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ARTICLE

Insights from the computation of the duration of day in *Vedāṅga-jyotiṣa*

Varuneshwar Reddy Mandadi[1] · **D. G. Sooryanarayan**[1] · **K. Ramasubramanian**[1]

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Abstract
Vedāṅga-jyotiṣa (c. 1400 BCE), the earliest known Indian astronomical text, introduces one of the first instances in Indian literature of a formula in the form of a linear equation connecting the length of the day with the number of days elapsed since the winter solstice. The paper critically examines this equation, highlighting its significance as an early example of abstract mathematical representation and for igniting a scholarly debate concerning the geographical origins of this text based on the latitude suggested by its day-night duration calculations. This study contests earlier hypotheses positing Mesopotamian influence and instead shows that the data optimally fits the lengths of the daytime as observed in the latitudes around 29° N, located well within India, and is totally inapplicable to Mesopotamian latitudes except at the solstices. The investigation extends to similar linear equations in subsequent *Siddhānta* texts, assessing their mathematical and geographical relevance.

Keywords Indian mathematics and astronomy · History of science · Length of the day · *Vedāṅga-jyotiṣa* · Linear approximation

1 Introduction

The *Vedāṅga-jyotiṣa* holds a crucial place in the annals of ancient Indian astronomy. The text, dated in the range 1150–1370 BCE (Sastry & Sarma 1984, p. 13), is the earliest extant Hindu treatise dedicated solely to astronomical calculations, marking a significant phase in India's intellectual tradition. Among its mathematical formulations is a linear equation designed to compute the varying lengths of day and night. This equation not only reflects the abstract mathematical reasoning present in early India but also provides a rare glimpse into the application of such concepts for practical celestial observations. This equation is of great significance for two reasons:

(1) It represents one of the earliest known linear expressions in Indian astronomy, offering evidence of them as early as 1350 BCE.
(2) It serves as a nexus for historical and geographic inquiry, particularly in the debate over the *Vedāṅga-jyotiṣa*'s place of origin.

This debate is fueled by the equation's implication of the latitude of the place of observation based on the proportion of daytime to nighttime at the winter solstice. The fact that this ratio was 2:3, which is valid around 35° N, led (Thibaut, 1899) to conjecture that this could be based on Babylonian sources. Once Babylonian tables with the ratio 2:3 were found, Kugler (1900) made this claim formally and it was supported by Neugebauer (1952) and later articulated more forcefully by Pingree (1963, 1973). This claim has been countered by the Japanese scholar (Ohashi, 1993, 2009, 2012, 2019) who has argued that the *Vedāṅga-jyotiṣa* formula better corresponds to the variation of the duration of the day as observed in latitudes 27–29° N.

Given the importance of resolving this debate, this study undertakes a systematic, data-driven analysis to evaluate the equation's accuracy across different latitudes. Using a mean absolute error analysis, the research validates that the equation aligns best with 27–29° N, reinforcing the argument

Varuneshwar Reddy Mandadi
varunesh.india.39@gmail.com

D. G. Sooryanarayan
soorya.narayan@iitb.ac.in

K. Ramasubramanian
mullaikramas@gmail.com

[1] Cell for Indian Science and Technology in Sanskrit (CISTS), Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

for an independent tradition of mathematical astronomy in India.

Furthermore, the study examines similar formulations in later *Siddhānta* texts, particularly the *Vasiṣṭha-siddhānta* as summarised by Varāhamihira (Sastry 1993; Thibaut and Dvivedī 1889) demonstrating a continuity of mathematical thought in Indian astronomical traditions. The *Vasiṣṭha-siddhānta*'s formulation, which is designed for 23–24° N (corresponding to Ujjain or Avanti), further supports the view that these mathematical models were adapted to specific geographic contexts. The research also challenges the 'solstitial fixation' hypothesis by demonstrating that the *Vedāṅga-jyotiṣa*'s equation was designed for a best-fit across the entire *ayana* (6-month period), rather than solely for solstices (Van der Waerden, 1974).

The paper is structured in the following way. Section 2 examines the astronomical knowledge during the Vedic period, emphasizing the significance of the *Vedāṅga-jyotiṣa*. Section 3 analyzes the text's linear relation for finding the day duration. The ensuing subsections also address debates on its geographic implications and conduct a data-driven analysis for the latitudes with the least mean absolute error. Section 4 explores a similar linear equation in the *Vasiṣṭha-siddhānta* and assesses its mathematical and geographical relevance. Section 5 concludes with key insights and broader implications of the study for the understanding of Indian mathematical astronomy.

2 Vedic astronomy, *Vedāṅga-jyotiṣa* and its significance

2.1 Prevalent astronomical notions of the Vedic period

Several terms pertaining to astronomical concepts and frameworks can be gleaned from the Vedic literature. In fact, the *Chāndogya Upaniṣad* (7.1) (Swahananda, 2022) explicitly refers to the science of astronomy by the word *Nakṣatra-vidyā* while enumerating almost a couple of dozens of different branches of knowledge. This not only indicates the antiquity of these branches, but their pursuit as independent disciplines.

K.S. Shukla's survey (1987) of Vedic literature highlights the Sun as the illuminator of the universe and refers to its role of instigating seasons and governing the world as the ultimate controller (*Ṛgveda* 8.58.2, 1.95.3, 1.164.14) and the Moon as "*Sūrya-raśmi*", shining due to sunlight (*Taittirīya-saṃhitā* 3.4.7.1). The Moon's path was divided into 27 segments, identified as *nakṣatras*, with the occasional inclusion of *Abhijit* constellation (Lyra) making 28. The *śatapatha-brāhmaṇa* (10.5.4.5) lists 27 *nakṣatras* and their *upa-nakṣatras*.

In Vedic literature, the day is referred to as *vāsara* or *ahan*. It typically represented the time interval between one sunrise and the next, acknowledging its variable duration. The division of daylight duration, measured from sunrise to sunset, included categorizations into two parts: *pūrvāhṇa* (forenoon) and *aparāhṇa* (afternoon). Further divisions encompassed three parts (*pūrvāhṇa*, *madhyāhna*, and *aparāhṇa*), four parts (*pūrvāhṇa*, *madhyāhna*, *aparāhṇa*, and *sāyāhna*), and five parts (*prātaḥ*, *saṅgava*, *madhyāhna*, *aparāhṇa*, and *sāyāhna*), as detailed in the *Śatapatha-brāhmaṇa* (2.2.3.9). Days and nights were further subdivided into 15 parts each, termed *muhūrta*. Specific names were assigned to the *muhūrtas* occurring during both the days and nights of the waxing and waning fortnights (*Taittirīya-brāhmaṇa*, 3.10.1.1–3).

2.2 Astronomy in the *Vedāṅga-jyotiṣa* and its significance

The *Vedāṅga-jyotiṣa* is preserved in two recensions, namely the *Ṛgvedic* recension (*ārca-jyotiṣa*) and the *Yajurvedic* recension (*Yājuṣa-jyotiṣa*). Both versions share substantial similarity (Sastry & Sarma, 1984). The former comprises 36 verses, while the latter consists of 43 verses, with 31 verses being common to both. Both recensions expound on the *tithi* (lunar day), *nakṣatra*, and the month when the Sun embarks on its northward and southward journeys (*uttarāyaṇa* and *dakṣiṇāyana*) in the 5-year cycle. Additionally, they delineate the variations in daylight during these solar journeys in terms of a certain measure called *prastha* of water (see Sect. 3.1). This implies that they were using water-clock measurements.

A third recension, the *Atharva-jyotiṣa*, emerged later, introducing the names of the seven *grahas*, weekdays, and, in addition to the already known *tithi*, *nakṣatra*, and *yoga*, the names of the seven *karaṇas* in the Indian traditional calendar (Shukla, 1969). Comprising 162 verses, the *Atharva-jyotiṣa* spans both astronomy and astrology, attributing its teachings to Svayambhū and Bhṛgu.

During the chronological span from around 500 BCE to 500 CE, not many scientific works have survived. This has created a substantial void in the knowledge tradition of astronomy literature during this period. Thus, the *Vedāṅga-jyotiṣa* presenting the body of prevalent knowledge around 1400 BCE forms a crucial text to appreciate the nuanced scientific thought process of those times. It is indeed interesting to learn how computations are dictated in this text and the following section helps understand one such equation presented in this ancient text.

3 Algebraic approach to the computation of a day's duration

3.1 Two verses of *Vedāṅga-jyotiṣa* that are of interest

Here are the verses that are relevant to the computation. The first verse (*Ṛgvedic* recension v. 7 and *Yajurvedic* recension v. 8) (Shamasastry, 1936) reads:

घर्मवृद्धिरपां प्रस्थः
क्षपाह्रास उदग्गतौ ।
दक्षिणेतौ[1] विपर्यासः
षण्मुहूर्त्ययनेन तु ॥
gharmavṛddhirapāṃ prasthaḥ
kṣapāhrāsa udaggatau ।
dakṣiṇetau viparyāsaḥ
ṣaṇmuhūrtyayanena tu ॥
The increase of daytime and decrease of night-time is [the time equivalent of] one *prastha* of water [in the clepsydra per day] during the northward course [of the Sun]. They are in reverse during the southward course. [The total difference is] 6 *muhūrtas* during a half-year.

The second verse (*Ṛgvedic* recension v. 22 and *Yajurvedic* recension v. 40) (Shamasastry, 1936) reads:

यदुत्तरस्यायनतो गतं स्यात्
शेषं तथा दक्षिणतोऽयनस्य ।
तदेकषष्ट्या द्विगुणं विभक्तं
सद्वादशं स्याद्दिवसप्रमाणम् ॥
yaduttarasyāyanato gataṃ syāt
śeṣaṃ tathā dakṣiṇato'yanasya ।
tadekaṣaṣṭyā dviguṇaṃ vibhaktaṃ
sadvādaśaṃ syāddivasapramāṇam ॥
[The number of days] elapsed in the northward course or remaining in the southward course is doubled, divided by 61, and added to 12. The result is the length of daytime [in terms of *muhūrtas*].

3.2 The linear equation given in the verse

The formula for the computation of daytime given above can be expressed as

$$T = 12 + \frac{2}{61} n, \tag{1}$$

where T is the length of daytime in terms of *muhūrta*s and n is the number of days elapsed from or remaining until the winter solstice. One *muhūrta* is $\frac{1}{30}$ of a civil day—or 0.8 h. Thus, from the above equation, we see that according to *Vedāṅga-jyotiṣa*, the duration of the day is a linear function, where the length of daytime changes by roughly one *muhūrta* during one solar month.

3.3 Significance of the expression and the way it has been expressed

It is quite fascinating that Eq. (1) not only presents a decent linear approximation that captures the variation in the duration of daytime, but it is perhaps the most antique algebraic equation presented in the Indian astronomy–mathematics tradition. Variable quantities used to be denoted by many terms in the Indian tradition. While investigating the earliest known sources of representing unknowns in the Indian mathematical traditions, Datta and Singh (1935) indicate that in *Sthānāṅga-sūtra* (before 300 BCE) the unknown quantity was called *yāvat-tāvat* (as many as or so much as, meaning a variable that can take arbitrary value).

The verse under discussion uses *yat-tat*. This opens up the scope for *Vedāṅga-jyotiṣa* to be a precursor to the *yāvat-tāvat* and many other terms, such as *yā*, *kā*, and *nī*. This extends the known history of algebraic abstraction in India significantly further back than previously acknowledged, indicating an early engagement with mathematical reasoning. There is also another ancient example of a similar algebraic formulation using *yāvat* in the *Kātyāyana-śulbasūtra* while presenting the method of constructing a square which is *n-times* a given square. Here, the variable nature of n is brought out by the use of *yāvat*.

3.4 Debate around the place of origin of the composition of *Vedāṅga-jyotiṣa*

In order to appreciate this debate, let us take a closer look at the import of Eq. (1). According to this, the duration of the daytime (T in *muhūrta*s) at the winter and summer solstice are given by

$$\begin{aligned} T &= 12 + 0 = 12 \qquad \text{(since } n = 0 \text{ at the winter solstice)}, \\ T &= 12 + 6 = 18 \quad \text{(since } n = 183 \text{ at the summer solstice)}. \end{aligned} \tag{2}$$

Thus, the ratio of daytime and night-time is exactly in the proportion 2:3 at the time of winter solstice. This proportion is akin to the one given by Mesopotamians and seems to be valid around the latitude 35° N. It was conjectured by Thibaut (1899, p. 27) that this formula was transmitted to

[1] In both the published editions of the *Vedāṅga-jyotiṣa* (Shamasastry, 1936; Sastry & Sarma, 1984), a typographical oversight occurs in the reading: " दक्षिणे तौ". The critical issue lies in the incorrect segmentation of the word, failing to recognize that it is a compound word formed from merging दक्षिण and इतौ. In the split form, it doesn't semantically fit with the content of the verse. Fortunately, we have got hold of a manuscript that correctly uses the compounded form, in line with the emended version of the verse as presented above.

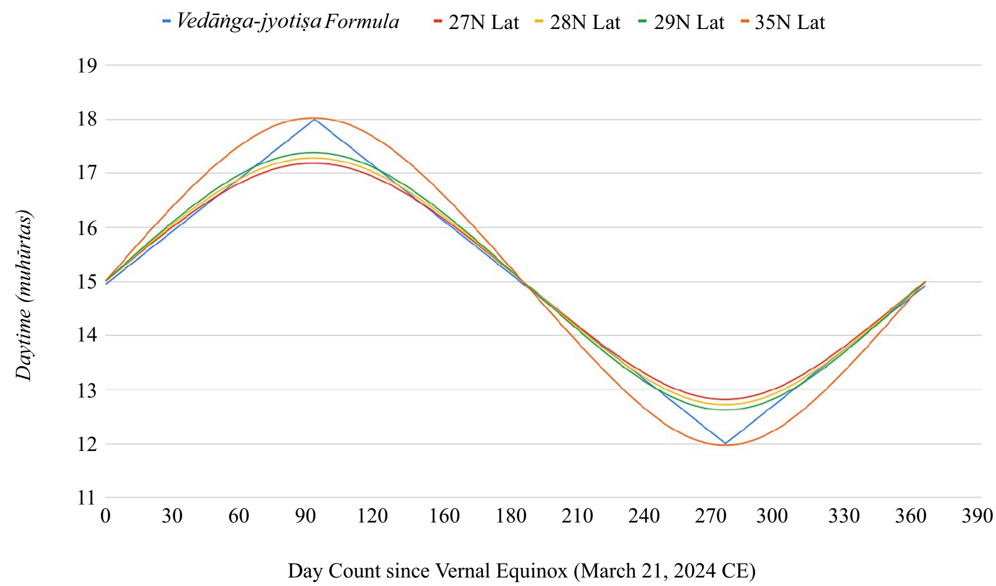

Fig. 1 Annual variation of the length of daytime in *muhūrta* starting from the vernal equinox for different latitudes compared to the linear expression in *Vedāṅga-jyotiṣa*

India from Mesopotamia. Once the Babylonian table for the duration of daylight with the ratio 2:3 between the minimum and maximum values was found, Kugler (1900, p. 82) made the formal claim and this was supported by Neugebauer (1952, p. 156) and more forcefully by Pingree (1963, 1973), who asserted that the *Vedāṅga-jyotiṣa* formula was based on the Babylonian tables. Although this latitude, which passes through Kashmir, was not inaccessible to ancient Indian people, the claim that the formula pertained to latitudes around 35° N and was based on Babylonian tables has been countered in several publications by the Japanese scholar (Ohashi, 1993, 2009, 2012, 2019). Some of the arguments of Ohashi are:

> ... that formula (1) was not obtained by the interpolation of the *Jyotiṣa-vedāṅga* observations at the solstices, but by the extrapolation from the observations of the length of daytime around the equinoxes. Practically, there are two possibilities:
> 1. If the formula were obtained from one *muhūrta*'s difference of the length of daytime during one solar month after the equinox, the most suitable latitude for this observation would be around 27° N.
> 2. If it were obtained from two *muhūrta*s' difference during two solar months, the most suitable latitude would be around 29° N.
> From the above consideration, I conclude that the *Jyotiṣa-vedāṅga* was produced at the latitude 27–29° N or so in north India (most probably the western part of the plain of the Gaṅgā where later Vedic people resided) without apparent foreign influence.

Essentially, what Ohashi conveys is that the latitudinal location proposed by Pingree is unacceptable. This is because a jarring difference between the calculation and the reality will be experienced by all, throughout the calendar year, if a latitude of 35° N is taken as the location where *Jyotiṣa-vedāṅga* was composed (see Sect. 3.5). Moreover, in the tradition, as time was considered an integral part of the ritual, the determination of time of rituals was crucial. Hence only those latitudes where the curve denoting the actual duration of the day is close to this linear approximation, given by (1), could have been proposed. In the next section, we investigate the latitudes where the error will be the least while using this linear function.

3.5 Analysis to identify the place of origin

We find that Ohashi's arguments are mathematically spot on, as we plot the linear function (1) and compare it to the actual durations of days for various latitudes. Figure 1 graphically represents the variations in the actual length of the duration of day from one vernal equinox to the next vernal equinox at specific latitudes[2] and also charts the formula given by (1).

In order to generate the graph depicted in Fig. 1, we need to know the actual duration of the daytime at a given location, which varies with the latitude of the observer. For this

[2] It may also be mentioned that Ohashi (2012) also made a graphical comparison of the linear function (1) with the daytime variations at these latitudes.

purpose, the length of the day for different latitudes was calculated using the well-known equation

$$\Theta(\varphi, \delta) = \text{Duration of day (in h)} = 12 \pm \sin^{-1}(\tan\varphi \tan\delta), \quad (3)$$

where φ is the observer's latitude and δ the Sun's declination. The declination was computed using the relation $\sin^{-1}(\sin\varepsilon \sin\lambda)$ where ε is the obliquity of the ecliptic and λ is the longitude of the Sun. The data of the longitude of the Sun for a given day was obtained from NASA Jet Propulsion Laboratory, California Institute of Technology (2025). For different latitudes, the length of a day's duration was computed using (3) and was plotted against the day of the year.

With the intent of determining the latitudes that best correspond to (1) given in *Vedāṅga-jyotiṣa*, the mean absolute errors are compared. The absolute difference between the derivative of the actual duration and the derivative of the linear function gives the estimate of error. The derivative of linear function yields a constant value of 0.026229 h (converted from *muhūrtas*). The derivative of the actual duration is estimated as the absolute difference between the duration of a day and its previous day. The absolute error at a given latitude (φ) and declination of Sun on the nth day (δ_n) is given by

$$\Delta T_n = \left| T_n - \Theta(\varphi, \delta_n) \right|, \quad (4)$$

where T_n is the duration found using the formula in the text (*Vedāṅga-jyotiṣa* in this case) and $\Theta(\varphi, \delta_n)$ is the actual duration of the day at a given latitude φ. Figure 2 shows that the mean absolute error is the least for latitude 29° N.

This empirical validation strengthens the argument that the formulation in *Vedāṅga-jyotiṣa* arose from indigenous observational traditions rather than being derived from Mesopotamian sources. In other words, the numerical analysis reinforces the existence of an independent Indian tradition of mathematical astronomy. This makes Ohashi's estimates highly tenable, compelling us to reconsider Pingree's arguments regarding the place of origin of *Vedāṅga-jyotiṣa*, as they appear less convincing in comparison.

Moreover, the findings challenge the assumption that the equation was intended primarily for solstices. Instead, the model exhibits a best-fit across the entire *ayana* (6-month period), suggesting that the formulation was designed for practical calendrical applications rather than only solstitial observations.

4 Similar linear equations in earlier Siddhānta texts

The only text that captures the astronomical schools of thought prior to 500 CE is a compilation by Varāhamihira (6th century CE). Varāhamihira in his work *Pañcasiddhāntikā* condenses the contents of the five *siddhānta*s from this period, namely

(1) *Paitāmaha-siddhānta*,
(2) *Saura-siddhānta*,
(3) *Vasiṣṭha-siddhānta*,
(4) *Romaka-siddhānta*,
(5) *Pauliśa-siddhānta*.

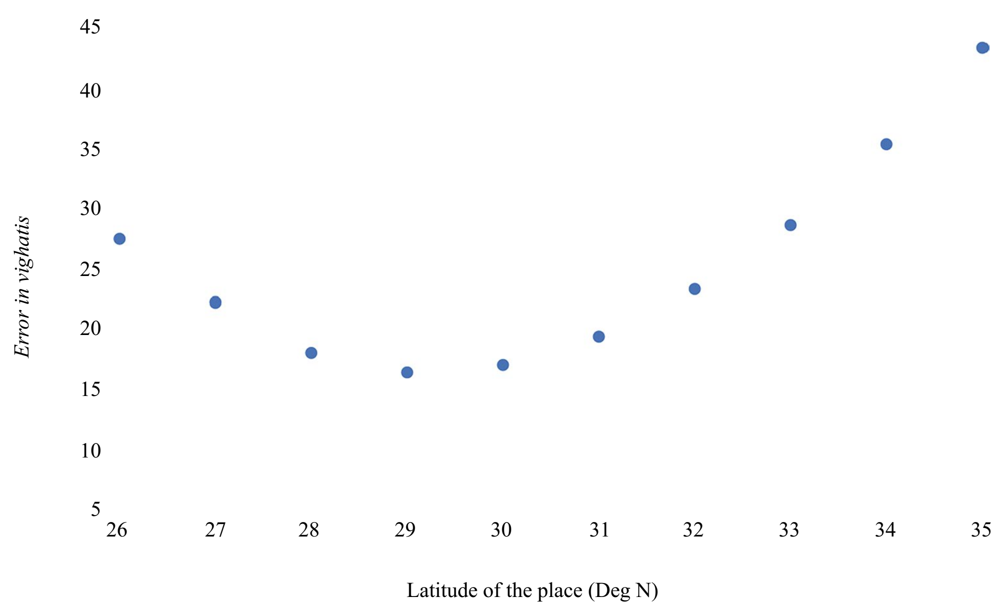

Fig. 2 Mean Absolute Error when *Vedāṅga-jyotiṣa* formula is applied to different latitudes

None of these five *siddhānta*s remain independently in their original form today. Within the extant summaries of these *siddhānta*s, only (1), (3) and (5) delve into the computation of day and night durations. Of these, the fifth verse of *Paitāmaha-siddhānta* (Sastry, 1993) essentially captures the same linear expression as stated in *Vedāṅga-jyotiṣa*. However, the eleventh and twelfth verses of *Pauliśa-siddhānta* (Sastry, 1993) present a relation which captures the shadow lengths and their variations since the equinoctial observation.

It is only in the *Vasiṣṭha-siddhānta* that we find an expression that is different from that of *Vedāṅga-jyotiṣa*. Here is the eighth verse Thibaut and Dvivedī (1889) of *Vasiṣṭha-siddhānta* that deals with the duration of the daytime (*dinamānam*)[3]:

मकरादौ गुणयुक्तो भूस्वर्गतितिथिमितो रवेर्दिवसः ।
कर्कटकादिषु षट्सु त्रयस्त्रिकाः शर्वरीमानम् ॥ ८ ॥
makarādau guṇayukto bhūsvargatitithimito raverdivasaḥ ।
karkaṭakādiṣu ṣaṭsu trayastrikāḥ śarvarīmānam ॥

The Sanskrit commentary on this verse presented by Thibaut and Dvivedī (1889) is:

इदानीं दिनमानमाह — मकरादाविति ।
मकरादौ दिने दिने भूस्वर्गतितिथिमित [4] एकनवपञ्चैकमितः (१५९१) त्रिभिस्त्रिभिः पलैर्युतस्तदा दिनमानं भवेदेवम् ।
कर्कटकादौ तदेव शर्वरीमानं रात्रिमानं भवति ।
अत्रोपपत्तिः । अवन्तिकायां मकरादौ परमाल्पदिनमानं पलात्मकं भूनवपञ्चैकमितं प्रकल्प्य ततः प्रतिदिनं पलत्रयस्य वृद्धिः कल्पिता दिनमाने । अथ मकरादौ यद्दिनमानं तदेव कर्कटकादौ रात्रिमानं भवतीति तदेव कर्कटकादौशर्वरीमानमुदितमाचार्येणेति सर्वं निरवद्यम् ।
idānīṃ dinamānamāha । *makarādāviti* ।
makarādau dine dine bhūsvargatitithimita ekanavapañcaikamitaḥ 1591 tribhistribhiḥ palairyutastadā dinamānaṃ bhavedevam ।
karkaṭakādau tadeva śarvarīmānaṃ rātrimānaṃ bhavati ।
atropapattiḥ । *avantikāyāṃ makarādau paramālpadinamānaṃ palātmakaṃ bhūnavapañcaikamitaṃ prakalpya tataḥ pratidinaṃ palatrayasya vṛddhiḥ kalpitā dinamāne* । *atha makarādau yaddinamānaṃ tadeva karkaṭakādau rātrimānaṃ bhavatīti tadeva karkaṭakādau śarvarīmānamuditamācāryeṇeti sarvaṃ niravadyam* ।

English explanatory notes corresponding to this commentary are captured in the same text by Thibaut and Dvivedī (1889) as follows:

> At the beginning of Capricorn the solar day (i.e., here, the *sāvana* day) is measured by 1591 *palas* to which three *palas* have to be added for each day; in the six signs beginning with Cancer three aggregates of three (added daily) give the measure of the night.
> A very rough rule for calculating the length of the day and the night at any time of the year. The shortest day, at Avanti, is supposed to have a length of 1591 *palas* = 26 *nāḍikās* 31 *palas*. In the period intervening between the shortest and the longest day the days are supposed to grow by regular increments of three *palas* and again to decrease by the same amount in the other half of the year. The measure of the days in the one period furnishes at the same time the measure of the nights in the other period.

The equinoctial day consists of 1800 *palas* or 30 *nāḍikās* or 15 *muhūrtas* or 12 h. The verse, by using the phrase *makarādau* ("at the beginning of Capricorn"), indicates that the Sun is at the winter solstice. Thus, the shortest day having 1591 *palas* is equivalent to 13.2583 *muhūrtas* or 10.6067 h. As per the commentary the location of *Vasiṣtha-siddhānta* points to Avanti or Ujjain in the latitudes of 23–24° N. This is evident from the very next verse (*Vasiṣṭha-siddhānta*, verse 9) which states that the zero-shadow day was observed when the Sun was at the zenith during the summer solstice.

In addition to making the observation of the time on the shortest day, probably using the time-measuring instruments such as clepsydra of the unknown era of *Vasiṣṭha-siddhānta*, there is a linear expression which is implied in the verse which states that the increment of 3 *palas* happens per day, all the way to the longest day over 6 months. The verse also states that in the ensuing 6 months the night elongates by the very same 3 *palas*. The rule given in the verse can be expressed as follows:

$$\Delta T = 3\,palas, \quad T_{\min} = 1591\,palas. \tag{5}$$

It should be noted here that T is 1800 *palas* on an equinoctial day. Figure 3 graphically represents the variations in the actual length of the duration of day from vernal equinox to the next vernal equinox at specific latitudes and charts the

[3] The edition brought out by Sastry (1993) contains a different reading of the verse, thereby yielding a formula for the duration of daytime that is essentially the same as the *Vedāṅga-jyotiṣa* formula. Thibaut and Dvivedī (1889) however make an emendation to the text as they felt the text was corrupted in both the manuscripts that were available to them (which is also mentioned in the Sastry (1993)'s edition). For our discussion, we have chosen this emended reading given by them, for it seemed to be fitting much better with the content of the subsequent verses.

[4] The term means planets and denotes 9 in the object-numeral system. The compound स्वर्गति is to be derived as स्वः गतिः येषां ते—those who have motion in the heavens or the space above.

linear function of the equation given in *Vasiṣṭha-siddhānta*. Figure 4 shows that the mean error of the derivative is the least for latitude 23° N. Since the observations were conducted from Avanti, known today as Ujjain whose latitude is close to 23° N, it establishes a clear and direct correlation between the two.

This makes it amply clear that the conjecture made by Pingree to propose the location of the composition of the text based on the fitting of the curve at the extreme solstitial points is a limited view. The verse from *Pañcasiddhāntikā* that has been analysed shows that such linear expressions were given in the early Indian texts to capture the non-linear change based on the best fit for the entire range of the two *ayanas* and not by merely looking at the lengths of day and night at solstices. In other words, the data-driven analysis is not in favor of Pingree's conjecture.

Moreover, the presence of similar linear formulations in the *Paitāmaha-siddhānta*, *Pauliśa-siddhānta*, and *Vasiṣṭha-siddhānta* in the *Pañcasiddhāntikā*—by virtue of this text being a mere compilation of earlier works—underscores a continued mathematical tradition in Indian astronomy.

5 Conclusion

This study undertook an empirical examination of the linear expressions in the *Vedāṅga-jyotiṣa* and *Vasiṣṭha-siddhānta* to analyze their mathematical and geographical relevance in computing day and night durations. A key finding of this research is the identification of *yat-tat* as an early instance of variable representation in Indian mathematics, which suggests that, in the tradition, *Vedāṅga-jyotiṣa* contains one of the earliest known conceptions of a formula containing a variable, which in this case is the number of days elapsed since the winter solstice.

Our data-driven analysis validates that the *Vedāṅga-jyotiṣa* equation aligns most accurately with latitudes of 27–29° N, reinforcing (Ohashi, 2009)'s argument that its formulation was related to the duration of the day in the latitude region around 29° N and has nothing to do with the duration of the daytime in the Mesopotamian region or with their tabulations in Babylonian texts as claimed by Kugler (1900), Neugebauer (1952), Pingree (1973), and Van der Waerden (1974) and others, by considering the values of the duration of the day merely at the two solstices. The study also provides the first systematic error analysis of the *Vedāṅga-jyotiṣa* equation, using mean absolute error analysis to demonstrate that its linear approximation was designed to best fit the entire *ayana* (6-month period) rather than just solstitial days.

Furthermore, our examination of the *Vasiṣṭha-siddhānta* shows that the formulae given therein for the variation of the day time expressions is different from the one in *Vedāṅga-jyotiṣa* and actually aligns well with the durations associated with latitudes around 23° N, corresponding to Ujjain or Avanti, further emphasizing how Indian astronomical models were adapted to specific geographical contexts. The presence of similar linear expressions in later Siddhānta texts, including the *Paitāmaha-siddhānta* and *Pauliśa-siddhānta*, demonstrates a continuous tradition of mathematical astronomy in India, providing insights into the evolution of these traditions.

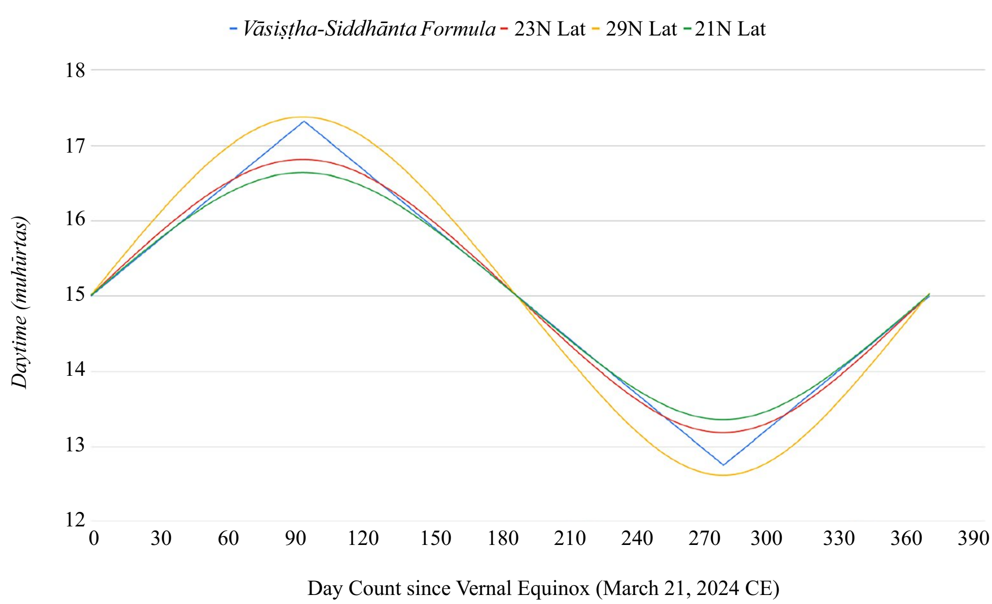

Fig. 3 Annual variation of the length of daytime in *muhūrta* starting from the vernal equinox for different latitudes compared to the linear expression in *Vasiṣṭha-siddhānta*

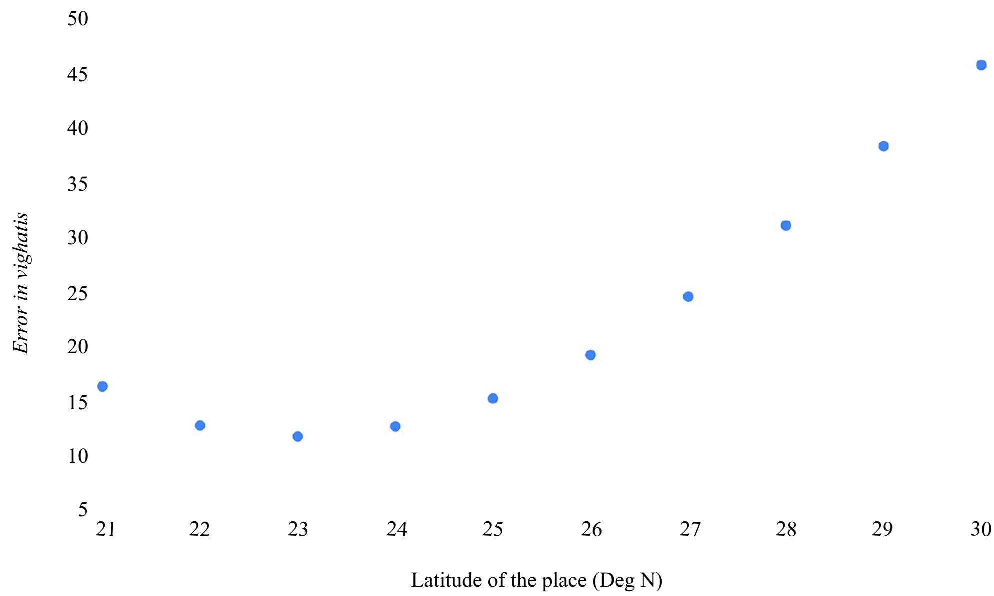

Fig. 4 Mean Absolute Error when *Vasiṣṭha-siddhānta* formula is applied to different latitudes

These findings collectively suggest that ancient Indian scholars were not merely recording observations but were actively engaged in mathematical modeling to approximate nonlinear celestial phenomena. By situating *Vedāṅga-jyotiṣa* within the broader tradition of Indian astronomical sciences, this research contributes to a deeper appreciation of the methodological rigor and indigenous foundations of early Indian mathematical astronomy.

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Author contributions All authors contributed significantly to the study. The historical study was primarily conducted by Sooryanarayan D G, while the deeper mathematical analysis was carried out by Varuneshwar Reddy Mandadi. Sanskrit content was analyzed from its roots by Ramasubramanian K. The first draft of the manuscript was prepared by Varuneshwar Reddy Mandadi and Sooryanarayan D G. Ramasubramanian K reshaped, commented on, and got the manuscript to its final form. All authors read and approved the final manuscript.

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Data availability The datasets generated during and/or analyzed during the current study are available at https://www.scidb.cn/en/s/vuiyei in the Science Data Bank.

Declarations

Conflict of interest The authors declare that there is no conflict of interest.

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