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Heavy Quark Dynamics in QGP: Boltzmann vs Langevin

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In this manuscript we review the basic concepts related to the study of the dynamics of the heavy quarks in quark-gluon plasma created in ultra-relativistic heavy-ion collisions. We discuss the relevant physical scale as well as the difficulties of the present theoretical approach with an aim to have a self-consistent description of the experimental data at both RHIC and LHC. In the second part we challenge the assumption of brownian motion for charm quarks and compare the dynamical evolution of charm and bottom quarks in the Fokker-Planck approach with the Boltzmann Transport calculationone. We show that while for bottom the motion appears quite close to a Brownian one, this does note seems to be the case for charms quarks. In particular the solution of the full two-body collision integral shows that the anisotropic flows are large with respect to those predicted by a Langevin dynamics. We show that using isotropic cross section one may describe the suppression R_{AA} and elliptic flow v_2 simultaneously.

Key Words : QCD; QGP; Heavy Quarks; Drag and Diffusion Coefficients; Langevin and Boltzmann Equation

Introduction

One of the primary aims of the ongoing nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies is to create a new state of matter whose bulk properties are governed by the light quarks and gluons (Shuryak, 2005; Jack and Muller, 2012). In this context, the heavy quarks (HQ), mainly charm and bottom, play a crucial role since they do not constitute the bulk part of the matter owing to their larger mass when compared to the temperature created in ultra-relativistic heavy-ion collisions (uRHIC's) (Rapp and Hees, 2010). Due to their large masses HQ can act as a type of external probe to

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investigate the bulk of the QGP medium and are affected by its density, temperature and collective expansion thereby carrying the information of the created plasma.

Let us now return to the heavy quarks which can be considered as the probe to the quark gluon plasma. Heavy quarks(HQ) are generally considered as favorable probes for two reasons: the first, typical of particles physics scenario, is that the mass $M_Q \gg \Lambda_{QCD}$ which makes it possible to evaluate the production cross section and p_T spectra within next-to-next-to-leading order (NLLO) (Cacciari *et al.*, 2005, 2012) in a perturbative QCD (pQCD) scheme; the second, more inherent to plasma physics is that, $M_Q \gg T$ and therefore the thermal production of heavy quark in the QGP is expected to be negligible as it is suppressed approximately by a factor $\sim e^{-M/T}$. Hence for HQ one has a nearly exact flavor conservation during the evolution of the plasma in both the partonic and hadronic stages. This remains true while going from SPS to LHC energies spanning a T range of $\sim 200-600$ MeV, as we can see in Fig. 1 where the ratio M/T_{max} (T_{max} is the estimated maximum initial temperatures at different colliders, from SPS (diamonds), RHIC (circles) up to LHC (squares)) remains larger than one for c and b quarks. We notice that even if the collision energy from SPS to LHC goes up by about a factor 10^2 the maximum temperature increases by at most a factor of three leaving the ratio M/T for charm and bottom quarks always larger than one. In Fig. 1 we have also indicated by a shaded area where the value of $M/T \sim 1/2$, so that one can expect that most of quarks are produced thermally and from this point of view can be considered to be light quarks with respect to the available energy. We also know that the strange quarks s at SPS energy are mostly thermally created, which is naively the reason why there is a strange enhancement with respect to pp collisions at energies around the maximum SPS $\sqrt{s_{NN}} = 17.8\, GeV$ (Antinori, 2004).

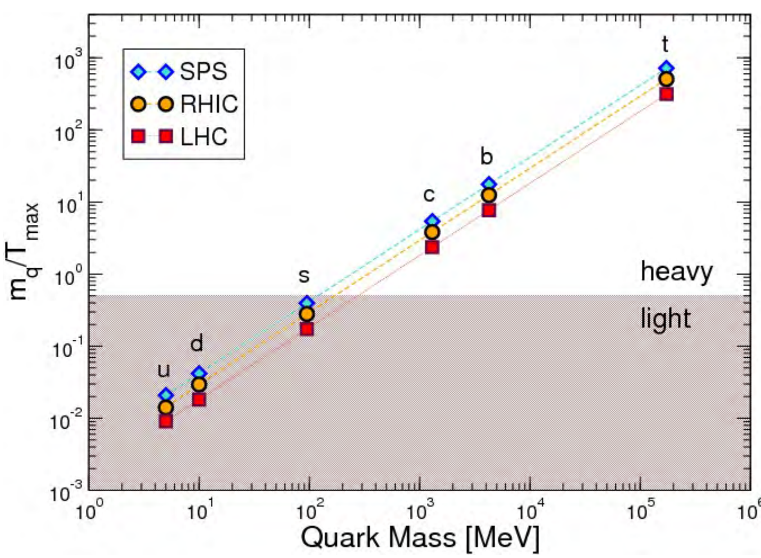


Fig. 1: Ratio of the quark mass to the maximum temperature reached in heavy-ion collisions at SPS, RHIC and LHC

To begin with, we will discuss briefly some of the early ideas about heavy quark as a probe of the QGP emphasizing the difficulties in describing simultaneously the modification of the spectra with respect to pp collisions and the large elliptic flow v_2, a measure of the anisotropic flow, observed experimentally. In the second part we will focus on the theoretical approaches to describe the dynamical evolution of the HQ comparing the most commonly used Fokker-Planck approach to the Boltzmann transport equation. In the following sections, we will discuss that while for the bottom quark, that the two approaches give very similar results with respect to the Langevin dynamics. While for the charm quark both the nuclear suppression factor R_{AA} and the elliptic flow v_2 using Boltzmann calculations are larger and more closer to experimental observations.

Boltzmann vs Fokker-Planck Dynamics

The propagation of HQ in QGP has been quite often treated within the framework of Fokker-Planck equation (Rapp and Hees, 20120; Svetitsky, 1988; Mustafa *et al.*, 1998; Moore and Teaney, 2005). The early ideas that HQ undergoes Brownian motion in the medium suggests that their interaction can be treated perturbatively and therefore generically leads to collisions sufficiently forward peaked and/or with small momentum transfer. Under such constraints it is known that the Boltzmann transport equation reduces to a Fokker-Planck dynamics (Svetitsky, 1988), which constitutes a significant simplification of the in-medium dynamics. Such a scheme has been very widely employed by many authors (Mustafa *et al.*, 1998; Moore and Teaney, 2005; Hees *et al.*, 2006; Cao and Bass, 2011; Hees *et al.*, 2008; Akamatsu *et al.*, 2009; Gossiaux *et al.*, 2011; Das *et al.*, 2010; Majumdar *et al.*, 2012; Alberico *et al.*, 2011; Young *et al.*, 2012; Lang *et al.*, 2012; Cao *et al.*, 2013; He *et al.*, 2013; Das and Davody, 2014; Xu *et al.*, 2014) in order to calculate the experimentally observed nuclear suppression factor ($R_{\rm AA}$) (Adare *et al.*, 2006; Abeleb *et al.*, 2007; Adare *et al.*, 2007; Abeleb *et al.*, 2012) and the elliptic flow (v_2) (Adare *et al.*, 2006) of the non-photonic single electron spectra. In parallel, other contemporary works with a description of HQ within a relativistic Boltzmann transport approach have been developed which include both collisional and radiative energy loss (Gossiaux and Aichelin, 2008; Gossiaux *et al.*, 2010; Uphoff *et al.*, 2011, 2012). The mentioned references give results those match data within the error bars and are more close to the possibility of predicting both R_{AA} and v_2 for $Pb + Pb$ at $\sqrt{s} = 2.76\,\mathrm{ATeV}$ simulatneously. Also other authors have in the past and more recently have undertaken the study of charm quarks within a Boltzmann approach (Younus *et al.*, 2013; Zhang *et al.*, 2005; Molnar, 2007; Das *et al.*, 2013).

The Boltzmann equation for the HQ distribution function can be written in a compact form as:

$$p^{\mu}\partial_{\mu} f_Q(x,p) = \mathcal{C}[f_Q](x,p) \tag{1}$$

where $\mathcal{C}[f_Q](x,p)$ is the relativistic Boltzmann-like collision integral where the phase-space distribution function of the bulk medium appears as an integrated quantity in $\mathcal{C}[f_Q]$, see for example Ref.s (Xu and Greiner, 2005; Lang *et al.*, 1993), while we study the evolution of the heavy quarks distribution function $f_Q(x,p)$.

For the purpose of focusing on the momentum transferred in the collisions the relativistic collision integral can be written in a simplified form (Rapp and Hees, 2010; Svetitsky, 1988) in the following way:

$$\mathcal{C}[f_Q](x,p) = \int d^3k \qquad [\omega(p+k,k) f_Q(x,p+k) - \omega(p,k) f_Q(x,p)] \tag{2}$$

where $\omega(p,k)$ expresses the collision rate of heavy quark per unit of momentum phase space which changes the heavy quark momentum from p to $p-k$, with the first term of the integrand being the gain of probability through collisions and the second term denotes the loss in the momentum space volume.

HQ interacts with the medium by mean of two-body collisions regulated by the scattering matrix of the process $g+Q \to g+Q$ ($\sigma_{g+Q\to g+Q}$). Therefore we can define the relative velocity between the two colliding particles as v_{rel}, the transition rate can be written as:

$$\omega(p,k) = \int \frac{d^3q}{(2\pi)^3} f_g(x,p) v_{rel} \frac{d\sigma_{g+Q\to g+Q}}{d\Omega} \tag{3}$$

where $\sigma_{g+Q\to g+Q}$ is related to the scattering matrix $|\mathcal{M}_{gQ}|^2$:

$$v_{rel} \frac{d\sigma_{g+Q\to g+Q}}{d\Omega} = \frac{1}{d_c} \frac{1}{4E_pE_q} \frac{|\mathcal{M}_{gQ}|^2}{16\pi^2 E_{p-k}E_{q+k}} \delta^0(E_p + E_q - E_{p-k} - E_{q+k}) \tag{4}$$

We recall that the scattering matrix is the real kernel of the dynamical evolution for both the Boltzmann and the Fokker-Planck approaches. Also all the calculations for both cases in the following sections, contain the same scattering matrices.

The Boltzmann equation is solved numerically dividing the phase-space into a three-dimensional lattice and using the test particle method to sample the distributions functions. The collision integral is solved by mean of a stochastic implementation of the collision probability $P = v_{rel}\sigma_{g+Q\to g+Q} \cdot \Delta t/\Delta x$ (Felini *et al.*, 2009; Greco *et al.*, 2009; Ruggieri *et al.*, 2013; Xu and Greiner, 2005; Lang *et al.*, 1993). The code has been widely tested regarding the collision rate and the evolution of non-equilibrium initial distributions towards the Bolztmann-Juttner equilibrium distribution both as a function of cross section, temperature and mass of the particles, including non-elastic collisions (Scardina *et al.*, 2013).

The non-linear integro-differential Boltzmann equation can be significantly simplified employing the Landau approximation whose physical relevance can be associated to the dominance of soft scatterings with small momentum transfer $|\mathbf{k}|$ with respect to the particle momentum $\mathbf{p}$. Namely one expands $\omega(p+$

$k,k)f(x,p+k)$ around k,

$$\omega(p+k,k)f_Q(x,p+k) \approx \omega(p,k)f(x,p) + k\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f) \tag{5}$$

Inserting Eq.(5) into the Boltzmann collision integral, Eq.(2), one obtains the Fokker Planck Equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})]\right] \tag{6}$$

by simply defining $A_i = \int d^3k\, w(\mathbf{p},\mathbf{k})k_i = A(\mathbf{p})p_i$ and $B_{ij} = \int d^3k\, w(\mathbf{p},\mathbf{k})k_i k_j$ that are directly related to the so called drag and diffusion coefficient. The Fokker-Planck equation can be solved by a stochastic differential equation i.e the Langevin equation, can be written as (Rapp and Hees, 2010; Moore and Teaney, 2005;l Cao and Bass, 2011):

$$\begin{aligned} dx_i &= \frac{p_i}{E}dt, \\ dp_i &= -Ap_i dt + (\sqrt{2B_0}P^{\perp}_{ij} + \sqrt{2B_1}P^{\parallel}_{ij})\rho_j\sqrt{dt} \end{aligned} \tag{7}$$

where dx_i and dp_i are the coordinate and momentum changes in each time step dt. A is the drag force and B the longitudinal and transverse diffusions, ρ is a stochastic variable Gaussian distributed. in terms of independent Gaussian-normal distributed random variables ρ_j, and

$$P^{\perp}_{ij} = \delta_{ij} - \frac{p_i p_j}{p^2}, P^{\parallel}_{ij} = \frac{p_i p_j}{p^2}. \tag{8}$$

are the transverse and longitudinal tensor projectors. We will employ the common assumption, $B_0 = B_1 = D$ (Moore and Teaney, 2005; Hees *et al.*, 2006; Cao and Bass, 2011, Hees *et al.*, 2008; Gossiaux and Aichelin, 2008; Gossiaux *et al.*, 2010; Das *et al.*, 2010; Majumdar *et al.*, 2012; Lang *et al.*, 2012). To achieve the equilibrium distribution $f_{eq} = e^{-E/T}$ with $E = \sqrt{p^2+m^2}$ as the final distribution one need to adjust the drag coefficient A in accordance with the Einstein relation (Walton and Rafelski, 2003) (see also (Majumdar *et al.*, 2012).

$$A(p) = \frac{D(p)}{ET} - \frac{D'(p)}{p}. \tag{9}$$

Numerical Results and Discussion

We now discuss the evolution of momentum distributions of charm and bottom quarks interacting with a bulk medium at $T = 0.4\,\mathrm{GeV}$ with scattering processes determined by the scattering matrices discussed in the previous section. The initial distribution of heavy quarks are taken from Ref. (Cacciari *et al.*, 2005) and given by $f(p,t=0) = (a+b\,p)^{-n}$ with $a = 0.70\,(57.74)$, $b = 0.09\,(1.00)$ and $n = 15.44\,(5.04)$ for charm and bottom quarks respectively. The above function gives a reasonable description of D and B meson spectra in the p-p collision at highest RHIC energy. Our purpose is to compare the time evolution of heavy quark spectra starting from the same initial momentum distribution and evaluating in each case, considering both the differential cross section $d\sigma/d\Omega$ which is the main ingredient of the Boltzmann equation, and the drag and diffusion coefficients which are the key ingredient of the Langevin equation originating from the same scattering matrix elements. For the details we may refer to Ref. (Das *et al.*, 2014).

Our purpose here is to compare between the Langevin and Boltzmann transport equations for various values of the transferred momentum that can be directly related to the angular distribution of scattering matrix or cross section. This has been achieved by using three different values of the Debye screening masses (m_D) needed to shield the divergence associated with the t-channel of the scattering matrix. We have chosen three values for m_D, one is 0.83 GeV that corresponds to $m_D = \sqrt{4\pi\alpha_s}\,T$ with $\alpha_s = 0.35$ at $T = 400\,MeV$ that is the main temperature we will consider for our study. The other two values correspond to a reduction factor of two ($m_D = 0.4\,\mathrm{GeV}$) simulating more forwad peaked modelings and an increase of a factor of two ($m_D = 1.6\,\mathrm{GeV}$) simulating more isotropic resonant-like conditions.

We have plotted the results as a ratio between Langevin to Boltzmann at different times to quantify how much the ratio deviates from unity. We started the simulation at $t = 0$ fm/c which corresponds to a ratio 1 as we start the simulation with the same initial momentum distribution for both Langevin and Boltzmann equations. So any deviation from unity would reflect how much the Langevin differ from the Boltzmann evolution.

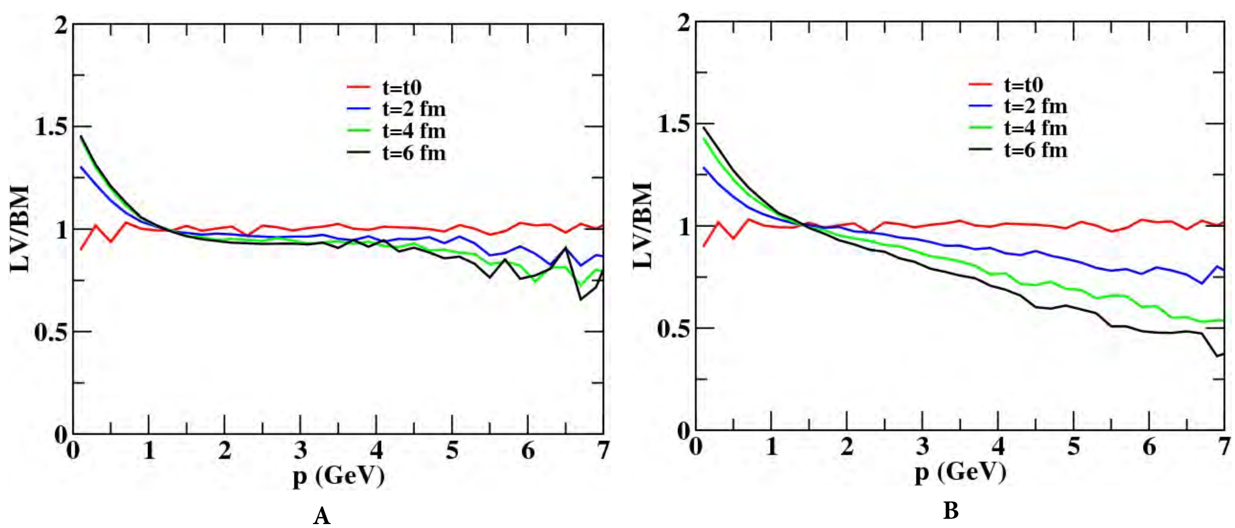


Fig. 2: (A) Ratio between the Langevin (LV) and Boltzmann (BM) p_T-spectra for charm quark as a function of momentum for m_D= 0.83 GeV at different time; (B) ratio between the Langevin (LV) and Boltzmann (BM) spectra for charm quark as a function of momentum for m_D = 0.4 GeV at different time

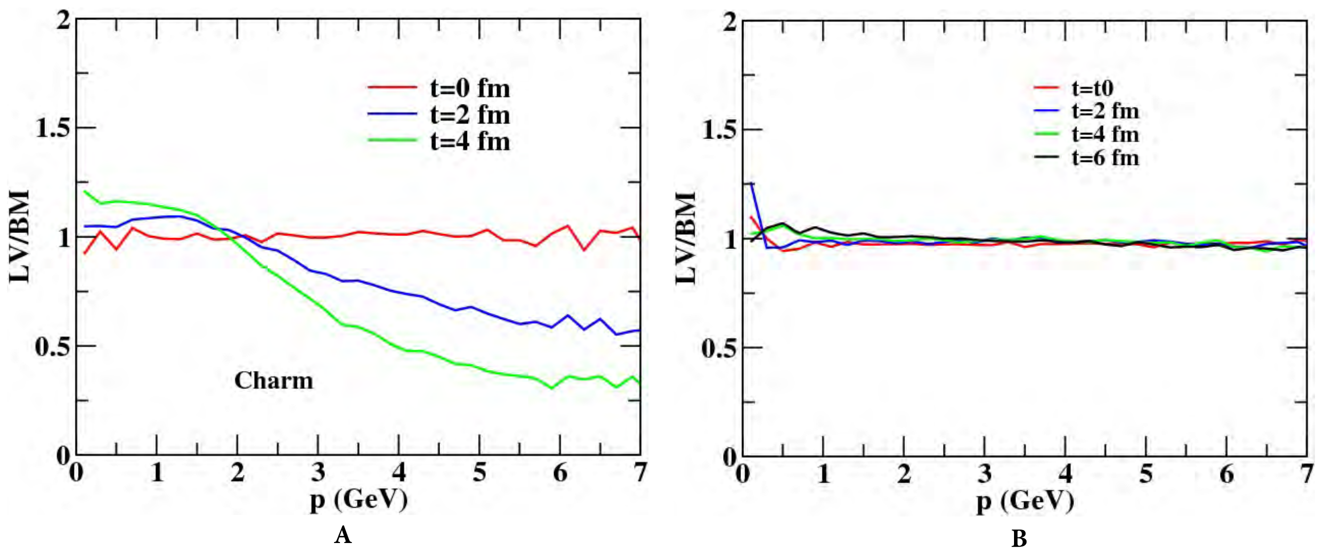


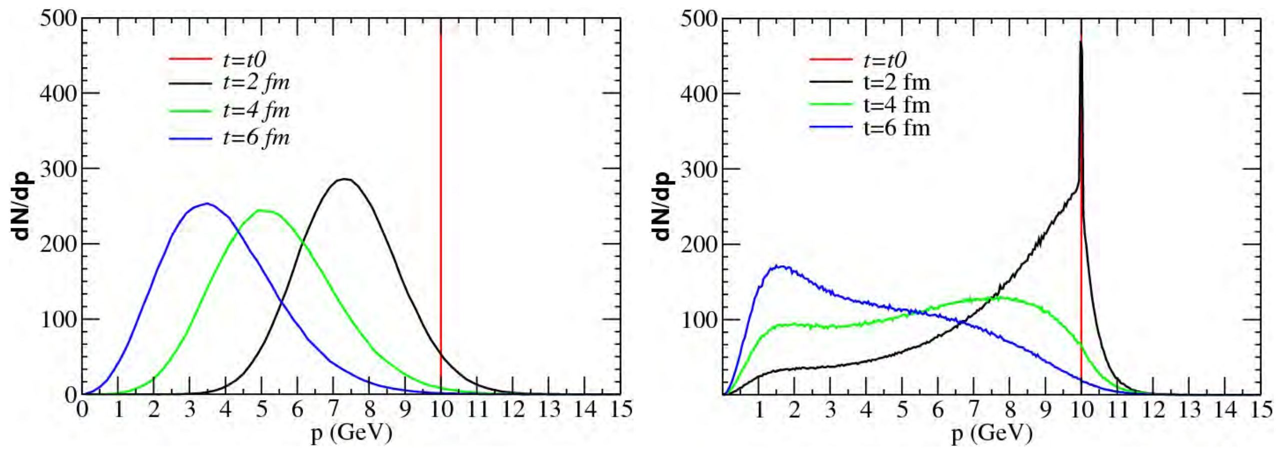
Fig. 3: (A) Ratio between the Langevin (LV) and Boltzmann (BM) p_T-spectra for charm quark as a function of momentum for $m_D = 1.6$ GeV at different time; (B) ratio between the Langevin (LV) and Boltzmann (BM) spectra for bottom quark as a function of momentum for $m_D = 0.83$ GeV at different time

In Fig. 2 the ratio of Langevin to Boltzmann spectra for the charm quark for $m_D = 0.4$ GeV (A) and $m_D = 0.83$ GeV (B) has been displayed as a function of momentum at different time. We remind that time scales of $4-6$ fm/c can be roughly taken as those corresponding to typical lifetime of a QGP in uRHIC's. For the smaller screening mass corresponding to more forward peaked cross section, we observe that the differences between Langevin and Boltzmann are quite limited and smaller than 15%. Instead at $m_D = 0.83$ GeV it is observed that at $t = 4$ fm/c a deviation of Langevin from Boltzmann is around 40% and at $t = 6$ fm the deviation is around a 50% at $p = 5$ GeV charm, which suggests Langevin approach overestimates the average energy loss considerably due to the approximations it involves.

When we consider a larger screening mass, $m_D = 1.6$ GeV to simulate a nearly isotropic scattering we see that the ratio of Langevin to Boltzmann spectra as shown in Fig. 3(A) can lead to differences as large as 75% at t=4 fm/c. However it is observed that the ratio stays practically almost unity for bottom quark (right) for all the time considered in our calculations.

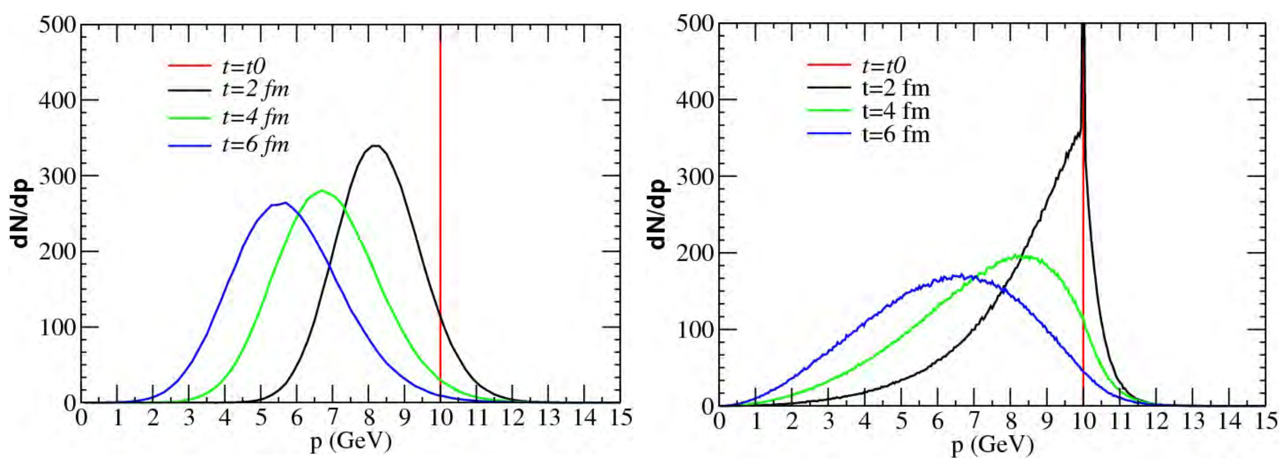
Heavy Quark Diffusion in Momentum Space

For a more thorough investigation of the heavy quark evolution implied by a Langevin and a Boltzmann approach, we study the heavy quark momentum evolution considering the initial charm and bottom quark distribution as a delta distribution at $p = 10$ GeV and with $m_D = 0.83$ GeV. The momentum evolution of the charm quarks are displayed in Fig. 4 within the Langevin dynamics. It is observed that both the charm and bottom (Fig. 5) quarks are Gaussian distribution as expected by construction. As known the Langevin dynamics consist of a shift of the average momenta with a fluctuation that includes the possibility of heavy quark to gain energy. We see such observation the momentum distribution that overshoots the initial momentum $p = 10\,\mathrm{GeV}$ at $t = 2\,\mathrm{fm/c}$, black solid line in Fig. 4.



A **B**

Fig. 4: Evolution of charm quark momentum distribution within Langevin dynamics (A) and Boltzmann equation (B) considering the initial momentum distribution of the charm quarks as a delta distribution at p=10 GeV



A **B**

Fig. 5: Evolution of bottom quark momentum distribution within Langevin dynamical (A) and the Boltzmann (B) considering the initial momentum distribution of the bottom quark as a delta distribution at p=10 GeV

In Fig. 4 we present the momentum distribution for charm quark within the Boltzmann equation, is evidently very different evolution of the particles momentum andes not have a Gaussian shape. Already at

$t = 2\,\mathrm{fm/c}$ it has very different spread in momentum with a larger contribution from processes where the charm quark can gain energy and a long tail at low momenta corresponding to some probability to loose a large amount of energy and in general a shape that is not of Gaussian form. This essentially indicates that for a particle with $M \sim \langle p \rangle \sim 3T$ as it is for the charm quark at a temperature $T = 0.4\,\mathrm{GeV}$, the evolution is not of Brownian type. For the bottom quarks, shown in Fig. 5, the momentum evolution gives a much better agreement between the Boltzmann and the Langevin evolution because $M_{bottom}/T \simeq 10$. It would be interesting to find observables that are sensitive to such details of the HQ dynamics. A first candidate could be the $D\bar{D}$ and/or $B\bar{B}$ correlation (Zhu *et al.*, 2008) that should be quite different in a Langevin dynamics with respect to the Boltzmann dynamics.

Comparison with Experimental Observables

The Langevin and Boltzmann equation have been solved for the heavy quarks with the initial condition mentioned previously. We convolute the solution with the fragmentation functions of the heavy quarks at the transition temperature T_c to obtain the momentum distribution of the heavy mesons (B and D). Peterson function has been used for heavy quark fragmentation given by:

$$f(z) \propto \frac{1}{[z[1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}]^2]} \tag{10}$$

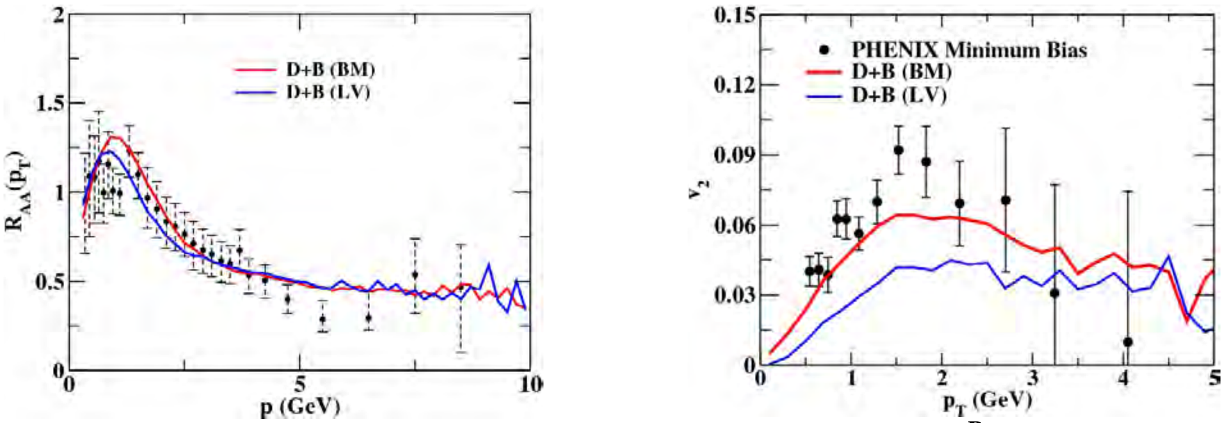
for charm quark $\epsilon_c = 0.04$. For bottom quark $\epsilon_b = 0.005$.

One of the key observable, investigated at RHIC and LHC energies, is the depletion of high p_T particles (D and B mesons or single $e^{\pm}$) produced in heavy-ion collisions with respect to those produced in pp collisions. We calculate the nuclear suppression factor, R_{AA}, using our initial $t = 0$ and final $t = t_f$ Heavy meson (D or B) distribution as $R_{AA}(p) = \frac{f(p,t_f)}{f(p,t_0)}$. The anisotropic momentum distribution resulting from spatial anisotropy of the bulk can be calculated to the quantity v_2 where:

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle , \tag{11}$$

is the the momentum space anisotropy.

As we mentioned earlier it is an contemporary issue for all the models to describe the R_{AA} and $v2$ simultaneously for the same set of inputs. In Fig. 6 we have shown the variation of R_{AA} as a function of p_T. In the present study we try to reproduce the same R_{AA} (almost) from both the LV and BM side and studied their corresponding v_2. In Fig. 6 we have plotted the variation of v_2 as a function of p_T calculated from both the LV and BM side. We have found that for the same inputs used in R_{AA}, BM calculations produce more v_2. The present calculation is performed at m_D=1.6 GeV (isotropic crose section) to demonstrated the maximum effect. It is also found that with the nearly isotropic cross section one may reproduce the R_{AA} and v_2 simultaneously within the BM approach where as the LV dynamics fail to do so.



A **B**

Fig. 6: (A) The nuclear suppression factor, R_{AA} as a function of momentum from the Langevin (LV) equation and Boltzmann (BM) equation at m_D = 1.6 GeV at RHIC energy; (B) the elliptic flow, v_2 as a function of momentum from the Langevin (LV) equation and Boltzmann (BM) equation and m_D = 1.6 GeV

Conclusion

We have briefly review the interest for the Heavy Quark dynamics in the QGP. After recalling that charm and bottom quarks can be considered heavy because both m_Q/Λ_{QCD} and m_Q/T are much larger than unity. However a more closer look into the physics involved tells that there is another scale to be considered $m_Q/ < p_{bulk} >= m_Q/3T$. For this last scale the charm cannot be considered really heavy at $T \sim 300$ MeV. In fact comparing the momentum evolution of a charm quark solving the full Boltzmann integral equation shows a dynamical evolution that appear to be quite far from a Brownian motion. This can lead to underestimation of the charm quark drag coefficient and the build-up of its elliptic flow $v_2(p_T)$. We found that using nearly isotropic cross section both the nuclear suppression R_{AA} and elliptic flow v_2 can be describe simultaneously within the Boltzmann approach where the Langevin dynamics fail to do.

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