HISTORY OF PLUS AND MINUS SIGNS

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(*Received December 12, 1965; after revision December 21, 1966*)

In this article a brief historical account of various notations adopted to indicate plus and minus signs has been given. The earliest work on the above topic, possible to trace, is that of the Egyptians (1550 B.C.). They used the symbols 𓂻 and 𓂽 for plus and minus signs respectively. But these were not used in the same sense in which we employ them at present. In the works of Diophantus (*c.* 275) the symbol ↑ has been used to designate subtraction. In the ancient Indian Bakshali manuscript (1881) we find that the negative quantity has been denoted by + and that the symbol has been placed after the number affected. The word *yut* (युत) has been used for addition and *yu* (यु) has been written after the affected quantity. In the works of the eighth century, in India, a small circle or dot has been placed above the negative quantity or the subtrahend has been enclosed by a small circle. In the earliest European works (1202) the symbols P^1, P^2 or $\bar{P}$ have been used for plus. In 1456, the word (et) was used for addition in Germany. Later on in the sixteenth century + and − symbols were introduced by the Germans to indicate addition and subtraction. The Dutch mathematician, Vander Hoecke (1514), was the first to use + and − to indicate operations.

In this article an attempt has been made to give a general survey of various notations adopted to indicate 'plus' and 'minus' arranged in a chronological order. The reader will find, in these lines, a kind of moving picture of the changes in notations. Through a consideration of the history of signs, it will be appreciated that mathematics has continually adjusted itself to human needs.

The history of symbols of operations, viz. addition and subtraction, goes back to the Egyptians. Ahmes[1] (*c.* 1550 B.C.) used the symbol 𓂻 to designate addition and 𓂽 for subtraction. These symbols are simply figures of ancient Egyptian writing. They were not used in exactly the same sense in which we employ plus and minus signs at present. It is notable that Ahmes wrote (𓂽), as usual, from right to left, whereas ancient Egyptian sentences are usually printed from left to right.

Diophantus[2] (*c.* 275) wrote a manuscript named *Arithmetica*, of which a copy of the thirteenth century is available. In this copy the addition is represented by a simple juxtaposition, as in $K^{\gamma}\bar{\alpha}\Delta^{\gamma}\,\overline{\iota\gamma}$ for x^3+13x^2 and

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the symbol ↑ has been used to designate subtraction. Since the original manuscript of *Arithmetica* is not available, we are not quite sure whether he wrote the aforesaid symbols as also the following passage:

'Minus multiplied by minus makes plus, and minus by plus makes minus. The sign of negation ψ is turned upside down, ↑.'

Bakhshali Manuscript:

In 1881, at Bakhshali, a village near the city of Peshawar in the north-western corner of India, which is now a part of Pakistan, a manuscript work on mathematics was dug out by a farmer in a ruined enclosure, where it lay between stones. The greater portion of the manuscript was already destroyed and the remainder consists of 70 leaves of birch-bark, quite a few of which are in mere fragments. The works of Dr. Hoernlé are worth reading in this connection.[3, 4, 5]

Regarding the period of the manuscript Hoernlé says, 'I am disposed to believe that the composition of the former (the Bakhshali work) must be referred to the earliest centuries of our era, and that it may date from the third or fourth century A.D.'. This period has also been accepted by historians of mathematics—Cantor and Cajori. Thibaut of Banaras is not quite sure but he has accepted the period between A.D. 700 and 900 as the probable time of the manuscript.

The manuscript is written in ancient Sharda script. The notations used there are generally found in the works of Brahmagupta and Bhaskar. The symbol used to designate a negative quantity is very remarkable. In the manuscript the negative quantity has been denoted by +, which is at present used to indicate a positive quantity and the symbol has been placed after the number affected. Thus

$$\begin{matrix} 13 & 6+ \\ 1 & 1 \end{matrix}$$

means 13—6, i.e. 7.

Nobody knows as to when this symbol was used to indicate a negative quantity. Dr. Hoernlé could not answer the question—from where did the symbol arise? Dr. Thibaut told Hoernlé that Diophantus used the letter ψ upside down (⋔) to indicate a negative quantity. There is some similarity between + and ⋔. Putting forward this plea Dr. Kaye concluded that Indian mathematicians were influenced by Greeks, which is a clear injustice. For, the symbol used by Diophantus was ↑ not ⋔, and ⋔ and + are not very similar. Besides, it has also been satisfactorily proved that Greek mathematicians were influenced by Indians and not vice versa.

It remains to explain the origin of the Bakhshali symbol +. It was customary among Indian mathematicians that they used the first letter of the word to denote its meaning. The word *yut* (युत) has been used for addition and, therefore, they wrote *yu* (यु) after the affected quantity. Thus

5 8
1 1 यु

means 5+8, i.e. 13. It seems that they used the Sanskrit character ऋ (for ऋण meaning loan) to denote subtraction, which was afterwards changed into +.

The letter *ka* (क) of the Ashok script is very remarkable in this connection. There is a great resemblance between *ka* of Ashok script and +. It is possible that this *ka* after a long practice might have reduced to + only, for the sake of convenience in writing. Dr. Hoernlé says that it may be the abbreviation *ka* of the word *kaniyas* (कनीयस) or *nu* of the word *nyun* (न्यून), both of which mean 'diminished' and an abbreviation of each in the Brahmi script would be a cross. This supposition seems very plausible because in the Bakhshali manuscript all other mathematical operations are generally indicated by abbreviations of the words denoting operations. The only anomaly in this supposition is that the words *kaniyas* and *nyun* have never been used in the Bakhshali work to indicate subtraction.

According to an Indian historian of mathematics B. B. Dutta[6], + is a slightly changed form of the letter *ksh* (क्ष) abbreviated from the Sanskrit word *kshaya* (क्षय) ('decrease'), which has been used in the Bakhshali work, more than any other word, to indicate subtraction. The symbol for *ksh*, in Brahmi and Bakhshali script, differs from + only in having a small circular knob at the lower end of the vertical line. The circle might have been dropped after a long practice for convenience of writing. This hypothesis appears to be a very plausible one.

In the manuscripts of Pṛthudakaswami[7] (A.D. 860), Bhaskar[8] (*c.* 1150) and other Indian mathematicians of that time, a small circle or dot is placed above the negative quantity (subtrahend) like thus: 8° or $8^{\cdot}$ to indicate -8, or the negative quantity is enclosed in a circle as

(8) standing for -8.

European Symbols for Plus and Minus:

The word 'minus' indicating an operation is found in the works of Fibonacci (1202). The word 'plus' indicative of addition is not found before the fifteenth century. The earliest European symbol for plus was p^1, p^2 or $\bar{p}$. The last one was commonly used in connection with the Rule of False Position. Similar to the plus symbol $\bar{p}$ or p the minus symbol, used in the fifteenth and sixteenth centuries, was $\bar{m}$ or $\tilde{m}$. As usual, the bar simply indicated an omission as by introducing a bar over *u*, one '*m*' just after *u* is omitted from 'Summa', similar to (¨) in über for ueber. In the fifteenth century the symbol **ry** was also used by some writers to indicate minus instead of $\bar{m}$. In the sixteenth century + and — symbols were preferred by the Germans to indicate addition and subtraction. The existence of the symbols mentioned in this passage is not found before the fifteenth century.

In a manuscript[9] of 1456 written in Germany the word 'et' (meaning 'and' in French) is used for addition and is generally written so that it closely resembles the symbol +; e.g. '4 et 5' for 4+5.

According to some historians of mathematics the sign (—) is the bar taken from **ry** or $\bar{m}$. There are others who believe that it comes from the habit of contracted writings such as suma for summa. The latter hypothesis is the more plausible of the two.

The symbol $\div$ is commonly written for *m* in Uncial writing and $\dot{-}$ in Visigothic writing. Therefore, it is possible that (—) would have been used for *m* (minus) dropping the dot above or below the dash for convenience in writing just as (+) is a symbol for *et*. In D. E. Smith's opinion, its use in this sense may have come from the habit of merchants in indicating a missing number in a case like 2 yd.—3 in., where the number of feet is missing. We have the same habit in writing certain words today, using either a dash or a series of dots.

The symbols + and — were used in Algebra for the operations of addition and subtraction long before their use in Arithmetic in the same sense. Wildman (1489) used the symbols + and — merely for juxtaposition of quantities. E.g. 'Was — ust/das ist minus—und das + das ist mer'.[10] He then wrote '4 cetner+5 pfound' and '4 cetner—17 pfound', thus showing excess or deficiency in the weights. He did not write the symbols to indicate operations, but wrote, for example,

$$\text{`}\tfrac{2}{5}\ \tfrac{4}{5}\ \tfrac{3}{5}\ \text{adir fa}\ \tfrac{9}{5}\ \text{ist}\ 1\tfrac{4}{5}\text{'},$$

as we write $1\frac{4}{5}$ instead of $1+\frac{4}{5}$, which is merely a juxtaposition indicating addition. We see that juxtaposition serves to express the excess, which is the use of the plus sign, but in the case of minus some notation must be adopted; a mere juxtaposition will not serve the purpose.

The Dutch mathematician, Vander Hoecke[11] (1514), was the first to use + and — signs to indicate operations. He wrote

$$\bar{R}\tfrac{3}{4}-\bar{R}\tfrac{3}{5} \text{ for } \sqrt{\tfrac{3}{4}}-\sqrt{\tfrac{3}{5}} \text{ and } \bar{R}3+5 \text{ for } \sqrt{3}+5.$$

After him these signs were frequently used by Grammateus (1518) in the Rule of False Position, where he expressed, with their help, excess and deficiency instead of an operation. He also used + and — in the modern sense in his algebraic works.

We have already mentioned that the aforesaid symbols indicative of operations were first employed in Algebra. In Arithmetic they first appear for operations in the works of Georg Walckl (1536), who wrote $+\frac{1}{3}$ 230 to indicate the addition of $\frac{1}{3}$ of 230 and $-\frac{1}{5}$ 460 to indicate the subtraction of $\frac{1}{5}$ of 460. Stifel was the first algebraist who employed these symbols in the most general sense. He wrote '3 sum +2' for $3x+2$ and '8 sum −18' for $8x-8$. From this time German and Dutch mathematicians commonly used these symbols. Definite shapes for these signs were still in confusion up to the eighteenth century. For example, in Bartjens

$$xx = \div\!\!-\, 2375 \maltese 1{,}785{,}000$$

is written for $x^2 = -2{,}375x+1{,}785{,}000$.

In England Recorde (*c.* 1542) used the symbols + and −, and Baker (1958) used X and − in connection with the Rule of False Position. They did not employ them for operations as in Arithmetic. At that time English mathematicians used these symbols indicating operations in Algebra only.

Various symbols for the plus sign appear in different early works on mathematics. The + sign has also its significance among the English people from the religious point of view. + is placed over churches indicating the cruel murder of Christ. The expression 'plus or minus' is also found on Roman tombstones indicating the age of the deceased person in some such form as AN. LXXXXIIII. P.M.; that is '94 years more or less'.

References

[1] Eisenlohr, A., *Ein Mathematisches Handbuch der alter Aegypter*, 2nd ed. (Leipzig, 1877), p. 47.

[2] Heath, T. L., *Diophantus of Alexandria*, 2nd ed. (Cambridge, 1916), p. 130.

[3] Hoernlé, A. F. R., *Indian Antiquary*, **12**, 89-90, 1883.

[4] ——— Verhandlungen des VII Internationalel Orientalisten Congresses, Arische Section, p. 127 et. Seq., 1886.

[5] ——— *Indian Antiquary*, **17**, 33–48, 275–259, 1888.

[6] Dutta, B. B., The Bakhsāli Mathematics, *B.C.M.S.*, **21**, 1–60, 1929.

[7] Pṛthudakaswami, Commentary on *Brahma-sphuṭa-Sidhā* of the Asiatic Soc. of Bengal.

[8] Colebrooke, H. T., *Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhaskara*, London, 1817.

[9] Karpinski, L. C., Robert of Chester's Latin Translation of the *Algebra of al-Khowarizmi*, New York, 1915.

[10] Smith, D. E., *History of Mathematics*, **2**, 399, 1953.

[11] Halliwell, J. O., *Rare Arithemetica*, 2nd ed. (London, 1838-39), p. 183.