

$$a_i \;=\; \left(\frac{\Lambda_j^2 - m_j^2}{\Lambda_j^2 - m_i^2}\right), \tag{13a}$$

$$b_i \;=\; \left(\frac{\Lambda_j^2 - m_j^2}{m_j^2 - \Lambda_i^2}\right), \;\; i \neq j, \; (i \; or \; j = \pi, \eta) \tag{13b}$$

$$\lambda \;=\; \left(\frac{m_\eta^2 - m_\pi^2}{\Lambda_\eta^2 - \Lambda_\pi^2}\right). \tag{13c}$$

These CSV $NN$ potentials given in Eq. (9) and Eq. (12) can be now used to estimate the contribution to the difference between $pp$ and $nn$ scattering lengths at ${}^1S_0$ state using the following relation

$$\Delta a = -a^2 M \int_0^\infty \Delta V_{vac}\, u_0^2(r)\, dr \tag{14}$$

where $\Delta V_{vac} = V_{vac}^{nn} - V_{vac}^{pp}$, $\Delta a = a_{pp} - a_{nn}$, $a^2 = a_{nn} a_{pp}$ and $u_0(r)$ is the zero energy wave function (Kermode *et al.*, 1990).

## Summary

In this work we have estimated the contribution of $\pi$-$\eta$ mixing to difference between $pp$ and $nn$ scattering lengths using the CSV two body $NN$ potential. We have computed $\Delta a$ and it is found that $\Delta a = 0.00082$ fm without form factors and with form factors it is $-0.0001$ fm.

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