AL-BĪRŪNĪ AND THE ARITHMETICAL SEQUENCE OF THE SANSKRIT *GAṆAS*

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Al-Bīrūnī's surmise regarding inaccuracy of the text of a maunscript on Sanskrit prosody, referred to by him in Chapter XII of his book *Kitāb-ul-Hind*, has been examined in the light of the material presented by al-Bīrūnī in the said chapter. Based on the text quoted, an arithmetical sequence of Sanskrit *gaṇas* has been worked out. This turns out to be the mirror image of the accepted form of *Prastara* in Sanskrit and other Indian Languages, used for depicting the permutations of metrical forms having a given number of letters. The result is found to be applicable to metres containing any number of letters.

Chapter XIII of *Al-Beruni's India* deals with the grammatical and metrical literature of the Hindus. An analysis of some of the material presented by al-Bīrūnī is discussed in this paper. An attempt has also been made to work out a logical sequence of the Sanskrit *gaṇas* based on the text presented by al-Bīrūnī.

The Long and the Short

The basic units in the classical Sanskrit prosody are the short and long letters or sounds, called *laghu* and *guru* respectively. A long letter is roughly double the short in respect of sound value, syllabic quantity, and the time taken for utterance. Metrical music is produced not by the mere presence of long and short letters in a line, but by their order of succession. In the Vedic metres, the music depended upon the modulation of voice in the pronunciation of letters, and so the essential features of these letters, i.e. whether they were short or long, did not matter, and were not taken into account.[1] In classical Sanskrit metres on the other hand, the music depended on the essential features of the letters themselves, on their variation, and their order of succession. A single letter could no longer form the unit of a metrical line as in the Vedic metres. A mere mention of the number of letters, which were all independent units, sufficed to give an idea of the metrical line in the Vedic metres. There was no need to give the essential features of these letters, and how they stood related to each other. Both these points are vital in the case of classical metres. Hence a new unit which would take into account these things had to be devised and adopted for the scanning of lines in these metres.

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It is possible to describe in detail the order of short and long letters in a line, as has been done by Bharata in his *Nāṭya Śāstra*, and by Vīrataṅka in his *Vṛtta-Jātisamuccaya*. That, however, is a very cumbersome process, and sacrifices brevity for no corresponding advantage.[2]

Doublets and Triplets

A unit of two letters, in its four different forms SS, SI, II, IS, is conceivable for this purpose where S stands for long and I stands for short. *The Ratnamañjuṣā*,[3] has adopted this system based on the four doublets with long and short in addition.

The unit of two letters is comparatively small to express the basic constituents of the music, specially in the case of long lines. So a new unit of triplets or *trikas* which is neither too short nor too long was adopted by the classical poet-prosodists.

In ancient India, the number 3 was generally admitted as the smallest among the large, and largest among the small numbers. Multiplicity was considered as starting with 3, not with 2. The smallest number which constitutes *bāhulya* is 3 according to Pāṇini. So both as a matter of principle, and for the sake of convenience, a new unit of three letters called *trika*. (or triplets) was adopted for metrical scanning. These triplets consisting of the permutations of longs and shorts taken three at a time are called *gaṇas*.

Sequence of the triplets (gaṇas)

'In a lexicographical work to which the author has given his own name' wrote al-Bīrūnī, 'the feet composed of three *laghu* or *guru* are called by single consonants, which in the following diagram are written on their left.

Diagram I

m	S	S	S
y	I	S	S
r	S	I	S
t	S	S	I
s	I	I	S
j	I	S	I
bh	S	I	I
n	I	I	I

'By means of these signs, the author teaches how to construct these eight feet by an inductive method (a kind of algebraic permutation) saying:

"Place one of the two kinds (*guru* or *laghu*) in the first line unmixed (that would be SSS if we begin with a *guru*). Then mix it with the second kind, and place one of this at the beginning of the second line, whilst the two

other elements are of the first kind (ISS). Then place this element of admixture in the middle of the third line (SIS), and lastly at the end of the fourth line (SSI). Then you have finished the first half.

"Further place the second kind in the lowest line unmixed (III), and mix up with the line above it one of the first kind, placing it at the beginning of the line (SII), then in the middle of the next following line (ISI), and lastly at the end of the next following line (IIS). Then the second half is finished, and all the possible combinations of three *mātrās* have been exhausted".

Diagram II

	First Half				Second Half		
1.	S	S	S	5.	I	I	S
2.	I	S	S	6.	I	S	I
3.	S	I	S	7.	S	I	I
4.	S	S	I	8.	I	I	I

Calculation of the position in the sequence

'This system of composition of permutation is correct, but his calculation showing how to find that place which every single foot occupies in this series of permutations is not in accordance with it. For he says: "Place the numeral 2 to denote each element of a foot (i.e. both *guru* and *laghu*). Multiply the left (number) by the middle, and the product by the right one. If this multiplier, (i.e. this number on the right side) is a *laghu*, then leave the product as it is; but if it is a *guru*, substract one from the product".

'The author exemplified this with the sixth foot, i.e. ISI. He multiplies 2 by 2, and from the product 4 he substracts 1. The remaining 3 he multiplies by the third 2, and he gets the product 6.

'This, however, is not correct for most of the feet, and I am rather inclined to believe that the text of the manuscript is corrupt.

'The proper order of the feet would accoringly be the followings:

Diagram III

	No. 1	No. 2	No. 3
1.	S	S	S
2.	I	S	S
3.	S	I	S
4.	I	I	S
5.	S	S	I
6.	I	S	I
7.	S	I	I
8.	I	I	I

'The mixture of the first line (No. 1) is such that one kind follows the other. In the second line (No. 2) two of one kind are followed by two of the other, and in the third line, four of one kind are followed by four of the other.[3]

'Then the author of the above mentioned calculation goes on to say: "If the first element of the foot is a *guru,* subtract one before you multiply. If the multiplier is a *guru,* subtract one from the product. Thus you find the place which a foot occupies in this order".[4]

Modes of representations

The number r of permutations of n things when each may be repeated up to r times, is n^r The permutation of *laghu* and *guru* taken three at a time can be done in 2 or eight ways. The eight permutations of *guru* (S) and *laghu* (I) taken three at a time, form the *gaṇas.* Each of these is referred to by a Sanskrit letter for convenience. The accepted names of the *gaṇas* are as indicated by al-Bīrūnī in Diagram I.

The sequence of these *gaṇas* has been given differently by different Sanskrit authors.[5] One of the common rules is the following, given by Halāyudha Bhaṭṭa among others.

I	S	S	*ya*	S	I	I	*bha*	S	S	S	*ma*
S	I	S	*ra*	I	S	I	*ja*	I	I	I	*na*
S	S	I	*ta*	I	I	S	*sa*				

This nomenclature stresses the group relations. The first group consists of the three permutations of the *laghu* and two *gurus.* The second group gives the permutations of one *guru* and two *laghus.* The third group gives the two permutations in which all the three are of one kind or the other.

Another system followed by Jayadeva (who is found mentioned by writers about 1000 A.D.) and others, is to group pairs of *gaṇas* in which the *laghu* and *guru* have changed places.[6]

S	S	S	*ma*	S	I	I	*bha*	I	S	I	*ja*	I	I	S	*sa*
I	I	I	*na*	I	S	S	*ya*	S	I	I	*ra*	S	S	I	*ta*

The symmetry aspect of the individual *gaṇas* finds greater stress in this presentation.

Piṅgala, the great master says that the eight *gaṇas ma, ya, ra, sa, ta, bha,* and *na* along with *laghu* and *guru* form the basic components of the entire literature of prosody.[7]

The sequence of eight *gaṇas* as given in diagram III by al-Bīrūnī corresponds to the sequence given by Piṅgala. The first row has one *guru* and one *laghu* alternating. In the second row, two *gurus* alternate with two *laghus.* In the third row, four *laghus* come after four *gurus.*

Diagram I given by al-Bīrūnī is closely related to the arrangement stressing the group relations as given by Halāyudha and others. The first triplet is all *guru*. The next three triplets have 2 *gurus* and 1 *laghu*. The next three triplets have 2 *laghus* and 1 *guru*. The last triplet is all *laghu*.

The visible difference between diagram I and diagram III is only that items 4 and 5 have been interchanged.

There is, however, a more fundamental difference. While diagram I indicates the triplets of Sanskrit prosody, classified according to the number and position of the fundamental elements *laghu* and *guru*, diagram III follows the broader rules of *prastara*.

Prastara

The proper delineation of all the permutations of longs and shorts in a class of metres containing a given number of letters is called *prastara*. Two metres are possible with 1 letter in each line, 4 with 2 letters per line, 8 with 3 letters each, 16 metres with 4 letter lines, and so on. They follow the series 2^1, 2^2, 2^3, 2^4,..... The number of metres with n letters in each line is 2^n.

The accepted form for working out the logical sequence of the various metres containing a given number of letters is to alternate *guru* and *laghu* in column one, 2 *gurus* and 2 *laghus* in column two, 4 *gurus* and 4 *laghus* in column three, 8 *gurus* and 8 *laghus* in column four etc. till all the metres are worked out. The first form will, by convention, have all *gurus*, and the last form, therefore, will have all *laghus*.

Diagram III given by al-Bīrūnī is the *Prastara* for metres containing 3 letters. In fact Jayakīrti who lived about 1000 A.D. refers to this, in his *Chando'nuśāsanam*.[8]

The class of metres having 3 letters in a line is called *Madhyama chandas*. The number of metres in *Madhyama chandas* is 2^3 or 8. *Ma, ya, ra, sa, ta, ja, bha,* and *na gaṇas* give the *prastara* (or logical sequence) of the various metres in *Madhyama chandas*.

Since the position of each individual permutation in a logical sequence is fixed, it should be possible to calculate the position of each in a unique way. The method given in current books on prosody in Sanskrit and Indian languages differs from the one given by al-Bīrūnī.

The current system follows a gradual change from a combination containing only *gurus* to one containing only *laghus*. The logical sequence for metres containing one, two, three or four letters in a line is given in Diagram IV.

Diagram IV

Prastara of metres having one, two, three and four letters

Metres with one letter in each line

S
I

Metres with two letters in each line

S	S
I	S
S	I
I	I

Metres with three letters in each line

S	S	S
I	S	S
S	I	S
I	I	S
S	S	I
I	S	I
S	I	I
I	I	I

Metres with four letters in each line

S	S	S	S
I	S	S	S
S	I	S	S
I	I	S	S
S	S	I	S
I	S	I	S
S	I	I	S
I	I	I	S
S	S	S	I
I	S	S	I
S	I	S	I
I	I	S	I
S	S	I	I
I	S	I	I
S	I	I	I
I	I	I	I

System in current use

The calculation of the position of a given permutation in the series appears to be based on the binary system of numbers. From the left, place on top of each *laghu* or *guru* the number 1.2.3.4.8, etc. in geometric progression, each number being double the previous one. Cross out the number above the *gurus*. Add the numbers above the *laghus*. The result obtained *plus* 1 denotes the position of the permutation in the series.

For example, in the case of S I I, we have the following calculation:

1	2	4
S	I	I

1 is over a *guru* and is to be omitted. 2 + 4 gives 6. This plus 1 gives the position of S I I as 7th in the series.

Mathematically, the system may be explained as a series of reversed binary numbers in which I stands for 1 and S for 0, numbers to be read from right to left. Thus S I I may be rewritten as 011 which if read from right to left is six according to the binary system.

This method of calculation can be extended to the metres with any number of letters in a line. Each permutation is easily and uniquely determined.

Application of the rules quoted by al-Bīrūnī

Let us now consider whether the rules enunciated by the Sanskrit lexicographer as quoted by al-Bīrūnī give a unique value to each of the combinations, and whether they follow a logical order.

If the *laghus* and *gurus* are represented by 2's, and multiplied from left to right, subtracting 1 from the product if the multiplier is a *guru*, we get the following values.

S. No.	*Representation*			*Calculation*	*Value*
1.	S	S	S	$(2\times2-1)\times2-1$	= 5
2.	I	S	S	$(2\times2-1)\times2-1$	= 5
3.	S	I	S	$(2\times2\ \ )\times2-1$	= 7
4.	S	S	I	$(2\times2-1)\times2$	= 6
5.	I	I	S	$(2\times2\ \ )\times2-1$	= 7
6.	I	S	I	$(2\times2-1)\times2$	= 6
7.	S	I	I	$(2\times2\ \ )\times2$	= 8
8.	I	I	I	$(2\times2\ \ )\times2$	= 8

The instance quoted by the lexicographer (serial No. 6) happens to be correct. But there is only one other case (serial No. 8) where the numerical value of the representation indicates the position in the list also.

Examination of diagram III shows that these are the only two cases in the arrangement due to al-Bīrūnī also where the values correspond to the position in the list.

The rules of the lexicographer as we have applied are obviously unsatisfactory.

One thing which deserves special notice is that it is not merely a case of lack of correspondence between the value of each permutation and its positional number. None of the values calculated is below 5. Several items have identical values. There are two 5's, two 6's two 7's and two 8's.

We have now to consider whether some important rule which could reduce the value of some of these permutations to 1,2,3 or 4 has been lost sight of. Each permutation ought to give a unique value.

The vital link

In the paragraph which al-Bīrūnī has added after his calculations proved unsatisfactory, and gave what he thought as the correct arrangements, there are two important provisos. "If the first element of a foot is a *guru*, subtract one before you multiply. If the multiplier is a *guru*, subtract one from the product. Thus you find the place which a foot occupies in this order."[8] Let us apply these two rules also and evaluate the positions again.

S. No.	*Representation*	*Calculation*	*Value*
1.	S S S	$(2-1\times2)-1\times2-1$	= 1
2.	I S S	$(2\times2)-1\times2-1$	= 5
3.	S I S	$(2-1\times2)\times2-1$	= 3
4.	S S I	$(2-1\times2)-1\times2$	= 2
5.	I I S	$(2\times2)\times2-1$	= 7
6.	I S I	$(2\times2)-1\times2$	= 6
7.	S I I	$(2-1\times2)\times2$	= 4
8.	I I I	$(2\times2)\times2$	= 8

The application of the new rules has resulted in bringing serial nos. 1 and 3 against their numerical values, in addition to serial numbers 6 and 8 which continue to be correct. This is true of both the arrangements, that given by the lexicographer, and that given by al-Bīrūnī.

There is however another important improvement. No two permutations have the same numerical value. Each permutation is uniquely determined.

The difference between the first set of rules and the second set of rules is the clause about subtracting one before multiplying, if the first element happens to be a *guru*. This is an essential condition which somehow appears to have got overlooked, before diagram III was worked out.

Let us now arrange the permutations according to the numerical value.

Diagram V

	No. 1	No. 2	No. 3
1.	S	S	S
2.	S	S	I
3.	S	I	S
4.	S	I	I
5.	I	S	S
6.	I	S	I
7.	I	I	S
8.	I	I	I

We now get a system in which 4 *gurus* are followed by 4 *laghus* in the first line, two *gurus* alternate with two *laghus* in the second, and one *guru* and one *laghu* alternate in the third.

A closer examination will reveal that the present arrangement is a mirror image of the arrangement given in al-Bīrūnī's *India*. If columns III and I are interchanged in diagram III, we get the correct arrangement given in diagram V.

Al-Bīrūnī and Metrology

The present instance highlights certain aspects of the character and credibility of al-Bīrūnī. Although he apparently failed to obtain correct results based on the formula he applied, he did not on that account seek to brand the lexicographer as incorrect. Al-Bīrūnī only says "I am inclined to believe that the text of the manuscript is corrupt."[9] This is truly the attitude of a seeker after truth, with becoming modesty.

Al-Bīrūnī did not dispose of the matter there. He gave additional material which had either escaped his attention earlier, or came to his notice later. He gives it, to keep the record straight.

He gives new data. But strangely he does not seem to apply it. This was obviously additional information, very relevant to the discussion. The condition regarding the character of the first element has a controlling influence on the whole calculation.

Metrology was not a subject which al-Bīrūnī had mastered, although his reference to Arabic and Persian Metricians[10-11] shows his basic knowledge of the subject. He refers to various Sanskrit books[12] on Metrics. "I, however, had not seen any of these books, nor do I know much of the chapter of the *Brāhma-siddhānta* which treats of metrical calculations and therefore I have no claim to a thorough knowledge of the laws of their metrics. Nevertheless I do not think it right to pass by a subject of which I have only a smattering, and I shall not postpone speaking of it until I shall have mastered it."[13] As Sachau says in his introduction, "Himself perfectly sincere, it is sincerity which he demands from others. Whenever he does not fully understand a subject, or only knows part of it, he will at once tell the reader so, either asking the reader's pardon for his ignorance, or promising, though a man of fifty-eight years, to continue his labours and to publish their results in time, as though he were acting under a moral responsibility to the public."[12]

Al-Bīrūnī criticizes manuscript tradition like a modern philologist. He sometimes supposes the text to be corrupt, and enquires into the cause of the corruption.[13] As in the case of chapters on Indian Astronomy and Chronology, in the chapter on grammatical and metrical literature, al-Bīrūnī continues a literary movement, which at his time had already gone on for centuries; but he surpassed his predecessors, by going back upon the original Sanskrit sources, trying to check his paṇḍits by whatever Sanskrit he had contrived to learn, and by his conscientious method of testing the data by calculation.. As Sachau says, "Al-Bīrūnī's work represents a scientific renaissance."[14]

Elegance of the rule quoted by al-Bīrūnī

The elegance of the rules quoted by al-Bīrūnī lies in the possibility of getting the position of the metrical permutation directly. In the current method of calculation, 1 has to be added to the number obtained by the binary procedure. The binary classes are one short of the power of 2, while the total numbers of metrical permutations are in powers of two.

The scheme for the representation of the Sanskrit *gaṇas* quoted by al-Bīrūnī can be extended to the unique representative of any number from 1 to 2^n in the form of a permutation of two different things taken '*n*' at a time.

Importance of proper translation

The examination of an Arabic reprint[17] appears to indicate that it may be possible to differ from the representation given by Sachau and to construe the representation given by al-Bīrūnī in Arabic as consistent with the mathematical text. This is a matter which is being further investigated. It may be mentioned that a Malayalam translation of *Al-Beruni's India* published by the Sahitya Akademi,[18] which appears to be from the Arabic original, does

give a representation different from that of Sachau, but agreeing with the Sanskrit text.

This highlights the need for a proper study of manuscripts in foreign languages by persons competent both in the language and the subject. The Malayalam translation referred to above shows lack of understanding, or studied disregard of even common knowledge, which makes the retranslation of Indian terms from Arabic into an Indian language look ridiculous. There is obvious need for closer study of the original and for better translations of the painstaking and invaluable works of al-Bīrūnī by individuals, or groups, with adequate mastery of the media and of the matter.

Al-Bīrūnī appears on the Indian horizon as a star of rare brilliance, shedding its light on many a dark page of medieval history. Being an eminent scholar himself, he developed great zeal for a comparative study of the religions, cultures, philosophies and scientific achievements of other nations. E.C. Sachau, his able translator, says about him: "The Hindus and their world of thought have a paramount fascinating interest for him." Al-Bīrūnī appreciates scientific statements wherever they are found. In mathematics and architectural constructions, he considers the Hindus to have reached a high degree of art, though he ridicules their superstitions and practices.

REFERENCES

[1] *Jayadamān*, ed. H.D. Velankar, Foreword: p. 16.

[2] *Vṛtta Jātisamuccaya, Vīraṭaṅka*, Ch. 5.

[3] *Ratnamañjuṣā*, Unknown Jaina author, referred to by D. H. Velankar in his Foreword to *Jayadaman*, p. 17.

[4] *Al-Beruni's India*, Tr. E. C. Sachau, p. 141.

[5] *"Ādi madhyavasāneṣu*
Ya ra tā yānti lāghavam
Bha ja sā gauravam yānti
Ma nau tu guru lāghavam"
—Commentary on Piṅgala *Chandas* by Halāyudha Bhaṭṭa, p. 2.
Ādi madhyavasānesu
Bha ja sa yānti gauravam
Ya ra ta lāghavam yānti
Ma nau tri guru lāghavam
—*Jayadevachandas*, ch. 6.

[6] *"Sarvādi madhyānta glau*
Trikau mnau bhyau jrau stau"
Jayadevachandas, ch. 1. *Sū* 2.

[7] *Ma ya ra sa ta ja bha na la ga*
Sammatam bhramati vāṅmayam jagati yasya.
Sa jayati Piṅgala naga: Śivaprasādādviśuddhamtiḥ
Trigurum Viddhi makāram
Laghvādi Samanvitam yakārākhyam
Laghu madhyamam to refam
Sakāramantya gurunibaddham.
Laghvantam hi takāram

Jakāramubhayor laghum vi-jāniyat
Adi gurum Ca bhakāram
Nakāramiha paiṅgale tri laghum.
Commentary on Piṅgala's *Chandas* by
Halāyudha Bhaṭṭa, p. 1.

[8] *Al-Beruni's India,* p. 142.

[9] *ibid,* p. 142

[10] "In counting the syllabus (*gaṇa chandas*) they use similar figures to those used by Alkhalid Ibn Ahmad and our metricians" *ibid,* p. 132.

[11] "The Persian metricians, for instance, call such a consonant *moved by* a *light vowel* (i.e. pronounced with a sound like the Hebrew Schwa)" ibid pp. 138-39.

[12] *ibid,* pp. 137-138

[13] *ibid, p.* EAB

[14] *ibid,* p. XX

[15] *ibid,* p. XXVI

[16] *ibid,* p. XXXVII

[17] *Al-Beruni's India* published by the Osmania Oriental Publications Bureau, p. 110

[18] *Al-Beruni Kanda India,* Malayalam translation by A. M. Mohiuddin Alwaye of al-Bīrūnī's *Kitāb-ul-Hind,* Sahitya Akademi, p. 93.