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A CRITICAL STUDY OF 'VEDIC MATHEMATICS' OF ŚANKARACĀRYA ŚRĪ BHĀRATĪ KṚṢṆA TĪRTHAJĪ MAHĀRĀJ

K. CHANDRA HARI*

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'Vedic Mathematics' alias 'Sixteen Simple Mathematical Formulae from the Vedas' by the Late Śankarācārya Swāmījī Srī Bhāratī Kṛṣṇa Tīrtha of Goverdhana Maṭha, Puri was first published in 1965. Even though the work had a euphoric reception, its real nature vis- a-vis the *Atharvaveda* and scope had been a matter of great confusion to students as well as to scholars alike since then. K. S. Shukla did contest the Śankarācārya's claim that the sūtras belonged to the *pariśiṣṭa* of *Atharvaveda*, as early as in 1950, when Swāmījī was undertaking black-board demonstrations of the 'new discovery' at different places. In the present paper, the attempt is to decipher and demonstrate Swāmījī's method of discovery of the sūtras with the help of a fundamental algebraic principle enunciated originally by ancient Indian mathematicians such as Bhāskarācārya II. Erroneous use of '*Katapayādi*' in the fabrication of certain Upasūtras and the śloka giving the value of π/10 to 32 decimal places provide concrete evidence towards refuting the Vedic origin propounded by Swāmījī.

Key words : Algebra, *Kaṭapayādi*, Sūtra, Vedic Mathematics.

INTRODUCTION

Srī Bhāratī Kṛṣṇa Tīrthajī has depicted his book *'Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas'* as arising out of the mathematical aphorisms contained in the *Pariśiṣṭa* of the *Atharvaveda*[1]. During the last three decades since its first publication there have been vehement criticisms of the book's title *Vedic Mathematics* by well known scholars as well as certain efforts to explain the rationale with which Swāmījī might have chosen the above controversial title. Being generally considered as the prerogative of an author, the title of a work seldom invites the fierce criticism as in this case where-in a sure misnomer has been adopted as the title with certain ulterior purpose. A well orchestrated effort to picture a 'Vedic' origin is apparent in Swāmījī's words in another of his works viz., *Vedic Metaphysics* at

* Dy. S. G.(W) Institute of Reservoir Studies, ONGC, Ahmedabad-380005

pages163 to 167. The sixteen sūtras have been described as part of the *Śūlbasūtras* which defied western translators like Colebrooke, H.H.Wilson etc., and His Highness claims to have found out their meanings, ... *after long years and years of meditation in the forest,...* [2] Despite such lofty claims by the late Śankarācārya of Goverdhana Maṭha, K.S.Shukla, an eminent scholar of ancient Indian mathematics, has chosen to describe the work as *non Vedic modern elementary mathematics upto the Intermediate standard'and the title Vedic Mathematics' as deceptive.* To quote K.S. Shukla[3]

> "..... V.S. Agrawala's verdict that the work of Śrī Śankarācārya deserves to be regarded as a new *Pariśiṣṭa* by itself is fallacious .The question is whether any book written in modern times on a modern subject can be regarded as a *Pariśiṣṭa* of a Veda. The answer is definitely in the negative."

From what has been said above it is evident that the sixteen sūtras of Swāmījī's *Vedic Mathematics* are his own compositions, and have nothing to do with the mathematics of the Vedic period. Although there is nothing Vedic in his book, Swāmījī designates his preface to the book as *A Descriptive Prefactory Note on the Astounding Wonders of Ancient Indian Mathematics* and at places calls his mathematical processes as Vedic Processes.

The deceptive title of Śwāmījī's book and the attribution of the sixteen sūtras to the *Pariśiṣṭas* of the *Atharvaveda* etc., have confused and baffled the readers who have failed to recognize the real nature of the book, whether it is Vedic or non-Vedic. Some scholars, in their letters addressed to me, have sought to know whether the sixteen sūtras stated by Swāmījī occurred anywhere in the Vedas or the Vedic literature.

Even the Rashtrīya Veda Vidya Pratishthan ... are under the impression that the sixteen sūtras were actually reconstructed from materials in the various parts of the Vedas and the sixteen formulae contained in them were based on an appendix of the *Atharvaveda,* which (appendix) was not known to exist before the publication of Swāmījī's book.

(after a description of the various chapters, K.S. Shukla continues)

> ".....From the contents it is evident that the mathematics dealt within the book is far removed from that of the Vedic period. Instead. it is that mathematics which is taught at present to High School and intermediate classes.It is indeed the result of Swāmījī's own experience as a teacher of mathematics in his early life. Not a single methods described is Vedic, but the Swāmījī has declared all the methods and processes explained by him as Vedic and Ancient".

The General editor V.S.Agrawala, who has tried to depict the Swāmījī's work as a "modern *pariśiṣṭa* by itself", writes in his foreword as follows (Ref. 1pp 6 and 7):

> "But this work of Srī Śankarācāryajī deserves to be regarded as a new *pariśiṣṭa* by itself and it is not surprising that the Sūtras mentioned herein do not appear in the hitherto known *pariśiṣṭas*".

A list of these main 16 Sūtras and of their sub-Sūtras or corollaries is prefixed in the beginning of the text and the style of language also point to their discovery by Srī Swāmījī himself. At any rate, it is needless to dwell longer on this point of origin..."

It must be noted here that the above views of the editor runs contrary to Swāmījī's own words at page xiii to xv of reference 1.

Authors like S.Das, Gaṇitānand etc., have also refuted the 'Vedic' connection of the Swāmījī's mathematics in sure terms. Perhaps out of his respect towards the pontifical authority of Swāmījī, Das has suggested[4]: 'the possibility that a secret version of Atharvaveda exists in the oral form and Swāmījī had knowledge about it.' In the words of Gaṇitānand[5]:

> "The objectionable things about the book or system are the name *Vedic Mathematics* given to it and the claim that the 16 formulae are from the Vedas. Both are deceptive and false and are responsible for creating lot of confusion and misunderstanding. The book or the sūtras have nothing to do with the Vedas. The sūtras (or formulae) were composed by the author himself who lived from 1884 to1960. Hence, as mildly stated by Mañjula Trivedi, a disciple of the author,"these formulae are not to be found in the present recensions of *Atharva-veda*; they were actually reconstructed (by the author) on the basis of intuitive revelation from materials scattered here and there in the *Atharvaveda*". Only one thing is that the author composed the 16 aphorism in Sanskrit sūtra style, and put a stamp "from the vedas" on them ......"

Despite the above inconsistency, the work had a euphoric reception by the intelligentsia of India and abroad. As an example of the appreciations, the following words of A.P.Nicholas may be noted[6]:

> "One of the most delightful chapters in 20th century mathematical history is the reconstruction by the former Śankarācārya of Puri, Srī Bhāratī Kṛṣṇa Tīrthajī, of the system vedic mathematics, starting from a few well disguised clues in the *Atharvaveda*—surely an undertaking to be compared with the reconstruction of a lost language!
>
> We are told that he wrote sixteen volumes on the subject, but all that has survived is an introductory volume written in 1957, and published posthumously in 1965."

In short, a wide class of readers have failed to comprehend the reasons behind the Śankarācārya's description of his own discoveries as Ancient Vedic Mathematics and a few others have mistaken the work as really evolving out of the vedas, relying on the words of Swāmījī. (S. Das[7] did undertake a painstaking effort to understand the etymology of the words with reference to the Vedic literature and other ancient texts). The element of antiquity is completely missing from the demonstration of the modern elementary mathematics except for the Sanskrit sūtras exclusively framed to describe the processes involved and the original source remains a mystery.

SWĀMIJĪ'S TEXT CONTRADICTS ANCIENT INDIAN TRADITION

Swāmījī has presented the alphabetic numerical system *Kaṭapayādi*, in his treatise as the vedic numerical code[8]. What he decribes as "concrete, interesting and edifying illustrations" appears on page 209 under the tittle 'The code language at work'. To quote Swāmījī[9]:

"The three samples read as follows :

(1) केवलैः सप्तकं गुण्यात्	(*kevalaiḥ saptakaṃ guṇyāt*) :
(2) कलौ क्षुद्रससैः	(*kalau kṣudrasasaiḥ*) : and
(3) कंसे क्षामदाहखलैर्मलैः :	(*kaṃse kṣāmadāha-khalairmalaiḥ*)

In the first of these *saptaka* means seven; and *kevalaiḥ* represents 143 ; and we are told that, in the case of seven, our multiplicand should be 143!

In the second, *kalau* means 13; and *kṣudrasasaih* represesnts 077; and we are told that the multiplicand should be 077 and in the third, *kaṃse* means 17 and *kṣāma-dāha-khalairmalaiḥ* means 05882353 ; and we are told that the multiplicand should be this number of 8 digits !..."

Swāmījī goes on to describe the application which is not so edifying as he described beforehand. I have desisted from quoting the same completely due to paucity of space.

It is widely believed that the *kaṭapayādi* system was developed in Kerala[10]. To justify the Swāmīji's prefix 'Vedic' it can be (for the time being) accepted that author of the astrological work, *Upadeśasūtra* was the author of the *Mīmāṃsasūtras* viz,. Jaimini itself, and as such the *Kaṭapayādi* notation existed as early as in the fifth century BC or even earlier at the *Mahābhārata* times, taking that the appellation 'Jaya' had the hidden meaning of '18'[10]. Whatever may be its antiquity, the one thing that is certain about the system is the alphabetical order from right to left i.e. , in the ascending order of place values. There is no traditional record of its use in the reverse order as Swāmījī has presented. As such ancient Indian mathematics forbids the identification of *kevalaiḥ* as 143 instead of 341 or *kalau* as 13 instead of 31 or *kaṃse* as 17 instead of 71.

Regarding the last of the three samples of special sub-sūtras Swāmījī says :[11]

> "And with regard to these formulae, I came to this conclusion, that there must be some kind of key. In king Kaṃsa's reign, famine, pestilence, unsanitary conditions prevailed, that seemed to be the meaning of the text-apparently nothing to do with mathematics .. That seemed to be an historical account of the king Kaṃsa.But here the heading is Gaṇita sūtrā mathematical formulae. So I said there must be something. And after long years and years of meditation in the forest, I took the help of lexicographies, lexicons of earlier times ..... etc"

In such lines, Swāmījī provides a funny story about the terms he has created with specific interpretatons suited for the purpose of befooling and mesmerising people. Except him, nobody has come across these abtruse phrases in *inverted-kaṭapayādī* notation so far in the Vedic or ancient literature. His effort did meet with partial success in mesmerising the people as can be seen from V.P.Dalal's (of the Heidelburg University, Germany) opinion about the articulated śloka of $\pi/10$ in the inverted *kaṭapayādī* given on pages 362 and 363 of reference 1. Various scholars have repeated Swāmījī's version that the śloka has three meanings but no one including the 'discoverer' did not bother to elucidate them. Undoubtedly it is another manipulation of sort meant to glorify the Hindu tradition as well as his personal achievement in unlocking the so called cryptic clues of the vedas . The stark gimmick that he has played in the above context castes a question mark over the validity of his other claims as well.

Empirical Deduction of Rationale and Generalization into Sūtra

The following salient features of the sūtras are noteworthy :

(a) Almost all the sūtras are descriptive of the arithmetical or the algerbraic process involved.

(b) Neither the sūtras represent a mathematical theorem nor a new technique like the logarithmic or the trigonometrical functions.Also the sūtras cannot be rightly described as identities, algorithms, or even axioms because of the following reasons :

(i) The sūtras are multipurpose and amounts to different outline of short-cut procedures in different contexts. As such no unique proof can be found for a sūtra; only the respective method and the involved rationale can be substantiated using a proof in a specific context. So they do not fall within the class of theorems as per modern scientific perceptions.

(ii) Sūtras do not function either as identities or axioms in the illustrations of Swāmījī.

(iii) Apparently they are similar to algorithms,but they are not algorithms because of the vague sense that necessitates a commentary for the right application. They are short-cuts rather than precise formulation of methods for solving mathematical problems. (An algorithm as we usually mean is neither the simplest nor the most efficient method of performing a task.)

In short, euphoric descriptions using modern scientific terminology are quite inappropriate and unwarranted.

(c) Phrases have been styled as sūtras to provide the antique look as well as Vedic connection. *e.g.* '*sankalana-vyavakalanābhyāṃ*'

(d) Neither the sūtras could give any answer to the unsolved problems of mathematics like an easy method for locating the prime* numbers nor could they prove any proposition that had no proof till such time like the Fermat's last ** theorem. (Along with 'divisibility' Swāmījī could have attempted 'primality' also very easily using one of the sūtras).

(e) As per the General Editor's foreword (ref,1, p-6, the term 'Vedic' attached to the sūtras "is not to be approached from a factual standpoint but from the ideal standpoint viz., as the Vedas as traditionally accepted in India as the repository of all knowledge should be and not what they are in human progression" as observed by K.S.Shukla this kind of argument is quite fallacious and serves no purpose.

(f) For many less spectacular applications of the sūtras Swāmījī has rendered algebraic explanations in much the same way as putting the cart before the horse (as if he is proving some Vedic abstractions algebraically)

These features clearly suggest that the sūtras evolved out of Swāmījī's own experience as a teacher and researcher in mathematics, and if we are able to glean his process of derivation, further studies can be made in this direction.

Process of Derivation- Illustrations

From the biographical sketch given in reference 1 we can understand that as a student Swāmījī was outstandingly brilliant and he had a string of M.A's. to his credit by the age of just twenty. Further the study of latest researches and discoveries in modern science continued to be his hobby all throughout his life. As such Swāmījī must have been well aware of the fact that algebra is a generalization of arithmetic made possible by the use of symbols, usually letters such as x, y, z etc., for the unknown numbers. In the algebraic method relations among arithmetical entities are reflected in relations among their symbols and new relations among arithmetical entities could be discovered by manipulation of their symbols in accordance with certain rules of identities meant for simplifying experessions. The "short-cut decriptions" that the

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* *Prime numbers :* Numbers having no smaller natural numbers as its factors eg: 11, 13, 17, 19, etc. Every whole number greater than 1 is either a prime or the product of a unique set of primes.This fundamental theorem of arithmetic was known to the ancient Greeks and as such primality has been engaging Man's attention since very early days of history. But no easy primality test could be developed yet

** Fermat's proposition that defied all attempts for a proof: "If n is a whole number greater than two and if x,y and z are non - zero whole numbers then $x^n + y^n = z^n$ has no solutions"..

Swāmījī's 'Vedic' sūtras enshrine as well as the algebraic explanations given for them point towards the possibility of a camouflaged algebraic origin of the sūtras.

As the first case of study, let us choose the 'spectacular' method for which he has given no explanation.

Spectacular Illustration with Which The Text Begins

1/19 = 0.52631578947368421=18 digit recurring decimal,

Sūtra = ekādhikena pūrvena i.e. "By one more than the previous one"

This sūtra produces the above result by taking either the first digit of the dividend or the last digit of the answer and a subsequent division or multiplication respectively, by '2'. '2' is obtained by incrementing the left digit 1 of 19 by 1 as per the sūtra. The decimal point is fixed at the beginning of the calculation in the division method and just where the whole decimal begins to repeat itself in the multiplication method.

The above process of obtaining the multiplier itself is non-mathematical and the sūtra does not reflect any sensible rationale involved in the process. The '2' involved in the real mathematical process arise out of 19+1= 20, rather than the addition of 1 to the left of the digit. Sūtra as such is not representative of the mathematical process involved—this may not be a demerit for a short-cut method, but the imperfection outlined above can't be the characteristic of a 'Vedic' sūtra evolved from the conscience of the great Sages.

Rationale behind the sūtra :(Using binomial theorem)

$1/19 = 1/(20\text{-}1) = 1/20.\ (1\text{-}1/20)^{-1}$

$= 1/20\ (1+1/20+(1/20)^2 + (1/20)^3 + \dots\dots\dots)$ (1)

$= 1/2.1/10+(1/2)^2.1/100 + (1/2)^3.1/1000 + \dots\dots\dots$

If we consider only the 18- recurring digits,

$1/19 = (1/2.10)^{18}\ [2^{17}.\ 10^{17} + 2^{16}.10^{16} + \dots\dots 10^3.2^3 + 10^2.2^2 + 10.2+1\ ]$

(a) Ratio of the $(n+1)^{th}$ term to the n^{th} term= 1/2.10 i.e. the first term divided by 2 and displaced by a decimal place to the left becomes the second term and so on.

Therefore, 1/19 = 0.05+
= 0.0025
= 0.000125
= 0.00000625+

....

0.052621.......
1

0.052631.......

(b) Similarly ratio of the n^{th} to $(n+1)^{th}$ term = 20 = 2.10 i.e., the right digit of the recurring part multipl;ied by 2 and shifted to the left by one place becomes the second term on the left side.

The increasing order of 2 and 10 towards the left of the series of 18 terms illustrates the true mathematical rationale involved. It is apparent that the sūtra is just a simplification invented by Swāmījī rather than a 'Vedic' formula for converting a vulgar fraction into its decimal equivalant.

ARITHMETICAL COMPUTATIONS

Due to paucity of space we shall restrict ourselves to a few typical examples only:

(a) Multiplication of : *nikhilaṃ navataścaramaṃ daśataḥ*

Sūtra "All from 9 and the last from 10" is a deceptive phrasing in Vedic sūtra-style of an elementary operation of arithmetic. For a 2- digit number it means just taking the complement of 100 i.e. for an n-digit number the complement of 10^n.

i.e. 100 = 9.10 +10

1000 =9.100 + 9.10 + 10 etc.

Sūtra applied to the number 899 (for example) as such means :

$$\begin{aligned} 1000- &= 9.100 + 9.10 + 10 \\ 899 &= 8.100 + 9.10 + 9 \\ \hline & (9-8).100 + (9-9).10 + 10-9 \end{aligned}$$

The Sūtra finds its illustration here.

Application of the idea in the multiplication was developed algebraically, as is evident from the explanation provided by Swāmījī on page 16 reference 1.

If the numbers A and B lie close to the base 10,

A.B =(10-a) (10-b) where a nd b are the complements of A and B respectively. Now the operations possible are:

(i) (10-a) (10-b) = (10-a) + (10-b) - 10 +a.b
= A+B -10 +a.b

(ii) A.B = 10 - (a +b) + a.b
= 10. (A- b) + a.b
= 10. (B -a) + a.b

Every step of Swāmījī's method is thus of algebraic origin.

(b) General Multiplication by *ūrdhva tiryak sūtra*

'Vertically and cross-wise' the short and terse formula, is just a description of the modus operandi, styled as above like a sūtra. Swāmījī 's algebraic explanation in fact provides the synonymous algebraic process out of which he developed the method.

$$\begin{array}{l} a\,x^2 + bx + c \\ dx^2 + ex + f \\ \hline \end{array}$$

$$ad.x^4 + (ae + bd).x^3 + (af + be + cd)x^2 + (bf + ce)\,x + cf.$$

Algebraic Rule of Cross Multiplication

A discussion on the "Urdhva Tiryak" sūtra shall reamain incomplete if we do not refer to the algebraic rule of cross multiplication.

The following two first degree equations in three unknowns

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0 \qquad \text{can be solved}$$

for the ratios (x/z) and (y/z) in the ordinary way to obtain :

$$x/z = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y/z = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Herein the method can rightly be described as based on a *tiryagbhyāṃ* sūtra or 'Upasūtrá belonging ot the Vedas. Had there been not an explicit mention of the method in the next books of algebra, it would have certainly found a place in the corridors of Swāmījī's 'Vedic Mathematics'. Infact we can see a corollary of the above under a different name *parāvatya* Rule in Chapter xv (p 140) of reference1 in the solving of simultaneous simple equations. When z becomes unity :

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y + c_2$$

Can be solved as

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Unfortunately S. Das [14] has credited this result to Swāmījī's sūtra with the remark that the cross-multiplication method suffers from the drawback arising due to a confusion in the sign convention. Swāmījī's camouflaged sūtra operations must have taken many people for a similar ride in the last three decades.

Partial Fractions

Swāmījī blames[15] the current method as a very cumbrous procedure and the so called easy Vedic method he has given can be located on page 262 of *Higher Algebra* by Hall and Knight, without any Vedic connection at all. The easy method and the general formula of Swāmījī can be obtained in simple algebraic method as :

$$\frac{lx^2 + m x + n}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$$

Multiplying by (x-a), we obtain

$lx^2 + m x + n \quad = A + (x-a) . \{B / (x-b) + c/ (x-c) \}$

Now, on putting x = a,

$$\frac{la^2 + m a + n}{(a-b)(a-c)} = A$$: similarly B and C can also be found out.

(d) ***'Differential Calculus' devoid of any sūtra***

In the XVII[th] chapter[16] on Quadratic equations no sūtra is apparent in the application of differential calculus for breaking a quadratic equation. The current solution for $ax^2 + bx + c = 0$ as $x = - b \pm \sqrt{(b^2 - 4ac)} /2a$ is referred to as a very crude and clumsy way of stating that the first differential is the square root of the discriminant'. The real truth is that Swāmījī obtained his so called 'Vedic' rule from the modern result itself as $2ax + b = + \sqrt{(b^2 - 4ac)}$

In the field of mathematical discoveries observation and recognition of patterns play a significant role. Patterns can be just lucky coincidences or can sometimes be reflective of a new theorem. Herein, by observation it is easy to realize that '2ax + b' is the first differential of $ax^2 + bx + c = 0$

(e) Factorisation of Quadratics

Earlier, on the factorisation of simple quadratics (Ref.1, ch. VII, p. 89), an additional sub-sutra of immense utility was also born algebraically as :

$ax^2 + b.x + c = (p.x + m) . (q.x + n)$

$= x^2 . p.q + x. (p.n + m.q) + m.n$ ------------(e.1)

Equating the coefficients and on adding the same.

$a + b + c = p.q + (p.n + m.q) + m. n = (p + m). (q + n)$

i. e. "The product of the sum of the coefficients in the factors is equal to the coefficients in the product"and in the Vedic style Swāmījī put it as *guṇita samuccayah-samuccayaguṇitaḥ* . The method[17] outlined is also of algebraic origin. i.e. splitting of the middle coefficient into two such parts that the ratio of the first coefficient to the first part is the same as the ratio of the second part to the last coefficient. In the case of equation $ax^2 + bx + c = 0$, if $b = p_1 + p_2$ in such a way that $a/p_1 = p_2/c$, then $(p_2x + c)$ or $(ax + p_1)$ will be one of the factors.

In the equation (e.1) above,

Let, $b = (p.n + m.q) = p_1 + p_2$ i.e. $p_1 = p.n$, $p_2 = m.q$

Then $a/p_1 = p.q / p.n = p_2 / c = m.q / m.n = q / n$ and (q.x + n) is a factor of $ax_2 + bx + c = 0$.

The *lopana -sthāpana* sub-sūtra can also be proved on similar lines. In the case of second degree homogeneous polynomials having three variables.

i.e. $Ax^2 + By^2 + Cz^2 + D\,xy + E\,yz + F\,zx = 0$, the three factors can be obtained by;

(i) Splitting the coefficient of xy, D into two parts D_1 and D_2 such a way that

$$A/D_1 = D_2/B$$

(ii) Similar to the above, in the case of y the ratio is

$$B/E_1 = E_2/C \quad \text{and}$$

(iii) For z, $C/F_1 = F_2/A$ is the ratio that give the factor of z.

In either of these methods no 'Vedic' magic is involved. All the short-cuts emerge out of algebra and commonsense.

(f) *Auxiliary Fractions and Recurring Decimals*

In the words of Swāmījī[18]: "the whole modus operandi is to replace the denominator by its '*ekādhika*'(i.e. to drop the last digit and increase the penultimate one by 1) and make a consequential alteration in the division procedure (as in the case of other *ekādhika* operations[18]")

Alteration in the division the procedure amounts to obtaining a new dividend successively by prefixing the remainders to the emerging quotient digits. For example page258, (ref. 1)

F = 6/29 Auxiliary fraction (A.F.) = 0.6/3 and

F is evaluted as = 0.20689655172413 79310344827586. In the usual notation we all know that the A.F. 0.6/3 means 0.2, which does not yield a recurring decimal. True answer can only be obtained through some mathematical magic. Swāmījī's technique achieves the magic by taking dividends successively as :

0.6 followed by 2, 20, 26, 28, 19, 16, ... etc., where 2,0,6,8,9,6, etc., are the successive digits of the quotient 0.206896 ...., suffixed to the remainders respectively 0,2,2,2,2,1,1 ....etc. Swāmijī has given the procedure but not the rationale, like a magician who keeps his secrets.

Rationale of the Method

Infact the fraction F = N / D = N / D. (D+1) / (D+1) = A.F / (D+1) : and hence the numerator of A.F is actually N. (D+1) / D. In the example given above the true form of the auxiliary fraction is A.F = .620689655......../ 3 rather then .6 / 3 and this explains the magic involved. A.F is completed by making use of the quotient digits.

(2) In the second type wherein the denominators are ending in 1, Swāmījī's procedure is different. eg. F = 70 / 71 A.F is 6.9 / 7 that yields the decimal as F = 0.98591549295774647887З .... here in the remainder is prefixed not to the successive

quotient digits but to its complement from nine. Similar to the above, the rationale is F =N / D = N / D. (D-1) / (D-1) = A.F / (D-1) and hence numerator of the auxiliary function (A.F) is actually N . (D-1) / D. A. F for 70/71 will therefore be 6.901408450..... / 7. It is appparent that the complete dividend of A.F can be constructed step-wise using the complement of the quotient digits.

In the application of certain sūtras similar explanations were provided by Swāmījī also, but invariably a deliberate attempt can be seen to distort the truth about the origin of the 'easy' methods.

(g) *Differenatial Calculus and Co-ordinate Geometry.*

Swāmījī's 'vedic' garb did not spare even topics like Calculus and Co-ordinate geometry. Just by accident $\int x^n dx$ had the index changed to (n + 1) in the result i.e.x^{n+1} / n+1 and Swāmījī could depict it as an application of the *ekādhika* sūtra. In the chapter on Analytical Conics Swāmījī describes the equation $y-y_1 = (y_2 - y_1/x_2 - x_1). (x-x_1)$ as 'cumbrous and confusing'. Simplification of the same yields the formula as: $x.(y_2-y_1) - y.(x_2-x_1) = x_1.y_2 - x_2.y_1$ and the rationale is presented by Swāmījī as the vedic at-sight, one-line, mental method by the *parāvartya* sūtra. Whatsoever his Highness has discussed in the context of analytical conics are related to the theory of equations and hence only algebraical in content. Neither of the sūtras are capable of defining the Cartesian co-ordinates nor do they have any relation to the Differential Calculus and Trigonometric or Logorithmic functions. The following words of Swāmījī in the context of his fifth proof to the Pythagoras theorem are really noteworthy[19] :

> "This proof is from Co-ordinate Geometry. And, as modern Conics and Co-ordinate Geometry (and even Trigonometry) take their genesis from Pythagoras theorem, this process would be objectionable to the modern mathematician. But, as the vedic sūtras establish their Conics and Co-ordinate Geometry (and even their Calculus), at a very early stage, on the basis of first Priniciples and not from Pythagoras' Theorem (sic), no such objection can hold good in this case".

The perceptions reflected here are sufficient to undermine the credibility of all the high claims by Swāmījī about 'vedic' mathematics and the sixteen formulae he had written on the sūtras. Nowhere did His Highness enunciate the first principles. He is talking about in the above paragraph. Moreover His words lead to utmost confusion as it contradicts the modern derivation of the distance formula (between two points) in co-ordinate geometry by use of the Pythagoras theorem.

In a nut-shell, without exception Swāmījī's procedure was to discover the implicit 'easy' rationale of the arithmetical process algebraically and then to camouflage the same as a 'Vedic'sūtra by phrasing in Sanskrit in the sūtra style. Nothing Vedic is perceivable in any of the sūtras and as observed by K. S. Shukla the title 'Vedic Mathematics' or 'Sixteen Simple Mathematical Formulae from the Vedas' is definitely "deceptive" and intended to conceal the actual algebraic process of derivation.

Pros and Cons of Swamiji's Approach

In view of the respect and admiration towards the pontifical authority of the author most of the researchers took his words for granted and quite unsuspectingly mistook the short-cuts provided by Him as class by itself emerging from the Vedas. Many have euphorically praised the so called Sūtras which are only vaguely phrased operations, adaptable to different situations by use of appropriate commentary. Sūtras of other ancient Sanskrit works related to a particular topic cannot be used in elucidating a different subject.

S. Das did not take notice of this aspect while praising the shortness and simplicity of the Sūtra processes. But the conclusion reached by S. Das after an analysis of the computational methods is really noteworthy[20]:

> "These results and observations show that Vedic Mathematics is usually much better when working special types of equations than with general ones. This means that one should be discrete in using the general formulae and use them only when the equations do not possess the characteristics that make one of the special formulae applicable .... Second, in Vedic Mathematics mental calculations play quite a significant role in the mathematical processess ...."

In contrast to these valid scientific observations. S. Das continues to make a few odd assessments of the impact of these sūtras in the field of computer algorithms. To quote[20]:

1. "These results are sufficiently encouraging to prompt one to explore the possibility of developing software based on these sūtra algorithms for use in high speed computers.... The process of developing software using sūtra algorithms may spark off clue(s) to the development of a new programming language. Finally efforts may be made to build a computer using the sūtra algorithms of Vedic Mathematics........."

2. "While the cryptical quality of Vedic Mathematics comes from the basic nature of sūtra style of composition, the applicability of the very same sūtra to more than one area of mathematics depends on how the sūtras is interpreted .... Nevertheless, vedic Mathematics appears to have an empirical base when one notes that Bhāratī Kṛṣṇa seems to have suppressed the proofs of many mathematical formulae and then abstracted them into the form of sūtras. But in this process he has gained in two respects :greater computational economy (by eliminating the intermediate mathematical steps) and simplification of learning and the oral transfer of information[21]......"

Conclusions of Das alone are sufficient to rule out the possibility of the use of Swāmījī's sūtras in the field of computers. Cryptic,abstractions having different meanings and very often conveying only a vague sense of the procedure cannot be referred to as algorithms. Applications in the field of computer algorithms infact requires the algebraic rationale rather than a multitude of special cases, special formulae and mental processes. Infact algebraic programs definitely have an edge over numerical evaluation programs because of the following reasons :

a) A result in algebraic form may yield better insight than a numerical value.

b) Algebraic answers are exact as compared to the approximated numerical values generated by a numerical evaluation program.

c) Algebraic simplification without numerical evaluation is more economical of computer time in many cases.

It is apparent from the above discussion that in the field of computer algorithms, the sūtras have no significance at all. The confusion that spread among the intelligentsia in this regard can be understood from the fact that the Govt. of India, more than a decade before, in November 1987 did instruct the 'Rastriya Veda Vidya Pratishthan' to prepare a project for examining the applicability of Ancient Vedic Mathematics to modern computer calculations[22].

In short, Swāmījī's book for the first time made use of the fundamental principle that the rationale of the arithmetical processes can be understood with the help of algebra and the implicit rationale can be of help in discovering short-cut methods. Certainly the creative approach and the hardwork involved deserves great appreciation. But Swāmījī's effort to conceal the methodology and to include the whole of modern mathematics into the 'Vedic' frame are not reflective of scientific approach and his good intentions. A word of caution is also necessary in the context of its adoption as a topic of study. Mathemetically the methods are incomplete without the associated algebraic rationale. Many of the short -cut methods have restricted operation only within a special group beyond which it becomes "cumbersome like the general methods". Methods like the 'one -line mental multiplication' by the *ūrdhva- tiryagbhyaṃ* and those involving so many "special cases" etc. are totally unfit to replace the prevailing general methods which derive the results in simple steps that can be easily grasped by the children of primary schools. The experts may find Swāmījī's methods as providing speed as well as recreation but that may not be the case with children who are trying to have a grip with the subject. Swāmījī's mathematics cannot be considered a systematically enunciated discipline like, the '*Yogasūtra*' of Patañjali—feat having no parallel in the history of Sūtra-literature.

Ancient Sources of Swamiji's Methodology

The algebraical analysis and derivation of rationale is nothing new to the Indian context of mathematics. Bhāskarācārya II in his *Siddhāntāsiromaṇi* expressess the inter relationship of arithmetic and algebra in the following words[23]:

pūryaṃ proktaṃ vyaktamavyakta bījaṃ
prāyaḥ praśna no vina vyakta uktyā !
jñātuṃ sakya mandadhibhirnnitāntham
yasmāltasmād vacmi bījakriyāṃ ca !!

"Earlier stated (arithmetial operations of Līlāvati)—'*vyakta gaṇitaṃ*'—have algebra implicit in it (*avyakta bījaṃ*). Without the implicit algebraic rationale, arithmetic cannot be properly understood by the less intelligent ones, and so I proceed to detail the algebra now"

Also he has stated in *Līlāvati* [24]:

pāṭīsūtropamaṃbījaṃ gūtamityavabhāsate !
nāsti gūtamamūdhānāṃ naiva ṣodhetyanekathā !!

"Algebra is equivalent to arithmetic even though it is apparently mysterious. Nothing is mystery for the intelligent and infact the computational methods (*gaṇitaṃ*) are many rather than the six"

Perhaps,even Bhāskarācārya can't claim any credit for these ideas as he might have only quoted the views of early Ācāryas like Skandasena or Śrīdhara (9th century AD). In *Pāṭīgaṇita*, Śrīdhara himself has employed algebraic rationale for arithemtical operations like multiplication,squaring etc. As such the application of algebra to simplify arithmetical operations is not a new discovery at all. Swāmījī was well aware of this fact and that is why he has described the work in his preface as *The Astounding Wonders of Ancient Indian —Vedic Mathematics"*.

Scope of Vedic Mathematics in Educational Programmes

It is apparent from the above that any incorporation of Swāmījī's sūtras of Mathematics in the modern syllabus will be a meaningless act that will lead to great confusion. Without the true facts of their origin commentary and explanatory proofs the insights or vague process of computation alone are of little significance to the students. It will be more ideal to include *'Līlāvati'* and *'Bījagaṇitaṃ'* in the syllabus supplemented by Swāmījī's short-cut methods in a special chapter on the history of Indian Mathematics. The institutions that have been sponsoring studies on Swāmījī's sūtras may better focus their attention onto a larger canvas of the creative approaches that can be deciphered from our scientific Vedic heritage. In view of the various reasons cited earlier under section V, the so called 'Vedic Mathematics' has absolutely no scope in the realm of computer. We may rather turn our attention to the creative development of Computer Algebra.

Conclusions

Swāmījī's work is an extended and creative application of an age old principle of Indian mathematicians. By clever manoeuvering with the use of "Vedic " appellation and sūtra style Swāmījī's attempt was to detach the technique from its true background for gaining the credit of a new Vedic discovery from the so called nonexistent *pariśiṣṭa* of the *Atharvaveda*. This is well evident from the confusing way in which he has presented the algebra underlying some of the techniques as proofs of certain Vedic mathematical principles. The Sūlbsūtras and π given in erroneous *kaṭapayādi* notation provide ample testimony for his manipulative claim of Vedic origin.

There are no reasons to believe that Swāmījī was unaware of the implications of Bhāskarācāya's statment referred earlier. To explain the different arithmetical operations like squaring, cubing etc., Late P.K. Koru had applied algebra in 1938 in his Malayālaṃ translation of *Līlāvatī*[25]. In Modern Mathematics also we can find use of algebraic numbers - as for a famous example : in 1837Gabriel Lame did use algebric number in his attempt to prove the Fermat's last theorem referred earlier. Swāmījī's efforts to conceal the algebraic origin of his techiques and to glorify the Vedas by confusing everyone certainly do not conform to the scientific spirit of the ancient Indian tradition. It is hoped that the present article will clarify some of the prevailing confusions and enable the students to have a more objective understanding of Swāmījī's so called Vedic Mathematics.

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