MYSTICAL MATHEMATICS OF ANCIENT PLANETS

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The traditional Indian navagrahas include Sun, Moon, Mars, etc. (in order of weekdays) plus Rāhu and Ketu. The belief that planets exert great influence on life and events of individuals and world gave rise to planet worship among various peoples.

For worship in a mandala (symbolic diagram) various geometrical figures and forms were used to represent the planets. These include square, circle, semi-circle, triangle, pentagon, bow-figure and the popular lotus-figure.

The present paper gives a mathematical discussion of the geometrical figures connected with planet worship. It also deals with the yantras (mystic diagrams) of the ancient planets as well as with the magic squares associated with them. Original Sanskrit verses have been appropriatively given and some possible inter-cultural connections mentioned.

Key words: Ancient and Medieval mathematics, Medieval geometry, Magic squares, Planet worship, Yantras.

INTRODUCTION

The modern heliocentric astronomy has nine planets in our solar system. These are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.

They are listed here in the order of their increasing distances from the Sun around which they revolve with different periods of revolution (ranging from 0.24 to 248.43 years). Uranus was discovered in 1781, Neptune in 1846, and Pluto in 1930.

The pre-heliocentric (or pre-Copernican) concept or ancient definition of planet (graha) was different. The ancient astronomy was geocentric and so...
Earth was not taken as a graha. In addition to the Sun and the Moon, the ancient peoples could find, by naked eyes' observations, only the following five planets (tārāgrahas): Mars, Mercury, Jupiter, Venus, and Saturn. These seven heavenly shining bodies were found to have angular motions relative to fixed stars. They constituted the early list of seven planets after whom the weekdays are named in order from Sunday to Saturday. The current universal practice of using almost the same and similar names of the seven weekdays and in exactly same order throughout the world indicates that the present concept of a 7-day week was most probably started in a single place and diffused to other places (according to diffusion theory).

In India, Rāhu (ascending node of the moon) and Ketu (descending node of the moon) were also taken as grahas later on, thus making the traditional list of navagrahas ('nine-planets') of the Indian Jyotisha-sāstra. Rāhu and Ketu are called chāyā-grahas. According to Prof. K. S. Shukla, Rāhu and Ketu are mentioned in the Maithriya- upaniṣad along with Śani (Saturn). According to traditional interpretation, the word graha, as a bhūta-samkhya (word-numeral) stands for 'nine'.

Ancient peoples believed that the positions and configurations of planets have great influence on the life of individuals and on the course of events in the world and nature. Since remote times, man has been worshipping objects of nature and was involved in their personification (e.g. cf. Pṛthvi-mātā or mother earth) and deification (e.g. cf. Śūryadeva or the Sungod). The confident belief of the ancients that the planets exert both beneficial and malefic influence on the life and activities of human beings led them to attach great importance to the planets. The planets were not only personified but accepted as deities. From the point of view of history of science, the note-worthy thing is that the practice of recognizing planetary deities was prevalent in almost all ancient cultures. Their nomenclature reflects connections with classical gods and goddesses. Jupiter was the highest and most powerful among the Roman gods. He was the god of heaven and determined the course of all human affairs. Among the Greeks he was called Jeus or Zeus (cf. Sanskrit word Jīva for Jupiter or Bṛhaspati who was suraguru of heavenly gods).

Venus (cf. Vena mentioned in the Rgveda) was Roman goddess of love. Interestingly the Indian graha Śukra (Venus) is a god or male deity who
was also called Bhārgava (son of Bhṛgu) and Sīta (‘white’). In the same mythological style Bhauma (Mars) is son of Earth, Budha (Mercury) is regarded son of Moon, and Śani (Saturn) that of Sun.

In ancient India, all the nine planets (navagrahas) were personified as male deities and were associated with attributes in analogy to other gods of Hindu pantheon. They were worshipped through various modes. Vedic, Smārtya, and Tāntrika rituals were devised to perform graha-yajñas to please and pacify them (graha-śānti) and to avoid their evil influence (anīśta). Elaborate procedures and details of the planetary rites appear in several texts and works like Grhya-sūtras, Purānas, Smṛtis, on Kuṇḍas and jyotiṣa, and in other monographs and manuals.

In the present paper, we are primarily concerned with those aspects which involve the history of exact sciences (geometrical and mathematical matters). Some other general and related relevant matters will also be mentioned.

**Geometrical Figures Associated with Planets**

A general and historical perspective of associating specific geometrical forms to certain objects and other aspects of speculative human thinking may be briefly mentioned first. The figures of square, circle and semicircle are associated with the three nitya-agnis (‘obligatory fires’) whose Aryan tradition is older than the Rgveda which is the oldest of the four Vedas.

The old concept of basic bhūtas (gross elements) is found not only in India but in some other ancient civilizations also. Among the Greeks, Empedocles (490-435 BC) was the first to declare four of these gross elements (or forms of matter) to be earth, water, fire and air which were the material principles from which the universe was evolved. The discovery of the regular solids of polyhedra led the Greeks to connect them with the elements. There are five such regular polyhedra which are also called by other names such as Platonic solids, mathematical figures, and cosmic figures. In the final tally, the Greeks believed that the figure tetrahedron (or triangular pyramid) corresponds to fire, cube to earth, octahedron to air, icosahedron to water, and the dodecahedron to the sphere of the universe (the vessel of the sphere itself was taken to be the fifth body or element).
In Ancient India, the traditional number of gross elements was five and these were called *pañca-mahā-bhūtas* (‘the five gross elements’). The whole *brahmāṇḍa* (universe or cosmos) is pervaded by these elements which are *kṣiti* (earth), *jala* (water), *pāvaka* (fire), *vāyu* (air), and *ākāśa* which is variously translated as sky, heaven, ether or empty space (which must be distinguished from the corporeal and invisible air). Their age-old geometrical association is described in the cognate text *Śāradātilaka*, I. 23-24 as follows:

“The circle is the *maṇḍala* (symbolic diagram) of the heaven or sky; the (circle) decorated by six dots is (the figure) of wind; the triangle with *svastika* is that of fire; the half-moon (i.e. semicircle) with lotus is that of water; and the square with *vajra* (thunderbolt weapon, figure or diamond-like figure) that of earth. The wisemen have thus spoken of the above *maṇḍalas* for the respective *bhūtas*.”

It is interesting to note that while the Greeks connected the gross elements with three dimensional solid figures, in India the elements were associated with two-dimensional plane figures. The case of elements is mentioned here just to point out that there was a tradition of connecting geometrical figures with them. The case of planets will be discussed now in detail.

The *Graha-Śānti* section in Ācāradhyāya chapter of the *Yājñavalkya-Smṛti* (about AD 300 or later) contains the main features for the worship of the planets (*grahas*) according to Hindu tradition. The names of these ancient nine planets (first seven in the order of weekdays) are mentioned in the work (I.295b-296a) as:

*sūryaḥ* *somo* *mahiputrah* *somaputro* *bṛhaspatih* // 295//
*sūkrah* *śanais’caro* *rāhuḥ* *ketuś* *ceti* *grahah* *smṛtāḥ* /

“The Sun, Moon, Mars, Mercury, Jupiters, Venus, Saturn, Rāhu, and Ketu are said to be the planets.”

The same work also states (I. 306b, p. 193)

*yaś ca yasya yadā duṣṭaḥ sa tamyatnena pūjayer* //306//
"One should specially worship that (planet) which, and whenever, it is cruel (malefic) to him."

Thus it seems that the planets may be worshipped separately (i.e., one at a time) or they may be worshipped more at a time, or collectively for the peace, progress and welfare of an individual or a group. For worship, either their idols (mūrtis) made of specified materials are used or their images are made (lekhyah or painted) on patas (cloth pieces or plates) with specified materials and in specified colours. In the absence of the above two type of objects (idols or picture-images), the specified figures (ākṛtis) associated with the planets should be used as their symbolic representations to serve the same purpose of worship.

In a collective worship of all the nine planets, their idols, images or their representative figures are to be placed in a maṇḍala (ritual diagram). The navagraha-maṇḍala consists of a square subdivided into nine equal square cells. The Brhat-Parāśara Saṃhitā calls the diagram as navabhuvanaṃcataśra-maṇḍalam (‘nine-celled square diagram’). The placement of the symbols of the planets in the nine cells is described in the same Saṃhitā as follows:

\[
\begin{align*}
&\text{madhye tu bhāskarāḥ sthāpyah pūrvadaksinatah śaśi} / \\
&\text{dakṣinena dharāśūnur budhāḥ pūrvottareṇa tu} // \\
&\text{uttarasyām surācāryah pūrvasyāmbhrgunandanaḥ} / \\
&\text{paścimāyām śaniḥ kuryāt rāhur daksīna paścime} // \\
&\text{paścimottaratataḥ ketur iti sthāpyāḥ grahāḥ kramāt} / \\
\end{align*}
\]

"Place Sun in the centre, Moon in the south-east (cell), Mars in the south, Mercury in the north-east, Jupiter in the north, Venus in the east. Do the placement of Saturn in the west cell, Rāhu in south-west (and) Ketu in north-west. Thus the planets should be placed in order."

The same placement is described in the Matsya Purāṇa but the description does not follow the weekday order of planets.

The placement of planets in navagraha maṇḍala is shown in Fig. 1
In ancient tradition, the direction of East was generally taken upwards. A Sanskrit passage describing the various figures associated with the planets is as follows:  

```
varttulo bhāskarḥ kāryo, hyardhacandro niśākaraḥ / 
aṅgārakstrikoṇas tu, budhaścāpakṭis tathā //  
padmākṛtir guruh kāryaś, catuṣkonaś tu bhārgavaḥ / 
sarpākṛtiḥ sāniḥ kāryo, rāhus tu makarākṛtiḥ // 
khadgākṛtis tathā ketuḥ kāryo maṇḍala pūjane / 
```

"In maṇḍala (symbolic diagram) – worship of planets, a circle should be drawn for the Sun, semicircle for the Moon, triangle for the Mars, bow-figure for Mercury, lotus-figure for Jupiter, square for Venus, serpent-figure for Saturn, makara or crocodile-figure for Rāhu, and sword-figure for Ketu."

The names of the maṇḍala figures representing the planets are also depicted in Fig. 1. As is often the case with so many matters, in a vast country like India which has a very long history and traditions, there are found some variations in the forms of the symbolic figures and in their association with the planets. For instance, in the description given by Jogesh Chandra Ray, the figures associated with Saturn and Ketu are mutually exchanged. Further, the word makara was translated as fish (instead of crocodile) as is clear from the accompanying Fig. 2 given by him:
The following set of similar verses is found elsewhere:

\[
\text{vṛttamaṇḍalamādityam, ardhaçandra niśākaram} / \\
\text{trikonaṁmaṅgalam caiva, budham ca dhanuśākrītim} / \\
\text{gurumaṣṭadalamproktam catuśkoṇam ca bhārgavam} / \\
\text{narākrītim sānim vidyādrāhum ca makarākrītim} / \\
\text{ketuṁ khaṅgasamam jñeyam grahaṃṇḍalake śubhe} //
\]

The noteworthy difference in this description is that Saturn is symbolized by \text{nara'Mi} or ‘figure of a man’ (instead of serpent). Jupiter’s lotus figure is called here \text{aṣṭadalam} or eight-petalled, Ketu’s \text{khaṅga} is written here as \text{khaṅga}.

The figures mentioned by M. Mishra seems to be a mixture from various sources and are shown in Table 1.

**MATHEMATIZATION OF PLANET FIGURES**

Out of various figures prescribed as ritual symbols of \text{grahas} (ancient planets of India) for their worship, the shapes of circle, semicircle, and square are uniquely defined (except for size). The form of triangle was usually taken to be equilateral and so it can also be assumed to be defined. It may be mentioned that in general tantra literature, the isosceles triangles with apex upwards were often called \text{Śiva} or \text{linga} or \text{agni} triangles, while those pointing downwards were called \text{Śakti} or \text{yoni} triangles. For instance, the central triangular complex of the famous Śrīyantra (Fig. 3) consists of 4 \text{Śiva} and 5 \text{Śakti} triangles. Interestingly the 9 parallel bases (starting from topside) were given the names of the 9 \text{grahas} in the weekdays order from Sun to Saturn plus Rāhu and Ketu.
<table>
<thead>
<tr>
<th>Planet or Graha</th>
<th>Placement in manḍaḷa</th>
<th>Planet’s ākṛiti for worship</th>
<th>Aṅkṛiti according to M. Mishra</th>
<th>Shape of fire-pit and and pīṭha (base)</th>
<th>Area of base in sq. angulas</th>
<th>Diameter of basic Circle for kuṇḍa racanā (from Kulkarni, ref. 18)</th>
<th>Talisman Magic Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>Centre</td>
<td>Circle</td>
<td>Circle</td>
<td>Circular</td>
<td>12</td>
<td>27.0937 ang.</td>
<td>6</td>
</tr>
<tr>
<td>Moon</td>
<td>South-east</td>
<td>semicircle</td>
<td>semicircle</td>
<td>Square</td>
<td>24</td>
<td>33.9375 ang.</td>
<td>9</td>
</tr>
<tr>
<td>Mars</td>
<td>South</td>
<td>triangle</td>
<td>triangle</td>
<td>triangular</td>
<td>4</td>
<td>42.125 ang.</td>
<td>5</td>
</tr>
<tr>
<td>Mercury</td>
<td>North-east</td>
<td>bow-shaped</td>
<td>arrow</td>
<td>arrow-shaped</td>
<td>4</td>
<td>58.875 ang.</td>
<td>8</td>
</tr>
<tr>
<td>Jupiter</td>
<td>North</td>
<td>lotus figure</td>
<td>lotus figure</td>
<td>rectangular</td>
<td>6</td>
<td>36.4844 ang.</td>
<td>4</td>
</tr>
<tr>
<td>Venus</td>
<td>East</td>
<td>Square</td>
<td>pentagram</td>
<td>pentagram (or pentagon)</td>
<td>9</td>
<td>39.25, or 20 ang.</td>
<td>7</td>
</tr>
<tr>
<td>Saturn</td>
<td>West</td>
<td>Snake figure</td>
<td>man’s figure</td>
<td>bow-shaped</td>
<td>6</td>
<td>29.3125 ang.</td>
<td>3</td>
</tr>
<tr>
<td>Rāhu</td>
<td>South-west</td>
<td>Crocodile figure</td>
<td>Crocodile (makara)</td>
<td>winnowing basket</td>
<td>25</td>
<td>39.0937 ang.</td>
<td>4</td>
</tr>
<tr>
<td>Ketu</td>
<td>North-west</td>
<td>Sword</td>
<td>flag</td>
<td>triangular flag</td>
<td>8</td>
<td>58.1875 ang.</td>
<td>4</td>
</tr>
</tbody>
</table>
Briefly stated, the *padma* of lotus figure is a flowery design. Most commonly used figure of lotus has 8 petals which are symmetrically arranged usually in a circle. The outer part of a petal may be relatively longer or broader and is of three types, namely, (i) round, (ii) simply-pointed, (iii) inflectional or ogee-form. In Fig. 3, the triangular complex is surrounded by a lotus figure of 8 petals of inflectional type, i.e., each of the two sides of every petal has a point of inflexion where the curvature changes.
The figures representing real pictures or forms of bow, sword, snake, man and *makara* are difficult to define exactly or uniquely in a mathematical sense unless there is some common and acceptable understanding. Moreover, such figures also need some sort of artistic talent to draw them with reasonable and good-looking forms (to avoid the wrath of deities). There was a need to geometrize and mathematize some figures, and this was done. The bow-figure was obtained by the following construction (see Fig. 6).

Two equal intersecting circles are drawn such that the distance between their centers (K and L) is equal to the radius, *r*, of either circle. UV is their common chord. The line KL is extended both ways beyond the circles to become YZ such that

\[ YA = BZ = \frac{r}{3} \]

If YS and ZT are tangents to the circles, then YSUTZY is theoretically an ideal bow-shaped figure (Fig. 7)\(^{17}\).

In another method,\(^{18}\) we locate two points P and Q (instead of S and T) on the circles such that the perpendicular distance of P and Q from the line YZ is same, and is equal to half the side of a regular octagon inscribed in either circle. In other words,

\[ \angle AKP = \angle BLQ = 22.5 \text{ degrees} \]

By this method we get the bow-shaped figure YPUQZY (see Fig. 8).
Even man's figure (narākṛti) was geometrized with the help of a circle (representing head) and 5 trapezia (representing other parts) for ritual purpose (see Fig. 9).

There was effort in another direction as steps towards well-defined geometrical forms and some sort of standardization. It seems that to make the matter easy and practical, simpler figures were accepted in place of non-mathematical forms (snake, man, and makara). Some new figures (arrow, flag, sūrpa etc.) were incorporated. Some other changes were also made. The star-like pañca-kona (pentagram) was taken to represent the beautiful white shining star-graha Venus.20

This new allotment to Venus made its older symbol (viz. square) free for others and it was allotted to Moon. Some symbols were dropped!

Thus we find the following verses describing the mandala figures of the grahas to be used for worship alternatively (athavā):21
vr̥ttamandalamā dityam caturasram niśākaram /
trikonaṃ maṅgalam caiva, budham vai bānasannibham //
gurave paṭṭīśākāram, pañakonāṃ bhrugum tathā /
mande ca dhanusākāram, sūrpaṃ kāram tu rāhave //
ketave ca dhvajākāram manḍalāni krameṇa tu /

“The circle is the mystic figure of the Sun, the square that of Moon, the triangle that of the Mars and arrow-like figure that of the Mercury. The paṭṭīśa figure is for Jupiter and pentagram for Venus. For Saturn it is bow-shaped figure and figure of sūrpa (winnowing basket) is for Rāhu. For Ketu it is the flag-shaped figure. These are respectively the manḍalas (mystical figures).”

According to the Sanskrit – English Dictionary by Monier-Williams, paṭṭīśa is a spear with a sharp edge. It may be same as the spear which is now called pata (in central India) and which has a blade with sharp edges. The shape of the blade is a long rectangular strip. This compares well with those sources where the figure of the Jupiter is mentioned to be a rectangle.

Thus we find that the above-mentioned graha-maṇḍalas are to be regarded same as the figures of the graha-piṭhas or seats (or bases) of the planets. These piṭhas are located on the 9 cells of a vedī exactly in the same manner as the 9 cells of a graha-maṇḍala are filled (cf. Fig. 1). The names of 9 piṭha-figures have been entered in Table 1.
According to Kulkarni, the height of each planet-seat (or base) should be 1/24 of the height of the vedi (which is square in shape). But its cross-sectional area (i.e., the area of a pīṭha figure) depends on the planet and is shown in Table 1.

More complicated mathematization of figures was needed when the worshipping of the planets was done through havana-yajña which needed agni-kundaś (fire-pits). Fortunately, the sectional figures of these pits for the planets are the same as their pīṭha-figures. In any case, the important thing was to specify each figure geometrically in a well-defined mathematical manner. To avoid wrath of deities, the fire-pits of various laid down shapes must be constructed accurately. For this correct calculation of dimensions of the sectional figures was needed. In addition to the construction of appropriate vedis, attention is needed to the accessories of kuṇḍas such as mekhalā (belt), kaṇṭha (throat), nābhi (navel), etc.

Here, we shall briefly talk about the main fire-pits (kuṇḍas) only as used in the graha-yajña. The shapes of the pits for Sun (circle), Moon (square), Mars (triangle) and Jupiter (rectangle) are familiar figures. Mercury’s figure of pit (arrow with triangular head) and that of Ketu (triangular flag with thin strip as mast) are simple geometrical forms. The figure of bow (for Saturn) is already discussed above. Rāhu’s fire-pit is in the form of sūrpa (winnowing basket) whose one method of construction is as follows: (Fig. 11):
Four equal chords, equidistant from the center O, are drawn on the four sides O. The length of each chord is equal to $\sqrt{3}r$. J and K are points of intersection of chords. Arc JPK is part of a circle with center at O and radius $=OJ = OK$. Then the figure CJPKDC is in the shape of a surpa.

The fire-pit of Venus is panca-kona (pentagram or pentam or pentacle; see Fig.10) whose methods of construction are found in Kulkarni. In fact he has carried out detailed calculations regarding fire-pits and has given various methods of construction. A critical examination of his work will need a separate long study. 27

**YANTRAS (MYSTIC DIAGRAMS) OF PLANETS**

In an earlier paper28 the author of the present article has pointed out that for the study of history of exact sciences in India, the subject of agni-kundas (fire-pits) still awaits attention. Another neglected area of study in the field is, as a whole, that of yantras which are used in worship, meditation, and other rituals especially in the Tantric system of India (both Hindu and Buddhist).

By yantras we mean here the variety of mystic diagrams and figures which comprise of some sort of geometrical drawings and designs inscribed frequently with mantras (mystic formulas consisting of peculiar symbolic letters, words, and numerals). These yantras are used in worshipping gods, goddesses and other deities and also in other rituals for peace and welfare as well as for malefic objectives (abhicāra).

Numerous such yantras are found described in various Sanskrit works and other sources. Only one yantra, namely Śrīyantra, has been studied in great details. The present author has collected a lot of original material which he expects to present in a separate study. In the present article, yantras related mainly to the grahas (ancient planets) will be discussed.

For the Sūrya-yantra (mystic diagram of the Sun), the Saura-Paddhati states. 29

\[
\text{likhed āṣta-dalāṃ padmaṃ tadbāhye śī-dalāṃ likhet} \\
\text{caturasraṃ tato bāhye sūrīyantram idaṃ smṛtam} \]


'Draw or construct (likhet) an eight-petalled lotus and, outside it, (again) make a lotus with eight petals. Then draw a square (i.e., bhūpura) outside (the figure). In this way we get the so-called sūrya-yantra (Sun's mystic diagram').

The diagram of the Sun's yantra as given in the Puras'caryārṇava is shown in Fig. 12. Other similar diagrams with slight variations are also mentioned in this work.  

According to a statement in the same work, the Moon's yantra will be same as Sun's yantra. But elsewhere, the Moon's yantra is given quite differently. It is shown in Fig. 13 after omitting the candra-mantra and some numerical figures. Unfortunately, the original source or other reference is not mentioned for the drawing of Fig. 13. 

The case of Mangala-yantra (for the next planet, Mars) is quite interesting and significant. We have seen above (see Table 1) that the Maṅgala manḍala, the figure (ākṛti) related to Mars is a triangle invariably. Historically, the Mars-yantra seems to be a natural and logical evolution from the basic triangular motive. The diagram (Fig. 14) consists of an original equilateral triangle ABC subdivided into 21 smaller triangles called kośṭhas or cells.
Its construction is concisely described in the Merutantra as follows:

\[
\begin{align*}
\text{trikona} & \text{mpurva-uddhriya panchadhā vibhajet tatah} / \\
\text{trīyarekhā} & \text{cihna bhyaṁ lāñchayet samabhāgataḥ} // \\
\text{adyarekha} & \text{ya-yugalam trīyācihmayor nyaset} / \\
\text{dvitīyāgre samākṛṣya trīyācihmayor-nyaset} // \\
yutārekhā trīyā tu samābhājya samam tatah / \\
turyā cihna-dvayenaṭha trihiś-cihnaśca pañcamī // \\
trīyāgre prakurvī tapaṇcabhyāmadhya-cihnaje / \\
turyāgre yojaset samyak pañcabhyā cihmayor dvayoḥ // \\
atha mīnadvaye dadyāt sūtrayugmaviccaskanah / \\
evam ekādhi kā samyak kośṭhānāmvimśaitir bhavet // \\
\end{align*}
\]

‘First construct an equilateral triangle (ABC). Then divide it into five parts (by drawing equidistant lines (parallel to the base). Mark the third line (DG) by points (E and F) of three equal division. Join (crossly) the ends of the first (ādyā) line to these points (E and F) of the third line. Join directly (samākṛṣya) the ends of the second line to the same points (E and F). With (yutā) third line, the upper part (ADG) is thus divided equally all around (in 9 parts).

Now take two suitable points (J and K) on the fourth line (YZ) and three points (U,M,V) on the fifth line (BC). Join ends of the third line to middle (madhya) point (M) of the fifth line. By joining the ends (Y,Z) of the fourth line and the remaining two points (U,V) of the fifth line suitably (samyak), the wiseman made the pair of zigzag line (USV and DMG ?) yield figures of two fishes (mīnadvaya). Thus we get twenty-plus-one cells.

The stringently interesting part in the above construction is the formation of two fish-figures in the lower part by the zigzag lines. These are joined back to back at S and M in Fig. 14 and are shown separated in Fig. 15. It can be seen that by the above construction, the original triangle ABC has been subdivided into 21 small triangles or kośthas (cells) as the text calls them. Perhaps the
number 21 here signifies 9 *grahas* and 12 *rāśis* (zodiacal signs). Out of 21, only 11 cells are equilateral.

Some mathematics of the Mars-yantra may be discussed. Let the length of each side of triangle ABC be $a$. E and F are points of trisection and S the mid-point of DG. For mathematical consistency of concurrency of three lines each at J and K, we can take J and K to be points of intersection of YZ with DM and MG (M being mid-point of BC). Then U and V may be obtained by joining S to J and K, and producing them to meet BC. Finally U and V are joined to Y and Z respectively. That is the order of mathematical construction in the lower part DGCB. Now, by symmetry, U and V will be vertically below D and G, and thus angles BDU and CGV (not shown) will be $30^\circ$.

$$BU = \frac{DB}{2} = \frac{a}{5} = VC$$

Hence, $$UM = \frac{a}{2} - \frac{a}{5} = \frac{3a}{10} = VM$$

Again, $$JK = \frac{DG}{2} = \frac{UV}{2} = \frac{3a}{10}$$

Of course, $$DG = (\frac{3}{5}) BC = \frac{3a}{5}$$

and $$YZ = (\frac{4}{5}) BC = \frac{4a}{5}$$

We can argue in another ways as well. The equidistant lines $L_1$ to $L_4$ are all parallel to the base BC whose length is $a$. Therefore the lengths of the first,
second, third and fourth lines will be $a/5$, $2a/5$, $3a/5$ and $4a/5$, respectively. Both S and M (being middle points) will be in the same vertical line (which passes through A also). Thus, we will have UV equal to DG ($=3a/5$). Finally, we see that:

$$YJ = KZ = (YZ-JK)/2 = a/4$$

by (1) and (2).

In this way, the bases of all the 21 small triangles (whether pointing upward or downward) have been found. It may be noted that J and K are not the points of equal trisection of YZ, and U is not the mid-point, BM, and so, about V. The exact locations of these intermediary points (J, K, U, and V) have been found above mathematically in order to give a consistent and coherent figure of 21 cells. Without using the exact lengths of the segments of YZ and BC (as calculated above), the figure will not be perfect (there may be defective intersections at J and K).

The diagram of the Mars-yantra frequently appears in various books and magazines.\(^{35}\) It is usually found to be inscribed with numerals and a mantra for the planet (such as \textit{Om kujåya namah}). But the writers of the articles on \textit{Maŋgala-yantra} are generally found to present their description and discussion without citing any original text, source, or mathematical calculation which are necessary for authority, reliance, and authenticity.

For other planets (Mercury to Ketu), it is stated that\(^{36}\):

\begin{quote}
\textit{buddhådinåmyantram tu bhûpurâvrtåstadal-kamalam}
\end{quote}

‘The \textit{yantra} of Mercury etc. is the eight-petal lotus surrounded by \textit{bhûpura} (square with four gates)’.

However, various \textit{yantras} of these planets do appear elsewhere (but usually without supported text or authority).\(^{37}\)

\textbf{Magic Squares Associated with Planets}

Let $n$ be a natural whole number (i.e., a positive integer). Suppose a square be divided into $n^2$ small squares (called cells) which are arranged in an array of $n$ rows and $n$ columns. (similar to arrangement of 64 cells in a chess board). If all the cells are filled with numbers (positive integers) in such a way
that the sum of the numbers along any row, any column, and along the two
main diagonals is the same, then the figure formed by the square-arrangement
is called a magic square of order n. The constant sum is called magic constant
or magic number of that magic square. (Fig. 16 is a magic square of order 3 and
constant 15).

In Sanskrit, a magic square is called yantra or better, anka-yantras (to
distinguish it from other types of yantras). There are anka-yantras which are
associated with the nine grahas (ancient planets). They are used (like other
type of yantras), in various ways as instruments to worship and pacify the planets,
and for avoiding their malefic influence.

The anka-yantras (magic square) associated with
the Sun is of the order of three and is shown in Fig. 16. The
corresponding magic squares for other eight planets (from
Moon to Ketu) can be consolidated and expressed in general
form as shown in Fig. 17, where x = 1 to 8 is to be taken
for the eight remaining planets, Moon to Ketu, respectively.
Of course, the case x = 0 gives Sun's magic square (Fig.
16). All these nine graha-magic squares have been
mentioned by Chawdhri 38 separately but without
mentioning the Sanskrit verses or his source. Elsewhere,
the name of the legendary Garga (a popular ancient authority
on Jyotisha) is mentioned in connection with the above nine
magic squares of the planets.39

Interestingly, Chawdhri 40 mentions another set of
magic squares of the ancient planets but they are called
talismans (instead of yantras) by him (pp.42-47) (but he
still translates this word as ‘mystic diagram’, p.313). The
noteworthy thing is that these talismanic magic squares are
not of one and the same order. For example, Jupiter talisman
is of order 4 and is as follows (Fig. 18) (one printing mistake
has been corrected by us). It is formed from numbers 1 to
16, and its magic constant is 34. It can be proved
mathematically that when a magic square of order n is
formed from numbers 1 to n², its magic constant will be
(n²+1)n/2.
The full list of the planets along with the order they have talismanic magic squares is: Sun (order 6), Moon (9), Mars (5), Mercury (8), Jupiter (4), Venus (7), Saturn (3), Rahu (4), and Ketu (4).

In this regard, I noticed a very significant and surprising fact recently. I found that the magic squares of exactly the same orders for the first seven planets are given in a Latin magic text which was ‘translated (or adapted) from an Arabic one of Moorish origin’ and copied in the 14th century. The orders of the magic squares do indicate that the earlier order of planets was not in weekdays order (as listed above).

In fact, the Latin text presents the planetary magic squares in the following order: Saturn (3), Jupiter (4), Mars (5), Sun (6), Venus (7), Mercury (8), and Moon (9). It is possible that the (Indian) talismans were also constructed in this very order. And those for Rahu and Ketu were added (perhaps the order of talisman was restricted to 9 as there were navagrahas).

As far as the magic squares themselves are concerned, they are not the same but comparable. For instance, the Saturn’s magic square from the Latin text (Fig. 19) and the Saturn talisman (Fig. 20) are shown here. The difference is only of the reversion of digits in each row. Perhaps this was also due to mode of writing – right to left (semitic) and left to right (Indian). Also cf. Fig. 16. For Jupiter the magic square from the Latin text (Fig. 21) may be compared with Jupiter talisman (Fig. 18). As we go to higher orders, the divergence increases. But the talisman for Rahu and Ketu is the same and it can be obtained by just reversing the numbers in each row in Fig. 21.

The most important thing to note is that the practical uses of magic squares as described in the Latin text read just like a typical Indian prescription in a tantric work on the so-called sat-karmas (stambhana, vasikarana, marana, mohana, santika,
etc). About the \textit{anka-yantras}, the Latin text says that these ‘seven figures’ contain ‘seven names of God’ hidden in them by ‘ancient philosophers and scholars.’ In India also the \textit{yantras} (mystic diagrams or figures) were of both deities themselves as well as instruments for worshipping them.

\textbf{REFERENCES AND NOTES}


2. Ibid., p.9.

3. According to the \textit{Sanskrit-English Dictionary} of M. Monier-Williams (Delhi reprint, 1972, p.1080), the word \textit{s'ukla} (‘white’ itself is a later form of \textit{s'ukra}.


6. \textit{Laks\textasharp man a De\textasharp ikendra, \textit{Sr\textasharp rad\textasharp tilaka}}, edited by Jivananda Bhattacharyya, Calcutta, 1892, p.3. The translation given here preserves mathematical meaning.

7. \textit{Y\textasharp ja\textasharp valy\textasharp ka-smrti} with \textit{Mit\textasharp ka\textasharp r\textasharp a-vy\textasharp ak\textasharp hy\textasharp a} (along with two commentaries) ed. By S. S. Setlur, Brahmapadin Press, Madras, 1912, p.189.

8. \textit{Br\textasharp hat-Par\textasharp a\textasharp ra-Sam hit\textasharp a} included in the \textit{Dharma\textasharp as\textasharp tra-Sa\textasharp ng\textasharp ra} ed. By Jivananda Bhattacharyya, Calcutta, 1876, part II, pp. 53-309, p252.

9. Ibid., p. 252.

10. \textit{Matsya-pur\textasharp a\textasharp a}, ed. with Hindi Translation by Shriram Sharma, two parts, Bareily, 1970; Part 1, p. 313.

11. Taranath\textasharp a Tarkavachasp\textasharp ati, \textit{V\textasharp ac\textasharp spa\textasharp y\textasharp am}, Vol. IV, Chowkhamba, Varanasi, 1990, p.2760 where the verses are quoted from \textit{S\textasharp antidipik\textasharp a} (\textit{S\textasharp antid\textasharp ipik\textasharp o\textasharp k\textasharp t\textasharp a\textasharp h}).

12. J. C. Ray, \textit{A\textasharp mader\textasharp a Jyo\textasharp ti\textasharp s\textasharp i O Jyo\textasharp ti\textasharp s\textasharp a} (in Bengali), Vol. I, Calcutta, 1903, p.482.

13. The five Sanskrit lines are found (quoted anonymously?) in the \textit{Graha-\textasharp S\textasharp anti Pray\textasharp og\textasharp a\textasharp h} with \textit{B\textasharp h\textasharp as\textasharp t\textasharp i\textasharp k\textasharp a} and notes by Daulatram Gaud, revised ed., Varanasi, 2001, p.191.

14. Makkhanlal Mishra, ‘Navagraha-up\textasharp as\textasharp ana\textasharp a, (in Hindi),\textit{Kaly\textasharp an\textasharp a}, 42(1) (Up\textasharp as\textasharp an\textasharp a-\textasharp an\textasharp k\textasharp a) (1968), 532-535.

15. For details see R. C. Gupta, ‘A Little-known Text and Version of \textit{Sr\textasharp i\textasharp yantra}, \textit{Ga\textasharp n\textasharp ita-Bharati}, 25 (2003), 22-28 (in press).

16. See \textit{Graha S\textasharp anti Pray\textasharp og\textasharp a\textasharp h} (ref. 13 above), pp. 170-171.
17. Ibid., p. 184
19. See Graha Śānti Prayogah (ref. 13), p.181 where dimensions of the trapezia and circle are also given. The man’s figure is stated to be drawn according to Saṃskāra Gaṇapati.
20. Ibid., pp. 172-173, and p. 242. Also see Kulkarni, op.cit. (under ref. 18 above), pp. 68-70, and p. 91.
21. Quoted in Graha Śānti Prayogah, p.191. We have corrected some printing mistakes. Also see p. 208 for a description of various planet figures (ākāras).
22. Ibid., p.208 (dīrgha-caturasra) and p. 242. Also see Kulkarni (ref. 18) pp. 90-91.
23. Kulkarni (ref. 18) pp. 89-91.
24. Ibid., pp. 89-90.
25. Ibid., p.89.
26. Ibid., pp. 72-73 where the method is stated to be taken from the Kuṇḍa Ratnāvalī (see ref. 18 above). Our Fig. 11 yields a figure of the sūrpa which resembles with that which is given in the Graha Śānti Prayogah (ref. 13, p. 187, quoting Laghupīṭha Mālā. The last work is also stated to contain a simpler sūrpa figure which is an isosceles trapezium whose base is 28, top 20, and altitude 24 aṅgulas.
27. For a critical review of Kulkarni’s book (ref.18), see Ganita Bhāratī 23 (2001) 138-143. Also see IJHS 38.1 (2003), pp.7-8.
29. As quoted in the Pratāpasiṁ ha Sāhadeva’s (King of Nepal), Puraścaryārṇava (1775) ed. by Murlidhar Jha, Chowkhamba Pratishthan, Delhi, 1985, p. 1140.
30. Ibid., plate-page 6.
31. Ibid., p.1141, where text for Sun’s yantra is quoted from Mahākramārcana Paddhati, and two more variant yantras are mentioned.
32. Ibid., p. 1158 where it sates: Aṣṭādala-kamala-dvayāmakam Candra-yantram (bhūpura is to be supplied invariably to all yantras).
33. Radheshyam Mishra, Camatkārī Yantra Elabama (in Hindi), Raja Pocket Books, Delhi, p. xxvii.
34. As quoted in Puraścaryārnava (ref. 29), p. 1158.
35. E.g., see R. Mishra (ref. 33), pp. xxvi; and S.L.Shukla, ‘Maṅgala-Yantra’ (in Hindi), Naṇasāyara (Delhi), No. 22 (June 1996), 43-46.
36. See Puraścaryārṇava (ref. 29), p.1158.
37. For instance, see, R. Mishra, (ref. 33), pp. xxviii and xxix where yantras for Jupiter and Venus appear (each being essentially a hexagram enclosed in bhūpura).


40. Chawdhri, op.cit. (ref. 38 above), pp. 42-47.