THE EXTANT *SIDDHĀNTA SĀRVABHAUMA*—AN ERROR IN THE SINE OF ONE-THIRD PART OF AN ANGLE

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The *Siddhānta Sārvabhauma*, an astronomical work of Munīśvara in its *Spaṣṭādhikāra* has discussed different trigonometrical formulae, tables and other topics related to trigonometry. The proof of the formula Sin A/3 as given in the commentary on *Siddhānta Sārvabhauma* is observed to be erroneous. A study of the relevant commentary portion reveals that this must be either due to defective copying of the portion concerned or printing mistake. The paper attempts to throw light on this aspect.

INTRODUCTION

Indian astronomer and mathematician Munīśvara composed his famous work entitled *Siddhānta Sārvabhauma* (=*SSB*)[1] in 1627 **A. D.** and he himself commented on it. In the chapter *Spaṣṭādhikāra* of *SSB* he gave different trigonometrical formula and other topics related to trigonometry. We find that some errors have crept in the proof of trigonometrical formula of Sine of one-third part of an angle in his commentary. We think that this error is either due to defective copying or a printing mistake.

DISCUSSION

Munīśvara gave the following formula of sin (A/3) in his *SSB*[2]

" *Jyātramaśastaddhanastri jyāvargāptaḥ svatribhāgayuka*
jyātraṃśe yojitastasmārduktarītyā ghanādikam
Phalaṃ purnarguṇatraṃśe yutamevaṃ muḥuhsputā
tṛtīyāṃśajyakā tristhatulyāṅkā bhihatirghanaḥ"

The meaning of these two *ślokas* substantially runs as follows :

"Divide four times of the cube of the sine of one-third of the desired arc by the three times of the square of the radius. Add this result to the one-third of the sine of the desired arc and this is equal to Sine of the one third of the desired arc".

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That is according to the meaning of the *ślokas*, we have

$$R \sin (A/3) = \frac{R \sin A}{3} + \frac{4(R \sin (A/3))^3}{3R^2}.$$

In the modern form, this becomes $\sin (A/3) = \frac{\sin A}{3} + \frac{4 \sin^3 (A/3)}{3}$

Munīśvara gave the proof of the formula in details in the *ślokas* starting from the number 73-95 of *Spaṣṭādhikāra*[3]. He further elaborates the proof in the commentary of *Siddhānta Sārvabhauma* (=*SSBC*)[4]. Now from the *śloka* and its commentary we have the following steps.

We are to find out sine of one-third part of a desired arc. Twice the sine of the desired arc is base (*bhūmi*) and the sine of the two-third part of the desired arc is *bhuja*. Take the front (face) as equal to this *bhuja*. Then we find a square.

Let the desired arc be A,

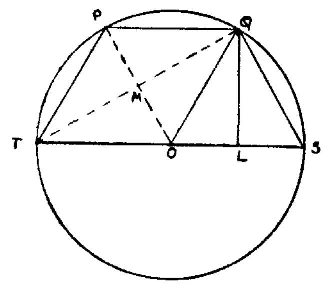
TS = base (=*bhūmi*) = $2 R \sin A$,

$QS = PT$ = sine of the two-third part of A,

$= R \sin (2A/3)$ = *bhu*,

PQ = front (or face = *mukham*) = PT,

QT = diagonal (*karṇa*).



Draw QO parallel to PT such that QOS is an isosceles triangle. From Q draw QL perpendicular on OS such that $OL = LS$.

Now

$$LS = OS/2 = \frac{TS - OT}{2} = (2 R \sin A - R \sin (2A/3))/2.$$

Squaring

$$(LS)^2 = \frac{4R^2 \sin^2 A + R^2 \sin^2 (2A/3) - 4R \sin A . R \sin (2A/3)}{4}.$$

Now

$$(QL)^2 = (QS)^2 - (LS)^2$$

$$(QL)^2 = (R \sin (2A/3))^2 - \frac{4R^2 \sin^2 A + R^2 \sin^2 (2A/3) - 4R \sin A . R \sin (2A/3)}{4}$$

$$\text{or, } (QL)^2 = \frac{4 R \sin A . R \sin (2A/3) + 3R^2 \sin^2 (2A/3) - 4R^2 \sin^2 A}{4} \quad \ldots \quad (i)$$

$$TL = TO+OL$$

$$=R \sin(2A/3)+\frac{2R \sin A - R \sin (2A/3)}{2}$$

$$=\frac{2R \sin A + R \sin (2A/3)}{2}$$

$$\therefore (TL)^2=\frac{4R^2 \sin^2 A+4R \sin A.\, R \sin (2A/3)+R^2 \sin^2 (2A/3)}{4} \quad \ldots \quad (ii)$$

$$TQ^2=\frac{8\, R \sin A.R \sin (2A/3)+4R^2 \sin^2 (2A/3)}{4} \quad [\therefore \quad TL^2+QL^2=TQ^2]$$

$$=2\, R \sin\, A.\, R \sin (2A/3) + R^2 \sin^2 (2A/3). \quad \ldots \quad (1)$$

Now in order to find out the square of TQ, draw perpendicular PM from P upon the diagonal TQ which is versed sine of the arc TQ. Now it is true that

$$TP^2-PM^2=TM^2$$

But TM=Sine of the half arc TQ.

$\therefore$ (Sine of the half arc $TQ)^2 = \dfrac{R \text{ versed sine of the arc } TQ}{2}$

or, 4 (Sine of the half arc $TQ)^2=2R.\, PM$

or, $(TP=\text{base})^2=2R.\, PM$

or, $PM=\dfrac{(R \sin (2A/3)\,)^2}{2R}$

or, $(PM)^2=\dfrac{(R \sin (2A/3)\,)^4}{4\, R^2}$

Now $TM^2=TP^2-PM^2$

or, $(TQ)^2/(2)=(R \sin (2A/3))^2-\dfrac{(R \sin (2A/3))^4}{4R^2}$

$$=\frac{4R^2(R \sin (2A/3))^2-(R \sin (2A/3))^4}{4R^2}$$

or, $(TQ)^2 = \dfrac{4R^2\, (R \sin (2A/3))^2-(R \sin (2A/3))^4}{R^2} \quad \ldots \quad (2)$

From (1) and (2) we have,

$$2R \sin A.\, R \sin (2A/3)+(R \sin (2A/3))^2 = \frac{4R^2\, (R \sin (2A/3))^2-(R \sin (2A/3))^2}{R^2}$$

or, $2R^2\, (R \sin A)\;\; (R \sin (2A/3))+R^2\, (R \sin (2A/3))^2$

$$=\{4R^2\, (R \sin (2A/3))^2-(R \sin (2A/3)^4)\,\} \quad \ldots \quad (3)$$

or, $R^2(R \sin A)+R^2(R \sin (2A/3))=4R^2(R \sin (2A/3))-(R \sin (2A/3))^3 \ldots \quad (4)$

or, $3R^2 (R \sin (2A/3) = 2R^3 (R \sin A) + (R \sin (2A/3))$ (5)

or, $R \sin (2A/3) = 2/3\ R \sin A + \frac{(R \sin (2A/3))^3}{3R^2}$

or, $\frac{R \sin (2A/3)}{2} = \frac{R \sin A}{3} + \frac{(r \sin (2A/3))^3}{2.3\ R^2}$

$$= \frac{R \sin A}{3} + \frac{4\ (R \sin (2A/3))^3}{8.3\ R^2}$$

or, $\frac{R \sin 2A/3}{2} = \frac{R \sin A}{3} + \frac{4 \left(\frac{R \sin (2A/3)}{2}\right)^3}{3\ R^2}$.

Now, $R \sin (2A/3) = TP = QS$

$$\frac{R \sin (2A/3)}{2} = TP/2 = QS/2 = \text{half of } bhuja.$$

i. e. $TP/2 = R \sin (A/3)$.

Hence we have

$$R \sin (A/3) = \frac{R \sin A}{3} + \frac{4\ (R \sin (A/3))^3}{3\ R^2}.$$

In the modern form this becomes

$$\sin A/3 = \frac{\sin A}{3} + \frac{4 \sin^3 (A/3)}{3}.$$

Now we pay more attention to the steps (3), (4) and (5).

For the step (3) Muniśvara gave for commentary on the part of the 89th śloka[5] as follows

bhuva triva $\dot{1}$ *bhujyā triba* 2
bhubava $\dot{1}$ *bhuva triva* 4

which means,

$$2R^2(R \sin A)\ (R \sin (2A/3)) + R^2(R \sin (2A/3))^2$$
$$= 4R^3\ (R \sin (2A/3))^3 - \sin (2A/3))^4.$$

Now after this step Muniśvara said

bhu gha $\dot{1}$ *bhu triva 4*
bhu triva 1 jyā triva 1

means

$$R^2(R \sin A) + R^2(R \sin (2A/3)) = 4R^2(R \sin (2A/3) - (R \sin (2A/3))^3$$

Now the question is that—how we get the step (4) from the step (3). It is impossible to get step (4) from (3).

Again on the commentary on 90 - 91 *ślokas* Munīśvara said [6]

bhu gha 1 jyā triva 2 *bhu triva* 3

means

$$3R^2(R \sin (2A/3)) = 2R^2(R \sin A)+(R \sin (2A/3))^3.$$

Again the question may arise—how we get the step (5) from (4).

On the examination of the commentaries we say that if the step (4) is of the form

$$2R^2 (R \sin A)+R^2(R \sin (2A/3))=4R^3 (R \sin (2A/3))-(R \sin (2A/3))^3,$$

it is possible the new step (4) can be deduced from the step (3) and step (5) can be deduced from the new step (4).

Thus the commentary on 89th *śloka* given by Munīśvara must be of the form.

bhu gha 1 bhu triva 4
bhu triva 1 jyā triva 2

We think that the error which is found in the commentary on 89th *śloka* is either due to defective copying of the number or a printing mistake when the text was printed.

REFERENCES

1 *Siddhānta Sārvabhauma* (with the author's own commentary edited by Muralidhara Thakkura, Saraswati Bhavana Texts No. 41, Part I, Benaras, 1932. Pages. 132—175.

2 Ibid. *ślokas* 71, 72 (*Spaṣṭādhikāra*)

3 Ibid. *ślokas* 73—95 (*Spaṣṭādhikāra*)

4 Ibid. PP 154, 155, 156, 157, 158, 159, 160

5 Ibid. P, 158.

6 Ibid. P. 158.