CIRCUMFERENCE OF THE JAMBŪDVĪPA IN JAINA COSMOGRAPHY*

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In Jaina cosmography, the periphery of the Jambu Island is taken to be a circle of diameter 100,000 *yojanas*. The circumference of a circle of this size, as stated in Jaina canonical and geographical works like the *AnuYogadvāra Sūtra* and *Triloka-sāra* etc. is equal to 316227 *yojanas*, 3 *krośas*, 128 *daṇḍas* and 13½ *aṅgulas* nearly. However, the *Tiloya paṇṇatti* (between the fifth and the ninth century A.D.) gives a value (apparently quoted from the canonical work *Diṭṭhivāda*) of the circumference of the Jambūdvīpa as calculated upto a very fine unit of length called *avasannāsanna skandha* where 8^{12} of these units make one *aṅgula* (finger-breadth). It is shown that the value was computed by making use of the following two approximate rules

(i) circumference $= \sqrt{10(\text{diameter})^2}$

(ii) $\sqrt{a^2+x} = a+(x/2a)$.

The correctly carried out long numerical calculations leave a fractional remainder whose true interpretation has been obtained here.

1. INTRODUCTION

According to Jaina cosmography, the Jambūdvīpa ('Jambu Island') is circular in shape and has diameter of 100,000 *yojanas*. Umāsvāti's *Tattvārth ādhigama-sūtra* (= *TDS*), III, 9, for example, states[1].

....योजनशतसहस्रविष्कम्भोजम्बूद्वीप : ||९||

....*yojana-śatasahasra-viṣkambho-jambūdvīpaḥ* ||9||

'The Jambūdvīpa is of diameter one hundred thousand *yojanas*'. That is,

$$D = 100{,}000 \text{ yojanas} \quad .. \quad (1)$$

Some other explicit references are :

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(i) *Tiloya-Paṇṇatti* (= *TP*), IV, 11 (Vol. I, p. 143) of Yativṛṣabha[2]

(ii) *Tiloya-Sāra* (= *TS*), *gāthā* 308 (p. 123) of Nemicandra (10th century A.D.)[3]

(iii) *Jambū-Paṇṇatti-Saṃgaho* (= *JPS*), I, 20 (p. 3) of Padmanandin[4].

The *Viṣṇu-purāṇa*, a non-Jaina work, also takes the Jambūdvīpa to be of the same shape and size[5].

The constancy of the ratio of the circumference of any circle to its diameter was recognized in all parts of the ancient world. This ratio is denoted by the Greek letter π(*pi*), so that the circumference C is given by

$$C = \pi D \quad .. \quad (2)$$

However, *pi* is not a 'simple' number. It is not only irrational but transcendental. Hence its true value cannot be expressed by an integer, fraction, surd, or by a terminating decimal. Thus, for any practical purpose, we can use only an approximate value of *pi*.

The simplest approximation to the exact formula (2) will be

$$C = 3D \quad .. \quad (3)$$

A rule equivalent to (3) is contained, for example, in *TS*, 17 (p. 9) which states

वासो तिगुणो परिही.... ||१७||

Vāso tiguṇo parihī...... ||17||

'Diameter multiplied by three is the circumference'.

Utilizing the crude formula (3), the circumference of the Jambūdvīpa will be given by

$$C = 300{,}000 \textit{ yojanas} \quad .. \quad (4)$$

However, the Jainas knew the inaccuracy of the rough value given by (4). That is why they attempted to find an accurate value which is far better than (4).

The purpose of the present paper is to describe those values of C which were intended to be more accurate and explain as to how they were obtained.

For the purpose of comparison, we first find the correct modern value of C. Taking the true modern value of *pi*, correct upto 27 decimal places, and using (2), we get[6]

$$C = 314159.265{,}358{,}979{,}323{,}846{,}264{,}338{,}3 \textit{ yojanas} \quad .. \quad (5)$$

correct to 22 decimal places.

However, the form in which ancient values were expressed should not be expected to be of the type (5) which utilizes decimal fractions. For expressing fractional parts, the Jainas employed a series of sub-multiple units to a very very fine degree. Starting with the *paramāṇu* ('extremely small particle') of an indeterminately small size and ending with a *yojana*, the *TP*, I, 102–106 (pp. 12–13) and I, 114–116 (p. 14), contains a system of linear units which we present in Table I below[7].

TABLE I

(Units of length from the *Tiloya-paṇṇatti*)

Infinitely many *paramāṇus*	=	1 *avasannāsanna skandha*
8 *avasa.* units	=	1 *sannāsanna skandha*
8 *sannāsannas*	=	1 *truṭareṇu*
8 *truṭareṇus*	=	1 *trasareṇu*
8 *trasareṇus*	=	1 *rathareṇu*
8 *rathareṇus*	=	1 *uttama bhogabhūmi bālāgra*
8 *ut. bho. bālāgras*	=	1 *madhyama bhogabhūmibālāgra*
8 *ma. bho. bālāgras*	=	1 *jaghanya bhogabhum ibalagra*
8 *ja. bho. bālāgras*	=	1 *karma-bhūmi bālāgra*
8 *ka. bālāgras*	=	1 *likṣa*
8 *likṣas*	=	1 *yūka*
8 *yūkas*	=	1 *yava* (barley corn)
8 *yavas*	=	1 *aṅgula* (finger-breadth)
6 *aṅgulas*	=	1 *pāda*
2 *pādas*	=	1 *vitasti* (span)
2 *vitastis*	=	1 *hasta* (fore arm or cubit)
2 *hastas*	=	1 *rikkū* (or *kiṣku*)
2 *kiṣkus*	=	1 *daṇḍa* (staff) or *dhanuś* (bow)
2000 *daṇḍas*	=	1 *krośa*
4 *krośas*	=	1 *yojana*

From Table I, it can be easily seen that

$$1 \textit{ yojana} = 5.3 \times 10^{16} \textit{ avasa.} \text{ units roughly,}$$

so that an *avasa* unit is of the order of about 10^{-17} of a *yojana* or of the order of about 10^{-22} with respect to the given diameter (1) That is why we must employ a decimal value correct to about 25 places in order to check or compare with another value which is specified upto the *avasa* unit together with the fractional remainder thereafter.

The value (5), which is in conformity with above consideration, can now easily be transformed and expressed in terms of the units of Table I, We have done this by successively changing the value of the fractional part left into sub-units at each stage. This transformed form of the correct modern value of the circumference of the Jambūdvīpa is shown in Table II.

TABLE II

(*Circumference of the* Jambūdvīpa *of Diameter* 100,000 yojanas)

Sl. No.	Denomination or unit	By $C = \pi D$, with actual value of *pi*	By $C = \sqrt{10}D$, with actual value of $\sqrt{10}$	As found in the *Tiloya paṇṇatti* (*TP*)	Area $= C.D/4$, with C from *TP*. (in square units)
1	*yojana*	314159	316227	316227	79056, 94150
2	*krośa*	1	3	3	1
3	*daṇḍa*	122	128	128	1553
4	*kiṣku*	1	0	0	0
5	*hasta*	1	0	0	0
6	*vitasti*	0	1	1	1
7	*pāda*	1	0	0	0
8	*aṅgula*	5	0	1	1
9	*yava*	5	7	5	6
10	*yūka*	4	3	1	3
11	*likṣa*	4	4	1	3
12	*ka. bālāgra*	3	7	6	2
13	*ja. bho. bālāgra*	2	4	0	7
14	*ma. bho. bālāgra*	3	3	7	3
15	*ut. bho. balagra*	6	5	5	7*
16	*rathareṇu*	7	5	1	4*
17	*trasareṇu*	4	2	3	2*
18	*truṭareṇu*	5	1	0	3*
19	*sannāsanna*	0	5	2	7
20	*avasa.* units	6	7	3	1
21	*kha-kha* fraction (or remainder)	43/100 nearly	71/100 nearly	23213 by 105409	48455 by 105409

2. THE JAINA VALUE OF THE CIRCUMFERENCE

Naturally, we need not expect the exact modern value of C (as calculated by us above) to be stated in any ancient Jaina work, because, like all other ancient peoples, the Jainas also used only approximate values of *pi* needed in the relation (2).

The Jainas commonly employed the following formula, which is better than (3),

$$C = \sqrt{10D^2} \quad \text{.. (6)}$$

or

$$C = \sqrt{10}D \quad \text{.. (7)}$$

There is no shortage of references to (6) or (7) in Jaina works. It occurs in the *Bhāṣya* (p. 170)[8] which accompanies the *TDS* under III, 11. Some other references are :

(i) *TP*, I, 117, first half (Vol. I, p. 14); *TP*, IV, 9 (vol. I, p. 143); etc.

(ii) *TS*, 96, first half (p. 41) and *TS*, 311, first half (p. 125).

(iii) *JPS*, I, 23 (p. 3).

(iv) *Jyotiṣ-karaṇḍaka* (*gāthā* 185)[9].

By taking the value of $\sqrt{10}$ correct to 27 decimal places, we get, from (7) which is theoretically equivalent to (6),

$$C = 316227.766{,}016{,}837{,}933{,}199{,}889{,}354{,}4 \text{ yojanas} \quad \ldots (8)$$

As before, we have converted this value in terms of the units of Table I. The result obtained is shown in Table II.

The value of the circumference of the Jambūdvīpa as found stated in the *TP*, IV, 50–57 (vol. I, p. 148)[10] is also given in Table II. The *TP* value is slightly *more* than

$$C = 316227 \text{ yojanas, } 3 \text{ krośas, } 128 \text{ daṇḍas, and } 13\tfrac{1}{2} \text{ aṅgulas.} \quad \ldots (9)$$

This simplified value which is rounded off to the nearest half of an *aṅgula* is found in many works including :

(i) *Anuyogadvāra-sūtra*, 146, where it is given as the circumference in a *palya* of diameter one lac *yojana*[11].

(ii) *Jīvājīvābhigama-sūtra*, 82 (without reference to Jambūdvīpa)[12].

(iii) *TS*, 312 (p. 126) as an accurate value.

(iv) *JPS*, I, 21–22 (p. 3).

A glance at the Table II will show that the *TP* value does not fully agree with that which is accurately found by the Jaina formula (6) or (7). The latter value is slightly *less* than

$$316227 \text{ yojanas, } 3 \text{ krośas, } 128 \text{ daṇḍas, and } 13 \text{ aṅgulas.} \quad \ldots (10)$$

Thus, there is a divergence even between the frequently met and rounded off Jaina value, givey by (9), and the one given by (10) which is based on the correct value of the square-root of ten to a desired degree[13].

Naturally, we are keen to know the cause of disagreement between the two sets of values, particularly because the values are intended to give accuracy to a very fine degree of smallness. Is there some arithmetical error of calculation in extracting the square root, successively, to the desired degree ? Or, the Jainas followed some different procedure ? This we answer in the following pages.

3. How the Circumference was Obtained

For finding the square-root of a non-square positive integral number N, the following binomial approximation was frequently used during the ancient and medieval times

$$\sqrt{N} \equiv \sqrt{(a^2+x)} = a+(x/2a) \quad .. \quad (11)$$

where a and x are positive integers, and the 'remainder' x is less than the 'divisor' $2a$; otherwise or alternately, we may use

$$\sqrt{N} \equiv \sqrt{(b^2-y)} = b-(y/2b) \quad .. \quad (12)$$

The appsoximation (11) was known to the Greek Heron of Alexandria (between c. 50–c.250 A.D.)[14] and even to the ancient Babylonians[15]. The Chinese Sun Tzu (between 280 and 473 A.D.)[16], while extracting the square-root of 234567 by an elaborate method, finally said:[17]

"Thus we get 484 for the square-root in the above and 968 for the *hsia-fa*, the remainder being 311".

He gave the answer

$$484+(311/968) \quad .. \quad (13)$$

Thus, whatever be the method of Sun Tzu, the result (13) is equivalent to what we get by using (11).

The *Jaina Gem Dictionary* (pp. 154–155) gives the same rule, as represented by (11), for finding the square-root[18]. The *TP*, I, 117 (vol. I, p. 14) implies that the circumference of a circle of diameter one *yojana* was calculated to be 19/6 *yojanas*. This is in agreement with the use of the rule (11), since

$$\sqrt{10} = \sqrt{(3^2+1)} = 3+(1/6) \quad .. \quad (14)$$

Now from (1) and (6) we get

$$C = \sqrt{(100{,}000{,}000{,}000)} = \sqrt{(316227)^2+484471}$$
$$= 316227+\frac{484471}{2\times 316227} \text{ yojanas} \quad .. \quad (15)$$

by applying the approximation (11).

In the present case, therefore, we have

'divisor' = 632454

and

'remainder' = 484471.

The fractional *yojana* remainder, namely

484471/632454

when converted into *krośas*, will give

$$484471\times 4/632454 \text{ krośas} = 3+(40522/632454) \text{ krośas} \quad .. \quad (16)$$

The fractional *krośa* remainder, namely

40522/632454

can, similarly, be converted into the next lower sub-units (*daṇḍas*). The process can be continued likewise.

We shall easily get 128 *daṇḍas*, 1 *vitasti* (= 12 *aṅgulas*), and 1 *aṅgula* with the fractional *aṅgula* remainder to be equal to

$$407346/632454 \qquad .. \quad (17)$$

which is equal to

$$67891/105409 \qquad .. \quad (18)$$

Thus, we see that the fractional *aṅgula*-remainder (18) is slightly more than half. In this way, we get the circumference of the Jambūdvīpa as given by (9).

However, if we want to carry out the evaluation to lower and lower units (as should be done in order to get a value comparable to that found in the *TP*), we easily have (putting 105409 equal to H);

(a) *aṅgula*-fraction, $67891/105409 = 5+(16083/H)$ *yavas*
(b) *yava*-fraction, $16083/H = 1+(23255/H)$ *yūkas*
(c) *yūka*-fraction, $23255/H = 1+(80631/H)$ *likṣas*
(d) *likṣa*-fraction, $80631/H = 6+(12594/H)$ *ka.bālāgras*
(e) *ka. bāl.* fraction, $12594/H = 0+(100752/H)$ *ja. bho. bālāgras*
(f) *ja. bho. bāl.*fraction, $100752/H = 7+(68153/H)$ *ma. bho. bālāgras*
(g) *ma. bho. bāl.* fraction, $68153/H = 5+(18179/H)$ *ut. bho.)bālāgras*
(h) *ut. bho. bāl.* fraction, $18179/H = 1+(40023/H)$ *rathareṇus*
(i) *rathareṇu* fraction, $410023/H = 3+(3957/H)$ *trasareṇus*
(j) *trasareṇu* fraction, $3957/H = 0+(31656/H)$ *truṭareṇus*
(k) *truṭareṇu* fraction, $31656/H = 2+(42430/H)$ *sannāsanna*
(l) *sannāsanna* fraction, $42430/H = 3+(23213/H)$ *avasa.* units.

Thus we have, finally, the *avasannāsanna* fractional remainder

$$= 23213/105409 \qquad .. \quad (19)$$

In this way, we see that the above long calculation yields a value which is in complete agreement with the *TP* value right from the whole number of a *yojana* down to the lowest submultiple units defined in the text.—Moreover, we have found out a meaning of the fraction (19), designated as *kha-kha* (or *ananta-ananta*, 'endlessly endless') term, which can yield measure in still smaller and smaller units of length (to be defined with the help of the infinitely small particles or *paramāṇus*) if desired.

That the above method is the actual one which was used by the Jainas is quite evident from the full agreement obtained above and is also confirmed by what is given by Madhava-candra in the commentary of his teacher's *TS* under the *gāthā* 311 (pp. 125–126) where the calculation has been carried out upto the fractional *aṅgula* remainder (17).

Once we know the circumference, the area of the Jambūdvīpa can be computed by using the well-known rule, for example see *TP*, IV, 9 (Vol. I, p. 143),

$$\text{Area} = C.D/4 \quad .. \quad (20)$$

The result of our computation of the area by using (20) and *TP* value of C is shown in Table II. The contribution of the fraction (19)

$$= 23213 \times 25000/105409 \text{ square } avasa \text{ units}$$

$$= 5505 + (48455/105409) \quad .. \quad (21)$$

The measures of various denominations (specifying the area) as found in the *TP*, IV, 58–64 (Vol. I, p. 149) agree with the corresponding value which we have computed, including the *kha-kha* fraction given by the bracketed quantity in (21). This again confirms our calculations and interpretations.

Incidently we have discovered that at least one line (or verse), which ought to be there to specify the numerical values (marked by asterisks in Table II) of the four denominations from *ut. bho. bālāgras* to *truṭareṇus*, is missing in the printed text in the *TP* (between verses 61 and 62 in the fourth *mahādhikāra*) which we have consulted if not in the original manuscripts.

The contents of the manuscript entitled *jambūdvīpa-paridhi*[20] ('Jambūdvīpa-Circumference'), which seems to be relevant to the subject of our present paper, are not known to me.

References and Notes

[1] The *Sabhāṣya-TDS* edited with the Hindi translation of Khubacandra, p. 163, Bombay, 1932 (Paramasruta Prabhavaka Jaina Mandala).

The date of Umāsvāti (or Umāsvāmin) is about 40–90 A.D. according to J. P. Jain, *The Jaina Sources of the History of Ancient India*, p. 267, Delhi, 1964 (Munshi Ram Manohar Lal); and about 4th or 5th century according to Nathuram Prmi, *Jaina Literature and History* (in Hindi), p. 547, Bombay, 1956 (Hindi Grantha Ratnakara).

[2] The *TP* (Sanskrit, *Triloka-Prajñapti*) in two vols. Part I (2nd ed., 1936) ed. by A. N. Upadhye and Hiralal Jain; Part II (1st ed., 1951) ed. by Jain and Upadhye. Both published by the Jaina Sanskrit Samrakshaka Samgha, Scholarpur (Jivaraj Jain Granthmala No. 1).

According to Dr. Upadhye (*TP*, Vol. II, Intr., p. 7), the *TP* is to be assigned to some period betwzen 473 A.D. and 609 A.D. However, the work may have acquired its present form as late as about the beginning of the ninth century (*TP*, Vol. II, Hindi Intr., p. 20).

[3] The *TS* (Sanskrit, *Triloka-sāra*) ed., with the commentary of Mādhava-candra, by Manohar Lal Shastri, Bombay, 1918 (Manikachandra Digambara Jain Granthamala No. 12).

[4] The *JPS* ed. by A. N. Upadhye and Hiralal Jain, Sholapur, 1958 (Jivaraj Jain Granthamala No. 7).

According to the editors (*JPS*, Intr., p. 14), Padmanandin might have composed the *JPS* about 1000 A.D.

[5] See the *Viṣṇu-purāṇa, aṃśa* 2, chapter 2 (pp. 138–40), ed., with Hindi transl., by Munilal Gupta, Geeta Press, Gorakhpur, 4th ed., 1957. Also cf. *TP*, Vol. II, Hindi Intr., p. 83.

[6] See Howard Eves, *An Introduction to the History of Mathematics*, p. 94, New York, 1959 (Holt, Rinehart and Winston).

[7] Cf. L. C. Jain, "*Mathematics of the TP*" (in Hindi), prefixed with the Sholapur ed. of the *JPS.*, p. 19.

[8] Premi, *op. cit.*, pp. 524–529, believes that the *Bhāṣya* is by the author of *TDS* itself, while J. P. Jain, *op.cit.*, p. 135, says that 'no evidence of the existence of such a *Bhāṣya* prior to 8th century A.D. has yet been discovered'.

9 As quoted by R. D. Misra, "Mathematics of a circle etc." (in Hindi), *Jaina Siddhānta Bhāskara*, Vol. **15**, no. 2 (January 1949), p. 105.

According to the commentator Malayagiri (*c.* 1200 A.D.), the *Jyotiṣ-karaṇḍaka* (of Pūrvācārya) was edited on the basis of the first Valabhi *vācanā* which took place *c.* 303 A.D.; *see* J. C. Jain, *History of Prakrit Literature* (in Hindi), pp. 38 and 131, Chowkhamba Vidya Bhavan Varanasi, 1961.

10 In this connection, the *TP* mentions the work *Diṭṭhivāda* (Sanskrit, *Dṛṣṭivāda*) from which the value is apparently quoted; (*see Babu Chotelal Jain Smriti Grantha*, Calcutta, 1967, English section, p. 292; and the *Anusandhāna Patrikā*, no. 2, April-June, 1973, p. 30 (Jaina Vishva Bharati, Ladnu).

11 *See* the *Mūlasuttāṇi* edited by Kanhaiya Lalji, pp. 561–562 (Gurukul Printing Press, Byavara, 1953).

12 Quoted by H. R. Kapadia in the "Introduction", p. XLV, to his edition of the *Gaṇita-tilaka*, Oriental Institute, Baroda, 1937.

13 The comparison made by Dr. C. N. Srinivasiengar, *The History of Ancient Indian Mathematics*, p. 22 (World Press, Calcutta, 1967) is wrong because he takes one *dhanuṣ* (or *daṇḍa*) to be equal to 100 *aṅgulas* (instead of 96).

14 D. E. Smith, *History of Mathematics*, Vol. II, p. 254 (Dover reprint, New York, 1958).

15 C. B. Boyer, *A History of Mathematics*, p. 31 (Wiley, New York, 1968).

16 *See ISIS*, Vol. 61, part 1, (1970), p. 92.

17 Y. Mikami, *The Development of Mathematics in China, and Japan*, p. 31, (Chelsea reprint, New York, 1961).

18 Quoted by Kapadia (ed.), *op. cit.*, p. XLVI.

19 *See* L. C. Jain, *op. cit.*, pp. 49–50, for his comments on these *kha-kha* fractions.

20 *See* the *Catalogue of Manuscripts at Limbadi* (in Devanagari), edited by Catura-vijaya, p. 61, serial no. 1014, Bombay, 1928 (Agamodaya Samiti).