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Beta Equilibrated Quark Matter in PNJL Model

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We present a first case study of the phase diagram of 2+1 flavor strongly interacting matter in $\beta-$equilibrium using the Polyakov $-$ Nambu $-$ Jona-Lasinio (PNJL) model. Phase diagram, charge neutral trajectories, isentropic trajectories and the equation of states are discussed and compared with the corresponding situation in the NJL model.

Key Words : Beta Equilibrium; Charge Neutrality; Phase Diagram; Isentropic Trajectories; Speed of Sound

Introduction

Under a variety of extreme conditions the occurrence of hadron to quark phase transition is now established. The quark gluon plasma phase is believed to have existed in the hot early Universe, a few microsecond after the Big Bang. Deconfined quark matter could also exist in the core of neutron stars (NS), where the density is high. The present study is expected to give us a better insight into the role that the superdense matter created in heavy ion collision experiments play in our understanding of the properties of matter inside the core of supermassive stars in the Universe (Bhattacharyya *et al.*, 2012).

Formalism

Following Witten's conjecture (Witten, 1984) that the strange quark matter, containing almost equal numbers of u, d and s quarks, may be the ground state of strongly interacting matter, there is a possibility of the existence of self-bound pure strange stars. The dominant reaction mechanism by which the strange quark production in quark matter occurs is the non-leptonic weak interaction process (Ghosh *et al.*, 1996).

$$u_1 + d \leftrightarrow u_2 + s \tag{1}$$

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Initially when the quark matter is formed, $\mu_d > \mu_s$, and the above reaction converts excess d quarks to s quarks. But in order to produce chemical equilibrium the semileptonic interactions,

$$d(s) \rightarrow u + e^- + \bar{\nu}_e \tag{2}$$

$$u + e^- \rightarrow d(s) + \nu_e \tag{3}$$

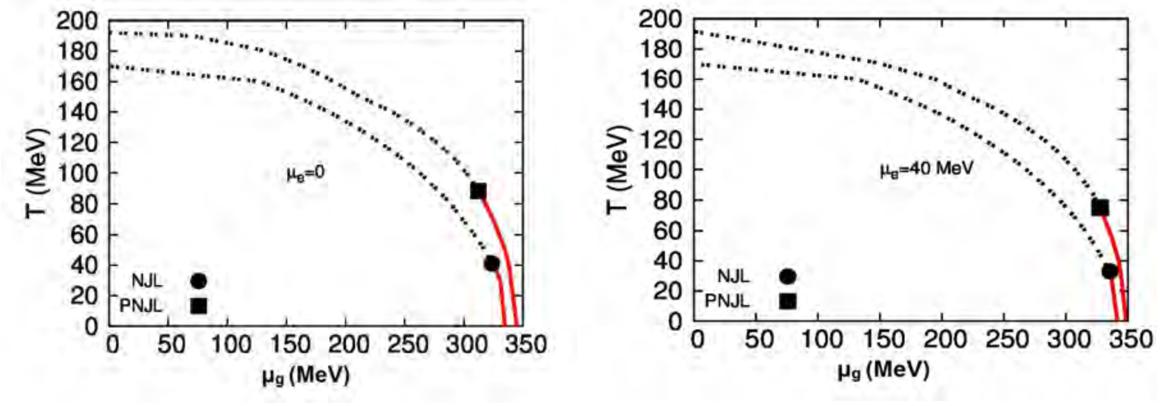
play important role along with the above non-leptonic interactions. These imply the $\beta-$equilibrium condition μ_d=μ_u+μ_e+$\mu_{\bar{\nu}}$; and μ_s=μ_d. We safely assume that neutrino gets enough time to leave the system; $\mu_\nu = 0$. The thermodynamic potential of 2+1 flavor PNJL model for non-zero quark chemical potential is (Bhattacharyya *et al.*, 2010)

$$\begin{aligned}
\Omega &= \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s - 6\sum_{f=u,d,s}\int_0^\Lambda \frac{d^3p}{(2\pi)^3}E_f \\
&\quad -2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1 + 3\Phi e^{-\frac{(E_f-\mu_f)}{T}} + 3\bar{\Phi}e^{-2\frac{(E_f-\mu_f)}{T}} + e^{-3\frac{(E_f-\mu_f)}{T}}\right] \\
&\quad -2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)}^3\ln\left[1 + 3\bar{\Phi}e^{-\frac{(E_f+\mu_f)}{T}} + 3\Phi e^{-2\frac{(E_f+\mu_f)}{T}} + e^{-3\frac{(E_f+\mu_f)}{T}}\right]
\end{aligned}$$

Electrons are described by the free non-interacting gas of fermions. Corresponding thermodynamic potential is:

$$\Omega_e = -(\frac{{\mu_e}^4}{12\pi^2} + \frac{{\mu_e}^2T^2}{6} + \frac{7\pi^2T^4}{180})$$

Thermodynamic potential is extremised w.r.t the fields ($\Phi, \bar{\Phi}, \sigma$) under beta equilibrium condition.



A

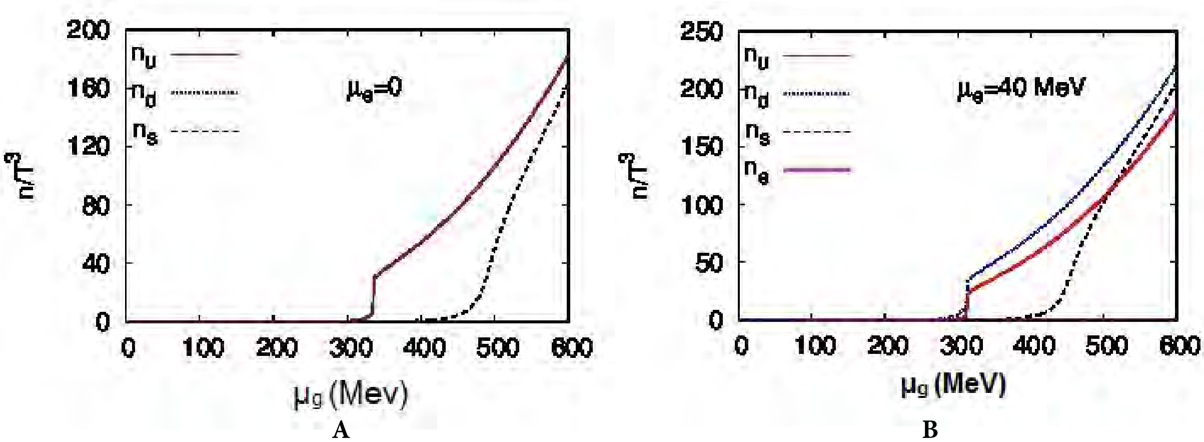
B

Fig. 1: Comparison of phase diagram in NJL and PNJL model at $\beta-$equilibrium for (A) μ_e=0 and (B) μ_e=40 MeV. The solid circle and square represent the CEP for NJL and PNJL model respectively

Results

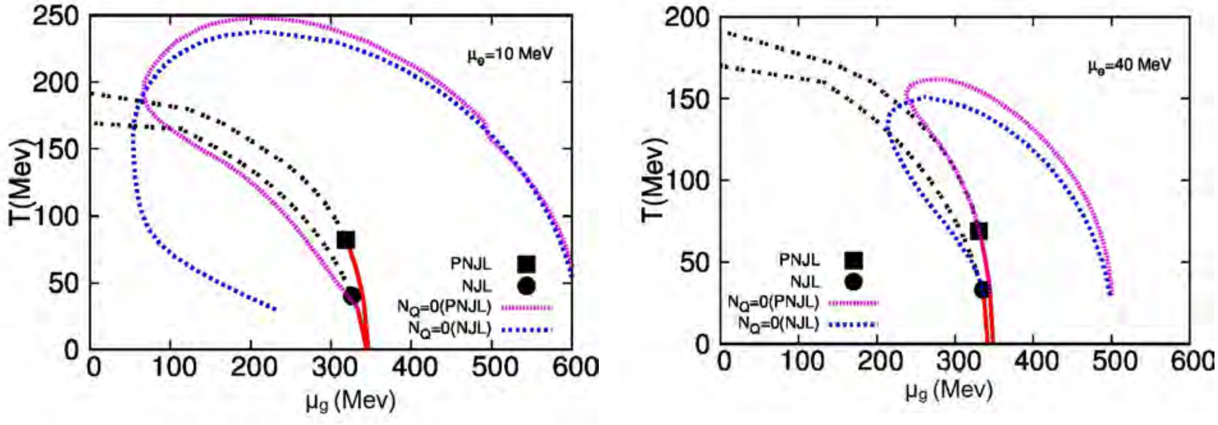
The phase diagrams for NJL and PNJL models are obtained from the behavior of the mean fields, and are shown in Fig. 1(a) and Fig. 1(b) for $\mu_e = 0$ MeV and $\mu_e = 40$ MeV respectively. While the broad features remain same, difference between the NJL and PNJL models arises mainly due to the Polyakov loop, whose presence is primarily responsible for raising the transition/crossover temperature in the PNJL model. Thus the CEP for PNJL model occurs at slightly higher T and lower μ_q compared to NJL model.

Next, we study the quark number densities for vanishing and non-vanishing electron chemical potential.



A **B**

Fig. 2: Quark number densities in NJL and PNJL model at $\beta-$equilibrium for (A) μ_e=0 and (B) μ_e=40 MeV



A **B**

Fig. 3: Comparison of charge neutral trajectory in NJL and PNJL model at (A) μ_e=10 MeV and (B) μ_e=40 MeV

From Fig. (2) it can be seen that for $\mu_e = 0$, $n_e = 0$ and we have n_u = n_d. At large μ_q, the number density n_s of strange quarks become almost equal to the light quark number densities as the constituent masses of strange quarks are reduced significantly. So the net charge density n_Q will be close to zero and the system will become charge neutral asymptotically. The charge neutral contours for NJL and PNJL models are shown in Fig. (3). In deconfined region differences subside as the Polyakov loop relaxes the confining effect, unlike hadronic phase.

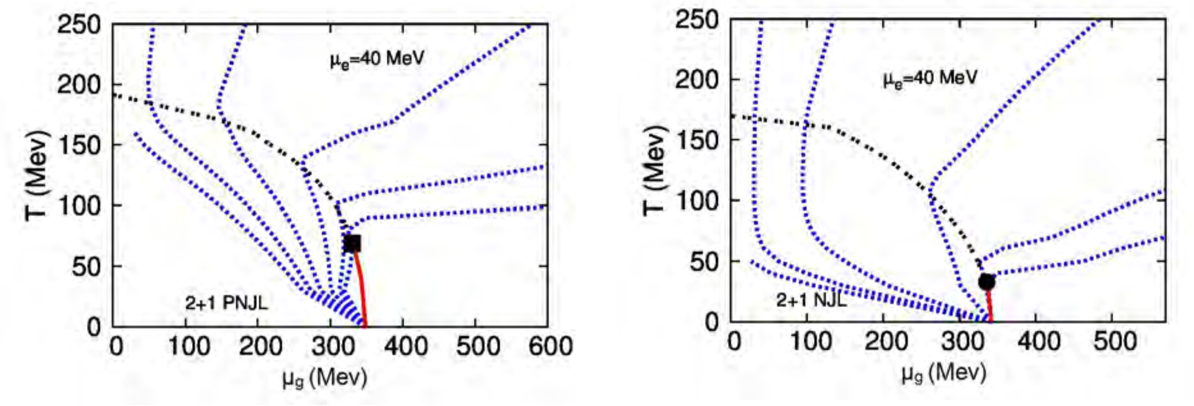


Fig. 4: The isentropic trajectories along with phase diagram for (A) at μ_e=40 MeV, 2+1-flavor PNJL, (B) at μ_e=40 MeV, 2+1-flavor NJL model. s/n_B=300,100,30,10,5,3.5 (from left)

Isentropic trajectories for 2+1 flavor PNJL and NJL model are shown. As seen from Fig. 4(B), even for low T and μ_q there is a significant entropy generation as there is no Polyakov loop to subdue the same. Similar differences continue to appear even in the partonic phase.

Next, in Fig. (5), the behavior of speed of sound along with chemical potential is shown for NJL and PNJL model as well.

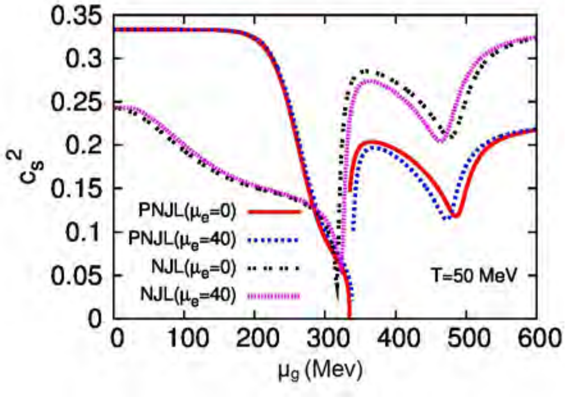


Fig. 5: Speed of sound for NJL and PNJL models at $T = 50$ MeV

As we see, for low μ_q quark d.o.f is suppressed due to Polyakov Loop and free electrons push c_s^2 to SB limit in PNJL model. At high μ_q, $n_q \gg n_e$ and residual intearaction inhibits c_s^2 to attain SB limit. But for NJL model those quasi-particles with heavy constituent masses tend to lower the c_s^2.

Summary

A comparative study of beta equilibrated 2+1 quark matter is done with NJL and PNJL model. Effect of Polyakov Loop strongly put imprints on thermodynamic properties which are found to be qualitatively same but quantitatively different.

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