JYOTIṢA IN KERALA

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This paper is a critical survey of the work of notable contributors to the science of *Jyotiṣa* in Kerala such as Bhāskara, Parameśvaran Namputiri, Mādhava, Comatiri of Kelallur, Comatiri of Putumana, Śrīpati, Parakkad Namputiri and Acyuta Piṣāroṭi, between fifth and fifteenth centuries A.D.

There has been a specific and traditional school of *jyotiṣa* exponents and experts in Kerala from ancient times. Whatever advances were made in the west in the field of *gaṇita siddhānta* by the western scholars during the eighteenth and nineteenth centuries had already been debated and discussed by the sixteenth century in this region by Vaṭasseri Parameśvaran Namputiri, Mādhavan of Saṃgama grāma, Comatiri of Kelallūr and Comatiri of Putumana. These scholars could be rightly ranked with Newton[1] and Leibnitz, especially in their contribution to the science of integral calculus.

The two aspects of *jyotiṣa* are *gaṇita* (recording the actual position and movement of the planets) and *phala*. In these two disciplines, the Ancient Indians have taken great strides by developing the *Brāhma*[2], *Saura* and *Vāsiṣṭha* schools of thought. Till about the sixth century *jyotiṣa* studies developed systematically in Northern India. The successive foreign invasions may perhaps account for the slackness in the development of this branch of science. In South India too, except in Kerala, there were no outstanding contributions. But in Kerala, the scholars specialised in *Siddhagaṇita* and *jyotirgaṇita*. It is difficult to determine the precise antiquity of this practise of *jyotiṣa* in Kerala. It must have come in the process of the aryanisation of the south especially through Āryabhaṭa (later half of the fifth century A.D.) From the *Āryabhaṭīya*[3], it is known that Asmaka was the place of his birth and that he went up to Pāṭalīputra (Kusumapura). The identity of Asmaka in Kerala is debatable, although the strong prediction for the Āryabhaṭa tradition in Kerala as also the large number of commentators on the *Āryabhaṭīya* being Keralites may support the belief that Āryabhaṭa was a native of the region.

In 682 A.D. there was a quinquinniel conference of *jyotiṣa* scholars at Tirunāvāya, near Ponnani to discuss the shortcomings of the methods (*gaṇita sampradāya*) of Āryabhaṭa and to reform the system which assumed a later name of *parahitam*. The contribution of Vararuci[4] to the development of this system was

considerable. *Vararucivākya* or *Girṇādi vākya* or *Pañcāṅga vākya,* is an important short treatise on the calculation of the position of planets.[5] His elucidations were utilised in determining and fixing the position of the moon (*Candragaṇita*), which method again is widely prevalent in Kerala. Bhāskara (sixth century A.D.) was another *gaṇita-paṇḍita* of Kerala whose important contributions were a detailed *Āryabhaṭīya bhāṣya,* the *Laghubhāskarīya*[6] (a manual of the *Āryabhaṭīyam*) and *Mahābhāskarīyam*[7], a *tantra grantha.* Haridatta (*circa* 650-700 A.D.) is considered to be the exponent of the *parahita* system of astronomical calculations, his basic text being the *Grahācāranibandhana.*[8] His *Mahāmārganibandha* is not extant, though it is referred to in the *Grahācāranibandhana.* In addition to the *Kaṭapayadi*[9] notation, Haridatta introduced corrections known as *Bījasaṃskāra* or *sabdasaṃskāra* in Āryabhaṭa's system.

In the ninth century Ravivarmadeva of Mahodayapuram (*Kodungallur* >*Cranganur*) established an armillary sphere. Śankaranārāyaṇa who was an astronomer of repute patronised by Ravivarman Kulaśekhara (Rāmavarman 885-913 A.D.) mentions his patron in his *vivaraṇa,* a commentary on the *Laghu-bhāskarīya,* gives the date of its composition as 869 A.D. : *Evam śakābdāh punariha candra randhra muni saṃkyayā asmābhiḥ avagataḥ* (śaka 791).[10]

Śrīpati, was a famous astronomer of the eleventh century. His important contributions were (1) *Sidhānta śekhara,*[11] a work on astronomy in 20 Chapters : (2) *Gaṇita tilaka,* a work on Mathematics and (3) *Jātakakarmmapaddhati.* These works are quite popular throughout Kerala.

Govindasvāmin who prepared a *bhāṣya* for the *Mahābhāskarīya* as also the *Govinda paddhati,* an astrological work and the *samprādayapradīpikā,* a commentary on *Parāśara Hora,* also flourished during this period. His methods were followed reverently later by Parameśvara and Nīlakaṇṭha, who refer to Govindasvāmin constantly.

No records have come down for sketching the history of *jyotiṣa* development during the eleventh to the thirteenth centuries. Probably the productions of this era had been lost, as by the beginning of the fourteenth century giant strides were made. Govinda Bhaṭṭatiri, a renowned *phalabhāga* expert lived during the last quarter of the thirteenth century. The chronograms for the day of his birth, i.e. *rakṣet govindaṃ arka* (1237 A.D.) and the day of his death : *Kālimdi prīyistuṣṭi* (1295 A.D.) are known. Tālakkulam, a hamlet of Ālatur, near Tirur was his native place. His mother's original home was Pālūr and Bhaṭṭatiri passed away at the *Paḍippura* (gateway) of his son's residence, according to a traditional account. The most important output of Bhaṭṭatiri was the *Daśādhyāyi,*[12] a commentary on *Bṛhajjātaka* of Varāhamihira. This is held to be the most critical out of the seventy other commentaries on *Bṛhajjātaka.* A less known work was the *Muhūrtaratna*[13]. About the same time was current the *Kṛṣṇīya* a treatise by Kṛṣṇa who is referred to in the *Uṇṇayacci-caritam campu*[14] of the fourteenth century.

The most illustrious exponent of *jyotiṣa* after Āryabhaṭa and Vararuci in Kerala seems to have been Parameśvaran Namputiri (A.D. 1360-1455) of Vaṭasseri illam : *aśvattha grāmajo bhārgavaḥ parameśvaraḥ* according to his disciple Nīlakaṇṭha Somayājin. His *Dṛg-gaṇita* gives the date of the composition as *śāke triṣu, viśvamite kṛtam* (corresponds to 1431 A.D.). The other less known contributions of Parameśvara were, the *Bhaṭṭadīpikā*, a commentary on the Āryabhaṭīya, *Grahaṇāṣṭaka*, *Laghubhāskarīya vyākhyā*, *Laghumānasagaṇita vyākhyā*, a commentary on the *Laghumānasa* of Mañjula, *Līlāvatīvyākhyā*, a commentary on the *Līlāvatī* of Bhāskarācārya,[15] the *Siddhānta-dīpikā*, a commentary on the *Mahābhāskarīya* of Bhāskara I, *Gola dīpikā*, which appears to be in two versions one consisting of the chapters of the *Golanibandhanavidhi*, *Grahācāranirūpaṇa*, *Bhūmyādi cintanā* and a fourth chapter, the other version is an abridgement without any chapter division, edited by T. Gaṇapati Śāstri.[16] The *Karmadīpikā*, *Jātaka karmapaddhati* and *Sūryasiddhānta vivaraṇa*, a commentary on the *Sūryasiddhānta*, were other less known works.

Two other contemporaries of Parameśvara have to be reckoned with namely Saṃgrāma Mādhava and Comatiri of Putumana. It cannot be stated precisely whether Mādhava was a *nambutiri* or a warrior of Iruñjālakuḍa. The main treatise of Mādhava, the *veṇvāroham*[17] is extant. It is an astronomical manual which enunciates a method for the accurate calculation of the longitude of the moon given for daily intervals correctly, without using the second and third differences. The author deals elaborately with the preparation of a table using which the longitudes of the moon can be read out at short intervals of one and half *nāḍikas* each, the same table being valid for a cycle of 248 days.

Before the time of Bhāskara II, the volume and surface area of spheres were correctly known, and derived. Indeterminate analysis had advanced. Mādhava in the fourteenth century discovered integration by summation of series and with its help discovered an infinite series for π, the arc of a circle and the sine and co-sine chords of arcs. The Kerala astronomer mathematicians of the later Āryabhaṭa school derived many trigonometrical results with the help of the cyclic quadrilateral. A new development was the discovery of the expression for the circumradius in terms of the sides of a cyclic quadrilateral. Parameśvara of the fifteenth century in his *Līlāvatībhāṣya* gives this new expression in the verse :

दोष्णांद्वयोर्घातियुतानां तिसृणां वधात्
एकैकोनेतरत्र्यैकयं चतुष्कवधभाजितात् ।
लद्वधमूलेनयद्वृत्तं विष्कम्भार्द्धेन निर्मितम्
सवंचतुर्भुजक्षेत्रं तस्मिन्नेवावतिष्ठते ॥

'The three sums of the products of the sides taken two at a time are to be multiplied together and divided by the product of the sums of the sides taken three at a time and diminished by a fourth. If a circle is drawn by the square root of this quantity

as radius, the whole quadrilateral will be situated inside it' ; that is, if a b c d are the sides and r the circumradius

$$r = \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(a+b+c-d)(b+c+d-a)(c+d+a-b)(d+a+b-c)}}$$

Parameśvara and other Kerala mathematicians understood clearly the significance and limitations of Brahmagupta's results about the cyclic quadrilaterals. The *Yuktibhāṣā* (sixteenth century) proves the following trigonometrical results :

$$\text{Sin}^2 A - \text{Sin}^2 B = \text{Sin}\,(A+B)\ \text{Sin}\,(A-B)$$

and

$$\text{Sin}\,A\,.\ \text{Sin}\,B = \text{Sin}^2 \frac{A+B}{2} - \text{Sin}^2 \frac{A-B}{2}, \text{ with}$$

the help of the cyclic quadrilateral using geometrical analysis.[18]

The Kerala mathematicians after the thirteenth century give logical proofs and demonstration of the following theorems :

(*a*) The area of a triangle is equal to half the product of the base and altitude.

(b) The product of the sides of a triangle divided by the circum-diameter is the altitude on the base (cf. *Yuktibhāṣā,* p. 231)

(*c*) The perpendicular bisectors of the sides of a triangle are concurrent.

(*d*) The square of the hypotenuse of a right angled triangle is equal to the sum of the squares on the other two sides.

Mādhava of Samgrāma Grāma, applied the theory of integration for calculating the circumference of a circle from its radius. An infinite series for π or circumference is obtained. Circumference $= 8R(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots)$. In the course of its derivation many integrals of the powers of a quality and the results of repeated integration are proved and used. These are

$$\int_0^x x = \frac{x^2}{2}$$

vargasaṃkalita $\displaystyle\int_0^x x^2 = \frac{x^3}{3}$

ghanasaṃkalita $\displaystyle\int_0^x x^3 = \frac{x^4}{4}$

vargāvargasaṃkalita $\displaystyle\int_0^x x^4 = \frac{x^5}{5}$

$$\int_0^x x^n = \frac{x^{n+1}}{n+1}$$

dvitīyasaṃkalita $\int_0^x\int_0^x x = \dfrac{x^3}{\underline{|3}}$

tṛtiyasaṃkalita $\int_0^x\int_0^x\int_0^x x = \dfrac{x^4}{\underline{|4}}$

anekavārasaṃkalita or
n^{th} order integral of $x = \dfrac{x^{n+1}}{n+1}$

For the series for π, by suitable manipulation many other infinite series for π are obtained. Similarly the infinite series for sin θ and cos θ (actually R Sin θ and R Cos θ) are derived by using the integration theory and subtle reasoning, the

$$\textit{ardhajyā}\ (R\ \mathrm{Sin}\ \theta) = a - \frac{a^3}{R^2\underline{|3}} + \frac{a^5}{R^4\underline{|5}} - \frac{a^7}{R^6\underline{|7}} + \cdots\cdots$$

$$\text{The } \textit{koṭijyā}\ (R\ \mathrm{Cos}\ \theta) = R - \frac{a^2}{R.\underline{|2}} + \frac{a^4}{R^3.\underline{|4}} - \frac{a^6}{R^5.\underline{|6}} + \cdots\cdots$$

(a is the arc, and R the radius).

These when the arc is expressed in radians become equivalent to

$$\mathrm{Sin}\ \theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \cdots\cdots$$

$$\mathrm{Cos}\ \theta = 1 - \frac{\theta^2}{\underline{|2}} + \frac{\theta^4}{\underline{|4}} - \frac{\theta^6}{\underline{|6}} + \cdots\cdots$$

The volume and surface area of a sphere are also correctly derived using the integration theory. All this is known from the *Yuktibhāṣa* (sixteenth century), a treatise expounding the ideas in the *Tantrasaṃgraha* of Nīlakaṇṭha Somāyajin (*circa* fifteenth century) and the *Kriyākramakari*, probably the work of Śaṅkara Warriar—a commentary on *Līlavatī*. The series for π, and the arc and for the cosine and sine are found in the *Trantrasaṃgraha*, and *Śadratnamālā*. Thus integration by fluxions was discovered and used in India at least two and half centuries earlier than Newton and Leibnitz.[19] One of Leibnitz's discoveries, sometimes ascribed to the Scottish Mathematician James Gregory (1638-1675) may be noted. If π is the ratio of the circumference of a circle to its diameter

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots\cdots,$$

the series continuing in the same way indefinitely. This of course is not a practical way of calculating the numerical value of π (3.1415926. . . .) but the simple connection between π and all the odd numbers is striking.[20]

The *Karaṇapaddhati*,[21] an astronomical treatise in ten chapters, the chief contribution of Putumana Comatiri, of Trichur seems to have been composed before the *Dṛggaṇita paddhati* came into vogue. Chapter six is of special importance

as it gives rules for trigonometrical sine, cosine, tan and π series. The value of π was determined in the eighteenth century in the west and this was acclaimed all over. But this had been achieved in India in the fourteenth century as the eighth Chapter of the *Karaṇapaddhati* refers : *vyāsa caturghanāt bahusaḥ pṛthaksthāt* regarding the circumference and diameter of a circle. Nīlakaṇṭha Somāyajin (1465-1545)[22], was the disciple of Dāmodaran Namputiri, the son of Parameśvaran Namputiri. He was known as Kelallur Comatiri and was born at Trikkaṇṭiyur, near Tirūr. He was a second *bhāṣyakāra* for the *Āryabhaṭīya*. The fundamental principles of calculus so ardently developed in the west during the eighteenth century are to be found, in Somayājin's work *Tantrasaṃgraha*. This contains 430 strophes in eight chapters. The conception of time, computation of the past *kali* days, the seasons, solar and lunar months are described in the first Chapter. Calculation of the mean solar day, determination of the orbit of the planets in the Hindu geocentric system are described in the second chapter. The third chapter deals with the fixing of the gnomon, calculation of the equator, meridian, latitude and declination by making use of the gnomon and its shadow. There are many illustrations and mathematical problems concerning the gnomon and its shadow. The fourth and fifth chapters deal with lunar and solar eclipses. *Vyātipāta* and its calculations are dealt with in the sixth and seventh chapters. The last chapter describes the *Sitamāna śṛṅgonnati* of the moon.[23] It is known that a Brahmin named Brahmadatta has based his Malayalam work *Yuktibhāṣā* on the *Tantrasaṃgraha*. There have been many commentaries on the *Tantrasaṃgraha* but none of them outstanding. Another important work of Nīlakaṇṭha was the *Siddhānta Darpaṇa* a short treatise on Astronomy in 32 strophes.[24] The *Candracchāya gaṇita*[25] deals with the methods for calculation of the time during night from a measurement of the shadow cast by the moon. The *Golasāra*[26] is a small metrical treatise on astronomy specially dealing with the earth and the planets in three chapters.

Nīlkaṇṭha had recourse to the summation of an infinite convergent series while deriving a good approximation formula for the area a in terms of the chord c and height h, viz : $a = \sqrt{(1+\frac{1}{3})h^2+c^2}$. He was aware that an infinite convergent series had a finite sum.[27]

The *Praśnamārga* of Panakkāḍ Namputiri is a unique treatise concerning itself with the *phala-bhāga*, perhaps composed in the seventeenth century. This is generally followed as a standard guide for matters relating to *praśna* even at present. Eḍakkad in North Kerala was the home town of Panakkāḍ Namputiri. Another scholar of repute of this period was Māttur Namputiri. The Māttur illam was situated in Pāñjal near Celakkara. His main work was the *Muhūrta śāstra saṃgraha* consisting of 35 strophes. Other notable works of the contemporary period are *a*) *Sadratnamāla*[28] a short treatise on Mathematics in five chapters by Śaṅkaravarma Thampuran of Kaḍattu naḍu written under the direction of Rāmavarman, brother of Udayavarman of Kerala. It is a discussion on trignometric series ; (*b*) *Gaṇita nirṇaya* of Puruṣottaman Namputiri of Puliyūr.

Achyuta Piṣāroṭi (1550-1621 A.D.), the teacher of Nārāyana Bhaṭṭa of Melputtūr, was an authority on astronomy and Grammar. He is referred to by Vāsudeva in his *Bhramarasandeśa*[29]

तस्मान् प्रत्यक्प्रहितनयन: कुण्डगेहाधिनाथं
सर्वज्ञं तं प्रणमगिरिशं शक्तिमानच्युतं च ।
एकस्तावद्वहति शिरसि ज्योतिषामेकमिन्दुं
ज्योतिश्चक्रंनिखिलमपशे धारयन्यन्तरङ्गो ॥

His patron was Ravivarman of Vettattunaḍu.[30]

लक्ष्मया प्रकाशविषयं राजयन्निजयानिजम् ।
नित्यमुद्यन्विजयते सुकृतालम्बनं रविः ॥

The notable works of Piṣāroṭi were (*i*) *Uparāgakriyānirṇaya* (*ii*) *Karaṇottama* (*iii*) *Sphuṭa nirṇaya*[30] (*iv*) *Horasāroccaya* (*v*) *Rāśigola sphuṭanīti*[32] and (*vi*) the Malayalam commentary on Mādhava's *veṇvāroha.*[33]

The foregoing is only a cursory survey of some of the notable contributors to the science of *jyotiṣa* in Kerala and is aimed only at stimulating further research in this field.

References

[1] Whiteside, D. H. "Before the Principia : The Maturing of Newton's thoughts on Dynamical Astronomy" *Journal for the History of Astronomy*, I, pt. I. February 1970, pp. 5-19 ; The Mathematical Principles underlying Newton's Principia Mathematica, *Ibid*, No. 2, pp. 116-139.

[2] Pingree, D. On the classification of Indian Planetary Tables, *J.H.A.*, I, Pt. II, 1970., pp. 95-108.

Chatterjee, Bina. *The Khaṇḍakhādyaka* (An Astronomical treatise of Brahmagupta with the commentary of Bhaṭṭotpala)2, Vols. Calcutta, 1970 ; also review by D. Pingree *J.H.A.*, 11, 1971, pp. 121-122.

[3] Nizami, K. A. Inda's Cultural Relation with Central Asia during the Medieval Period. *Central Asia*, ed. by A. Guha, Indian Council for Cultural Relations, Delhi, 1970, pp. 157-166. esp. p. 159.

[3] Clark, E. *The Āryabhaṭīya of Āryabhaṭa*, Chicago, 1930.

Yano, Michio. Aryabhata's Method of Expressing Numbers. *Journal of Indian and Buddhist Studies*, (Indogaku Rukkyogku Kenkyo), XIX, I, December 1970, pp. 434-438.

[4] *Candrakāvya*, ed. by C. Kunhan Raja, Adyar, 1948.

[5] Trivandrum Manuscripts ed. by K. Sambasivasastri, iv, 954, 15 ff. Malayalam ; Also S. N. Sen. A. K. Bag, and S. Rajeswar Sarma, *A Bibliography of Sanskrit Works on Astronomy and Mathematics*, New Delhi, 1966, p. 237.

[6] *Ed.* by P. K. Narayana Pillai, *T.S.S.*, 162, 1949.

[7] *Ed.* by T. S. Kuppanna Sastri, Madras Government Oriental Series, No. 130, 1957.

[8] *Ed.* by K. V. Sarma, Kuppusvami Sastri Research Institute, Madras, 1954. Also S. N. Sen, *op. cit.* p. 87., A Descriptive catalogue of Sanskrit MSS in the H.H. the Maharajah's Palace Library, Trivandrum, ed. by K. Sambasivasatri, Vol. III & IV, Trivandrum, 1935.

[9] L'Inde Classique, II, Paris, 1953, p. 708 also pp. 177-194 ; Also J. Filliozat, Ancient Relations between Indian and Foreign Astronomical Systems, *Journal of Oriental Research,* XXV Madras, pp. 1-8.

[10] *T.S.S.*, 162, p. 4 ; K. Kunjunni Raja, *The Contribution of Kerala to Sanskrit Literature*, Madras, 1958, p. 16. *Golan Mahodayapure Ravivarmadeva sambaddha Yantra Valayāṅkita rāśi cakrat.* Cf. *Laghubhaskariya*, p. 43.

[11] Sen, S. N. *op. cit.*, pp. 206-207.

Śrīpati, a North Indian Astronomer, most probably of Rohilkand was the author of *Dhruvamanasam.* The last verse of this work reads '*Bhaṭṭa devasya putrasya nāga devasya nandanah' Śrī pati Rohinikhaṇḍe manasam kritavan dhruvaṃ..*

[12] The Curator's Office Library, Trivandrum, (TC) 819 A.

[13] *TC* 748-50.

[14] Ullur S. Paramesvara Iyer, *Kerala Sahitya Caritam.*

[15] Sen, S. N. *Op. cit.*, pp. 167-170.

[16] Ibid, p. 168. Ed. by K. V. Sarma, Adyar Library, 1954.

[17] Ed. with Malayalam Commentary of Acyuta Pisaroti, by K. V. Sarma, *Ravivarma Sanskrita Granthavali*, Trippunithura, 1956. Also *Quarterly Journal of the Mythic Society,* XLIX, Oct. 1958, pp. 183-186.

[18] *Yuktibhāsā*, pt. I, Ed. by Ramavarma Thampuran and A.R. Akhilesvara Iyer Mongalodayam, Press Ltd., Trichur, 1952, pp. 224-27.

[19] Sarasvati, T. A. Development of Mathematical Ideas in India, *Indian Journal of History of Science*, IV, 1969, pp. 59-78.

[20] Bell, E. T. *Men of Mathematics*, New York, 1961 ; Chapter VII, Leibniz pp. 117-130.

Glorgio de Santillana. The Bruno a Leibnita *Reflections on Men and ideas,* Massachussets, 1968, pp. 284-299.

[21] Ed. by K. Sambasiva Sastry, Trivandrum Sanskrit Series, No. 126, Trivandrum, 1937 ; also *Karanapaddhativyākhyā*, an anonymous commentary on the *Karaṇapaddhati*(1817 A.D.): S. N. Sen, *op. cit.*, p. 105.

[22] *Āryabhaṭiya Bhāṣya, T.S.S.*, 101, pp. 113-156.

[23] Sen, S. N. *op. cit.*, pp. 156-157.

[24] Edited and translated by K. V. Sarma in Adyar Library Bulletin, XIX, pts. 3-4.

[25] Sen, S. N. *op. cit.*, p. 155.

[26] Nīlakaṇṭha Somayajin, *Golasāra*, ed. by K. V. Sarma, Hoshiarpur 1970 ; (review) *Indian Journal of History of Science*, No. 1, May 1971, **6** ; pp. 113-114.

[27] *Āryabhaṭīya bhāṣya*, p. 106. R. 14850, Govt. Oriental MSS Library Madras.

[28] Sen, S. N. *op. cit.*, p. 191.

[29] *TSS*, 128.

[30] *Praves'aka*, Ed. by P. S. Anantanarayana Sastri, Cochin Sanskrit Series, No. 2, Trippunithura, 1938.

[31] *T.C.* 655b ; 657c, 697c ; *QJMS*, XXI, p. 213.

[32] *Ed.* by K. V. Sarma, *Adyar Library Bulletin*, 1954, pt. 2.

[33] Edited by Ravi Varma Granthavali, Trippunithura, 111, 1955 ; Also S. V. S. Iyer in *J.O.R.*, 1952-53.

SELECT LIST OF OTHER PAPERS CONSULTED

1. Das, S. R. The Origin and Development of Numerals, *Indian Historical Quarterly*, Calcutta, 1927, pp. 97-120.
2. Fleet, J. F. Aryabhata's system of Expressing Numbers, *Journal of the RoyalAsiaticSociety of Great Britain and Ireland*, 1911, pp. 109-126.
3. Gupta, R. C. Fractional Parts of Āryabhaṭa's Sines and Certain rules found in Govindasvâmi's *Bhāṣya* on the *Mahābhāskarīya*, *Indian Journal of History of Science*, VI, 1971, pp. 51-59.
4. Michel, Henri. *Astronomical Jade*, *Oriental Art*, 1949, pp. 156-159.
5. Hiskett, M. The Arab Calender and Plaentary system in Hausa Verse, *Bulletin of the School of Oriental and African Studies*, Vol. 30, 1967, pp. 158-176.
6. Rajagopal, C. T. and Vadamurthi Aiyar, V. Y. A Hindu Approximation to π, *Scripta Mathematica*, 18, 1952.
7. Rodet, L. Notation Numerique d' Aryabhatiya, *Journal Asiatique*, 1880, pp. 433-440.
8. Scriba, C. J. Report on the Fifteenth Colloquium on the History of Mathematics at the Mathematical Research Institute, Oberwolsach Black Forest, Germany, 25 to 30 October 1970, *JANUS, Revue Internationale de l'histoire des Sciences, de la medicine de la pharmacie et de la technique*, LVII, 4, 1970, Leiden, pp. 293-298.