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MEAN SUN AND MOON IN ANCIENT GREEK AND INDIAN ASTRONOMY

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A study is made of the values of the length of the year and month used in the theories of Ptolemy, Āryabhaṭa and Brahmagupta. In comparing these values, we must take into account the fact that Prolemy used tropical coordinates and the Indian astronomers used sidereal coordinates. The correct values are based on accurate modern formulae.

INTRODUCTION

The length of the year and the month are fundamental parameters in astronomical theories. In this paper, I consider specifically the values of these parameters which occur in the theories of the Greek astronomer Ptolemy and the Indian astronomers Āryabhaṭa and Brahmagupta. To assess their accuracy, we must compare them with the corresponding values obtained by using modern formulae in a recent ephemeris[1].

THE MODERN VALUES

The values calculated from the modern formulae are given in Table 1 for the period 0 AD to 600 AD. For the sun's motion, we need the values of the tropical and sidereal year, and for the moon, the tropical, sidereal, synodic, anomalistic and draconitic months. Ptolemy's observations recorded in the Almagest range from AD 127 to 141[2] (p.1). The dates of the *Āryabhaṭīya* and *Brāhmasphuṭasiddhānta* are AD 499 and 628[3] (pp 92, 96).

PTOLEMY

The source for Ptolemy's parameters is the Almagest, of which there is a recent translation by G.J. Toomer[2].

It is important to emphasize that Ptolemy had a strong preference for tropical, rather than sidereal, coordinates. This is clearly stated in the following two extracts[2] (p. 132):

"The only points which we can consider proper starting-points for the sun's revolution are those defined by the equinoxes and solstices........"

"........it seems unnatural to define the sun's revolution by its return to (one of) the fixed stars........"

Table I

A.D.	Tropical Year	Sidereal Year	Tropical Month	Sidereal Month	Synodic Month	Anomalistic Month	Draconitic Month
0	365.242436	365.256484	27.32158845	27.32166705	29.53059449	27.55457996	27.21222233
100	365.242423	365.256477	27.32158808	27.32166672	29.53059415	27.55457840	2721222220
200	365.242410	365.256471	27.32158771	27.32166639	29.53059381	27.55457683	27.21222207
300	365.242397	365.256464	27.32158735	27.32166606	29.53059346	27.55457527	27.21222193
400	365.242384	365.256457	27.32158698	27.32166572	29.53059312	27.55457371	27.21222180
500	365.242371	365.256450	27.3215661	27.32166539	29.53059277	27.55457215	27.21222167
600	365.242358	365.256443	27.3215624	27.32166506	29.53059243	27.55457059	27.21222154

Ptolemy's observations confirmed Hipparchus' measure of the tropical year. He quotes Hipparchus as having written [2] (p.139):

"I have also composed a work on the length of the year in one book, in which I show that the solar year (by which I mean the time in which the sun goes from a solstice back to the same solstice, or from an equinox back to the same equinox) contains 365 days, plus a fraction which is less than ¼ by about 1/300th of the sum of one day and night,.....".

Thus, the Hipparchus - Ptolemy tropical year is:

(1) $Y_t = 365 + 1/4 - 1/300 = 365.246666... = 365;\ 14,48$ days.

Since the date of Ptolemy's observations is around AD 150, the modern value for that date found from Table I is $Y_t^\circ = 365.242417$ days. The difference $|Y_t - Y_t^\circ|$ divided by Y_t° is 1.16×10^{-5}. These results are given in Table II.

After completing the theory of the sun in Chapter III of the Almagest, Ptolemy takes up the motion of the moon in Chapter IV. For the parameters of the lunar mean motion, Ptolemy follows Hipparchus who in turn used Babylonian period relations. He accepts Hipparchus value of the synodic month [2] (p.176):

(2) $M_y = 29;\ 31,50,8,20 = 29.53059413$ days.

This is a Babylonian parameter, and is remarkably close to the modern value for AD 150 (Table 1), $M_y^\circ = 29.53059398$. From M_y and the tropical year Y_t, the tropical month can be derived.

Instead, Ptolemy calculates the daily motion in longitude[2] (p.179):

(3) $13;10,34,58,33,30,30 = 13.176382215$ degrees per day.

Dividing 360° by (3), we get the tropical month M_t and the corresponding modern value M_t^0 is obtained from Table 1:

(4) M_t = 27.32161181 days, M_t^0 = 27.32158789 days.

For the anomalistic and draconitic motion, Ptolemy makes small corrections to Hipparchus' values. His mean daily anomalistic and draconitic motions are [2] (p.179):

13; 3,53,56,17,51,59° = 13.06498286°
13; 13,45,39,48,56,37° = 13.229350999°

The corresponding months, their modern values and the relative errors are given in Table III.

Āryabhaṭa and Brahmagupta

Āryabhaṭa and Brahmagupta used sidereal co-ordinates. Their year length and months follow from the data in Table II which gives the number of revolutions in one *yuga*, of the earth, sun, moon, moon's apogee and node.

Table II

	Earth (R_1)	Sun (R_2)	Moon (R_3)	Apogee of Moon (R_4)	Node of Moon (R_5)
Āryabhaṭa	S 1,582,237,500 M 1,582,237,800	4,320,000	57,753,336	488,219	232,226
Brahmagupta	1,582,236,450	4,320,000	57,753,300	488,105.9	232,311.168

Āryabhaṭa had two systems, the sunrise system (S), given in the *Āryabhaṭīya*[4] and the Midnight system (M), given in the *Khaṇḍakhādyaka*[5]. These differ only in R_1, the number of earth's revolutions. The source for Brahmagupta's number (divided in Table 2 by 1000) is the *Brāhmasphuṭasiddhānta*[6].

The lengths of the year and the months follow from the following formulae:

Number of civil days in a *yuga:* $D = R_1 - R_2$

Sidereal year : $Y_s = \dfrac{D}{R_2}$

Sidereal month : $M_s = \dfrac{D}{R_3}$

Synodic month : $M_y = \dfrac{D}{R_3 - R_2}$

Anomalistic month : $M_a = \dfrac{D}{R_3 - R_4}$

Draconitic month : $M_d = \dfrac{D}{R_3 + R_5}$

In the last of the above formulae, the retrograde motion of the node is taken into account. The results and the corresponding errors are tabulated in Table III. For the modern values, I have taken AD 500 and 600 for the dates of Āryabhaṭa and Brahmagupta.

Table III

		Ptolemy	Āryabhaṭa (S)	Āryabhaṭa (M)	Brahmagupta
Tropical Year	Y_t	365.246667			
	Y_t°	365.242417			
$\dfrac{\lvert Y_t - Y_t^\circ \rvert}{Y_t^\circ}$	$\times 10^5$	1.16			
Sidereal Year	Y_s		365.258681	365.258750	365.258438
	Y_s°		365.256450	365.256450	365.256443
$\dfrac{\lvert Y_s - Y_s^\circ \rvert}{M_s^\circ}$	$\times 10^5$		0.61	0.63	0.55
Tropical Month	M_t	27.32161181			
	M_t°	27.32158789			
$\dfrac{\lvert M_t - M_t^\circ \rvert}{M_t^\circ}$	$\times 10^6$	0.88			
Sidereal Month	M_s		27.32166848	27.32167368	27.32166733
	M_s°		27.32166539	27.32166539	27.32166506
$\dfrac{\lvert M_s - M_s^\circ \rvert}{M_s^\circ}$	$\times 10^6$		0.11	0.30	0.08
Synodic Month	M_y	29.53059413	29.53058181	29.53058742	29.53058205
	M_y°	29.53059398	29.53059277	29.53059277	29.53059243
$\dfrac{\lvert M_y - M_y^\circ \rvert}{M_y^\circ}$	$\times 10^6$	0.005	0.37	0.18	0.35
Anomalistic Month	M_a	27.55457117	27.55460187	27.55460711	27.55454644
	M_a°	27.55457762	27.55457215	27.55457215	27.55457059
$\dfrac{\lvert M_a - M_a^\circ \rvert}{M_a^\circ}$	$\times 10^6$	0.23	1.08	1.27	0.88

Draconitic Month	M_d	27.21221926	27.21224811	27.21225328	27.21220693
	M_d°	27.21222214	27.21222167	27.21222167	27.21222154
$\frac{\lvert M_d - M_d^\circ \rvert}{M_d^\circ} \times 10^6$		0.11	0.97	1.16	0.54

Comments

From Table III, we see that Ptolemy's errors for the tropical year and month are larger than the errors of the Indian astronomers for the sidereal year and month. However, his values for the synodic, anomalistic and draconitic months (for which the credit must go to Babylonian astronomers) are much better than those of Āryabhaṭa and Brahmagupta.

P.C.Sengupta[7] (p.x lvi) has compared Ptolemy's parameters with those of Āryabhaṭa and Brahmagupta. His modern values do not agree with those in Table 1 of the paper.

In O. Pedersen's *A survey of the Almagest*[8] (pp. 423,424) the modern values calculated for Ptolemy's time are again different from those in this paper.

Acknowledgement

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