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POLAR LONGITUDES OF THE *SŪRYASIDDHĀNTA* AND HIPPARCHUS' COMMENTARY

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Present paper is an investigation into the genesis of the Polar longitudes, the spherical astronomical coordinate used in recording the positions of stars in the *Sūryasiddhānta* and the Hipparchus' *Commentary to Aratus and Eudoxus.* Historical background and the state of modern researches have been profiled in exploring the Greek origin and its adaptation by Indian astronomers. An attempt has been made to conceive the original method of computation of the polar coordinates via the observation of meridian transit time and the use of rising time of zodiacal signs. The method suggested in combination with the Hipparchus's epoch of fixed star observations removes the confusion prevailing between the polar longitude and right ascension in the Hipparchus' Commentary. Evolution of the equatorial coordinates, right ascension and declination, from the polar coordinates is explained.

Comparison has been made between the polar longitudes of *Sūryasiddhānta* and the respective modern computed values. Also contrast is provided of the ancient values with the approximate out put of the method suggested - computation of polar longitude using transit time. Further, it has been demonstrated that the *Sūryasiddhānta* values had their origin at the epoch of 522 AD and not as shown by Saha and Lahiri that the values were measured across a span of 300 years from 285 AD to 585 AD.

Key Words: Hipparchus, Meridian transit, Polar coordinates, Ptolemy, Star Catalog, *Sūryasiddhānta*

INTRODUCTION

Significant mention of the polar longitudes is traceable to the antiquity (BC 150), the times of the great Greek astronomer, Hipparchus. The lone surviving work of Hipparchus, *Commentary to Aratus and Eudoxus* contain a mention of 400 stars but many lacking a complete system of coordinates. The popularly known *Almagest Star Catalog* (*ASC*) of Claudius Ptolemy is suspected to be a plagiarism of an earlier star catalog of Hipparchus known to have existed

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by the attestation of Pliny and from the words of Ptolemy himself, "the fixed star observations as recorded by Hipparchus, which are our chief source for comparisons have been handed down to us in a thoroughly satisfactory form" [1]. Further, Ptolemy himself has suggested [2] the epoch of fixed star observations of Hipparchus as 265 years before the first year of Antoninus (137/138 AD) which means BC 129/128.

Neugebauer in his classic history of mathematical astronomy has relied upon the *Commentary to Aratus and Eudoxus* in describing the technical details of the spherical astronomy of that period. A definite system of spherical coordinates for stellar positions were not prevalent during the days of Hipparchus and even different norms existed for the positions of the cardinal points. In the second part of the Commentary where a systematic treatment of stellar observations is available. Neugebauer has found the coordinates mentioned as the polar longitude - the *dhruvaka* of *Sūryasiddhānta*. Little is known about the early history or evolution of this method of representing stellar positions except for their appearance in the above texts belonging respectively to Greece and India.

Controversy had been ranging in the western world as to the real nature of the Hipparchus' coordinate system and its relation to the *Almagest Star Catalog* while in India the polar coordinates available in *Sūryasiddhānta* had been in the midst of controversies in connection with their use to find out the zero point of the zodiac employed in Siddhāntic astronomy. Present paper is an attempt to provide a brief about the status of the modern studies on the nature and place of polar coordinates in the evolution of spherical astronomy and to offer some insight so as to clarify some of the prevailing controversies about its genesis in Greek as well as their appearance in *Sūryasiddhānta*.

MODERN ASSESSMENT OF THE POLAR LONGITUDES OF *SŪRYASIDDHĀNTA*

Merits vis-a-vis accuracy of the polar longitudes available in Indian Siddhāntic astronomical texts had been a subject of study by modern astronomers since the days of Whitney

1. Quoting Al-Bīrūnī, W. Jones and Colebrooke, Whitney remarks:

"It's evident that for centuries past, as at present, the native tradition has been of no decisive authority as regards the position and composition of the groups of stars constituting the asterisms. These must be determined upon the evidence of the more ancient data handed down in the astronomical treatises".

2. Indian Calendar Reform Committee did undertake a detailed study of the same in 1955 to reach the assessment that the longitudes were measured at three different epochs viz.,

- When vernal equinox was 22°21′ ahead of the present (1955) vernal equinox → 340.
- When vernal equinox was 20°08′ ahead of the present (1955) vernal equinox → 500.
- When vernal equinox was 19°21′ ahead of the present (1955) vernal equinox [4] →560.

Saha and Lahiri analyzed the polar longitudes after converting them to celestial longitudes and then comparing the same with the modern values of 1950 to derive the three classes that gave the above mean values of precession arc. Apparently the method is scientific and the Indian scholars have generally accepted the above assessment about the polar longitudes of *Sūryasiddhānta* as correct and the above chronology of *Sūryasiddhānta* measurements has become very popular in India. Indian National Calendar itself is founded on the assumption that the initial measurements leading to *Sūryasiddhānta* account of the junction stars had their beginning in 285 AD when Citra or α-Virginis coincided the autumnal equinox. Put it briefly, Saha's conclusion puts the polar longitudes of *Sūryasiddhānta* over a spread of 300 years from 285 AD to 585 AD - across the autumnal equinox on α-Virginis to vernal equinox on ζ- Piscium.

3. One of the recent studies by Pingree and Morrissey [5] has followed almost the same method in assessing the merits vis-a-vis accuracy of the stellar longitudes available in Siddhāntic works and has attempted to demonstrate that:

- There is no basis for identifying the *nakṣatras* mentioned in the Vedic literature
- Polar longitudes available in *Paitamahasiddhānata* of *Viṣṇudharmottarapurāṇa* is the Indian adaptation of a Greek star catalogue

- Ineptitude with which the Indians historically tried to correct these coordinates is reflective of the poor observational capabilities of the medieval Indian astronomers.
- Pingree, also points out that the only with the astronomical texts of the fifth century AD and later appear either the polar coordinates or ecliptic coordinates or a mixture of both.
- Pingree had also reduced the polar coordinates to ecliptic coordinates to explore the relationship of the Indian catalogues with Ptolemy's and found that the Indian works had no dependence on Ptolemy[6].

4. A new approach to polar longitudes is given by Pickering[7] in his study of the "Polar longitudes" contained in the *Commentary* of Greek astronomer Hipparchus (2nd century BC). Pickering infers an ecliptic coordinate basis for the positions of stars contained in his *Commentary to Aratus and Eudoxus.* As described by Pickering the salient features of these longitudes are:

- These positions are expressed in a manner that is quite different from modern forms. For example, Hipparchus will state that a star rises at the same time as a certain degree of the ecliptic rises, or that a star sets at the same time as a certain degree of the ecliptic rises. Positions of this type are collectively referred to as "phenomena."
- The most common phenomena are those where Hipparchus states that a star culminates (i.e., transits the meridian) at the same time as a certain degree of the ecliptic culminates. There are over 200 stars described this way in the *Commentary* - many of them more than once, and frequently with different (conflicting) results. These simultaneous culmi nation phenomena he called "mid-heaven" phenomena; here, we call these data "polar longitudes".[8]
- Pickering discusses two possibilities of derivation: (i) Use of an armillary astrolabe to find the degree of the ecliptic that rises at the same time as the star[9] (ii) if Hipparchus observed and recorded the stars ecliptically, then the positions in the *Commentary* could have been derived by using spherical trigonometrical. This is especially true of the polar longitudes, because while the spherical trigonometrical conversions for rising and

setting phenomena are cumbersome (and differ by latitude), the spherical trigonometry required for polar longitudes is fairly straightforward.

- Method of Hipparchus according to Pickering: Converting ecliptically observed coordinates into a polar longitude is a two-step process: (i) Star's α is computed from its ecliptic latitude and longitude and (ii) Then the degree of the ecliptic that has the same α is found probably tabularly.

It is apparent from the above discussion that the methods of observation and derivation of the "Polar longitudes" remains a topic of interest even after nearly two hundred years of study in the light of modern astronomy. An aura of confusion we can see around this parameter, which in Indian astronomy had received the name *dhruvaka*, rendered into English as 'Polar longitude'.

We noted above the remark of Pingree that the dependence of Indian Polar λ (s) to those of Ptolemy could not be established. Pickering's reference to the Ptolemy's catalog as derived from the Hipparchus's observations is also cited above. Both these ideas taken together leads to the possibility of the origin of Indian polar longitudes from a Greek or Hipparchan source. Pingree had taken the polar longitudes of *Paitamahasiddhānata* of *Viṣṇudharmottarapurāṇa* (*PVP*) as the basic data but here in *Sūryasiddhānta* shall be taken as reference:

(a) Table-I shown as Appendix-I of the *Sūryasiddhānta* (vide ref. 3) gives the polar longitudes (*dhruvakas*) and latitudes (*vikṣepas*) of the 27 *yogatā ras* of Indian astronomy.

(b) As noted by Pingree in the case of *PVP*, in *Sūryasiddhānta* too the longitudes are mostly in integral numbers of degrees and are therefore not intended to be of higher accuracy than half-a-degree.

(c) Fractional parts with some stars suggest that if they are original then the list originally had more accurate data and the fractional parts of others have been lost in course of transmission.

(d) Going by R. Newton's 'Crime of Ptolemy...' and the arguments used to demonstrate the plagiarism in the star catalogue of Ptolemy, it may be suggested that the *Sūryasiddhānta* data may be the adaptation of some Greek catalogue with the removal of fractional parts and some magnitude changes to account for precession.

(e) Another notable feature of the data is the lesser accuracy of the polar latitudes -approximate values are given and they do not differ very much from the ecliptic latitude wherever the star is properly identified.

(f) *Sūryasiddhānta* values are contrasted with the polar longitude and latitude computed using the modern values of right ascension and declination. Also provided is a contrast with the celestial longitude.

(g) It strikes our attention that the λ-Orionis and α-Orionis values of polar longitude [63/ 65.40, 67.33/ 70.37] given in the *Sūryasiddhānta* are different from the computed polar values, and the polar values of *Sūryasiddhānta* agree well with the ecliptic longitudes 62.78 and 67.83. With α-Scorpii too we find the same situation. Pickering has interpreted similar feature of the Hipparchus data as evidence for the ecliptic coordinate basis for the polar longitudes.

COMPARISON OF *SŪRYASIDDHĀNTA* VALUES WITH λ^*, β^* COMPUTED[10]

In the past studies, no attempt was ever made for comparing the polar longitudes of *Sūryasiddhānta* with the values computed for its epoch that was around 500 AD. Table-I gives a comparison of the *Sūryasiddhānta* values with λ^* derived from the values of Right Ascension.

1. Method of Computing λ^*

As we saw earlier the measurements could have been with an armillary astrolabe or transit instruments of the type we find described in *Almagest*. So the best choice for us will be to use the right ascension for the meridian transit of the star and compute the mid-heaven point which will be equal to λ^*. We can see the following observations of Saha and Lahiri in this regard:[11]

> "We are not aware how the Hindu savants determined *dhruvakas* and *vikṣepas*. It appears that they had a kind of armillary sphere with an ecliptic circle which they used to set to the ecliptic with the aid of standard stars like Puṣya (δ-Cancri), Maghā (α-Leonis)... They could also calculate the *dhruvaka* and *vikṣepas* of a star during the moment of its transit over the meridian of the place of observation. They calculated the *daśama-lagna* for the moment of transit from tables already constructed for the latitude of the observer and this *daśama-lagna* was the required *dhruvaka* of the star. By two vertical poles (i.e. gnomon) situated in

lagna was the required *dhruvaka* of the star. By two vertical poles (i.e. gnomon) situated in the north-south line, the zenith distance of the star at transit could be determined from which the declination of the star was deduced from the relation $\delta = \phi$ - zenith distance and... *vikṣepa*, which is thus: $\beta^* = \delta - \mathrm{Sin}^{-1} (\mathrm{Sin}\ \lambda^* . \mathrm{Sin}\ \omega)$".

Data is provided in Table-I for the epoch 500 AD.

2. *Sūryasiddhānta* λ^* versus the Modern Computed λ^*

We can see remarkable agreement between the modern values; and the *Sūryasiddhānta* values - the average error being only quarter of a degree despite the difficulty in the proper identification of all stars. It must be noted that:

- Some of the errors may be the result of manuscript errors that have crept in during transmission by the Indian oral tradition. For example, see the remarkably close values for η and α- Tauri and then we find the λ replacing λ^* for the two Orions followed by good agreement. Why should then be the values of β and 41-Arietis be in big error - their λ^* could have been 9 and 23 instead of 8 and 20 as available in later manuscripts.

- Agreement between Col.2 & 8 and the almost uniform error where the values differ point towards their measurement at the same epoch and the analysis made by Saha and Lahiri falls through because of their inappropriate or incorrect method.

- Pingree has given a comparison of the polar longitudes of *Paitāmaha* with those of *Sūryasiddhānta*. Values differ little and both match remarkably well with the polar longitudes computed.

- Accuracy of Indian data has a remarkable case in *Paitāmahasiddhānata* used as reference by Pingree which gives for Agastya (Canopus) $\lambda^* = 87$ and $\beta^* = 76$ S. Modern values computed for 500 AD are $\lambda^* = 87°.94$ and (-) 76.42.

3. Computation of β^* [$\delta = \phi - ZD$, $\beta^* = \delta - \mathrm{Sin}^{-1} (\mathrm{Sin}\ \lambda^* . \mathrm{Sin}\ \omega)$]

Appendix-II (of the *Sūryasiddhānta*) presents the computation of β^* via the use of azimuth data for the meridian transit of stars. Zenith distance is computed from altitude using the relation Sine of altitude = Cosine ZD. Place of observation is taken as Ujjayini [23° 09′ N, 75° 43′ E] as per the S*iddhāntic* astronomical tradition but it's not necessary that the λ*, β* data of *Sūryasiddhānta* were determined at Ujjayini itself. On comparison, it is evident that the ancient values of β* are not that accurate. But good agreement is possible in the case of certain values if a different *yogatārā* is chosen. For example:

Mṛgaśīrṣa is generally taken as λ-Orionis and *Sūryasiddhānta* gives λ* = 63 and β* = 10 S. These λ * and β* values may be compared with that of 35 Orionis which has the computed values λ * = 64.43 and β* = -9.39 and gives a better fit for the ancient data.

β - Arietis, 35-Arietis, η- Tauri, α - Tauri, 35-Orionis, β-Geminorum, α - Cancri, δ, β Leonis, δ - Corvi, α of Virginis, Bootes, Scorpii, Lyrae, δ - Sagittarii, α-Aquilae, β-Delphini, α-Pegasi have values that facilitate identification of stars and this agreement suggests determination at Ujjayini whose latitude was taken to be equal to ω or 24°.

THE INTEGER VALUES - ANCIENT METHOD?

It is evident that the values are not as accurate as found from the use of armillary sphere and trigonometric conversion of the parameters. Whole degree frequency of the data suggests alternate methods - even the Hipparchus data derivable from *Almagest Star Catalog* (*ASC*) has a high vale of whole degree occurrence - is there any other crude method that could have given the integer values available in *Sūryasiddhānta*?

Basis of the Ancient Star lore: To cite an example the ancient star lore prevalent in Kerala says:

- When the Revatī is overhead, you know that the rāśi Mithuna is above the horizon.
- Karka rāśi rises by two and a quarter *nāḍikas* when Aśvinī transits the meridian.
- When Bharaṇī is overhead, you know that Karka has risen

- When the six Kṛttikās are overhead, one *nāḍikas* of the Lion has expired
- Rohini is overhead when three *nāḍikas* expire in Leo.
- When Mṛga is overhead, Kanyā expires by its quarter.

Verses of *Nakṣatrappāna* (the Song of Stars) thus speak of all the twenty-seven *nakṣatras* of Indian Zodiac. Dikshit also has quoted similar data with a mention of the prevalence of such data among traditional Paṇḍits.[12]

"Overhead phenomena" thus formed a part of our way of life and obviously there's the possibility that the ancient method of observations and measurement had their genesis related to the overhead phenomena of polar longitudes.

The Indian method of computing *madhyalagna* may be understood as - "*madhyalagnam natā laṅkodayayair hi ūnādhiko* raviḥ" [13] which means:

"Ecliptic point on the meridian is the longitude of Sun at noon ± *natanāḍikās* (sum of the Laṅkarāśimānas[14])

i.e. taking Sun as the ascending point, ecliptic arc for the elapsed time from noon is to be subtracted or added according as the time is forenoon or afternoon.

This in turn gave an easy method for the determination of the polar longitudes on any day, especially equinoxes or solstices. All the 27 stars of the path of Moon could be observed at night by a round the year four-stage activity at equinoxes and solstices having sun respectively at 0, 90, 180, 270 and 360 degrees. Accuracy of the method here depended on the time measurement to fix the interval between noon and the moment of meridian transit. Transit time on any day when the equinox or solstices coincide with noon can be used conveniently to have Sun's 'λ as integral or zero. We can have the λ * simply by multiplying the fractional day of transit over the place[15] by 360.

An illustration for 21 March 500 AD is provided below: (Mean sun is taken as zero at noon)

Star	*Day-fraction 'd'*	*'d' x 360°*	SS $\lambda*$	*Star*	*Day-fraction 'd'*	*'d' x 360°*	SS $\lambda*$
β-Arietis	0.023	8.20	8	α-Bootis	0.547	196.86	199
41-Arietis	0.058	20.87	20	α-Librae	0.563	202.62	203
η -Taurus	0.097	34.84	37.5	δ-Scorpii	0.607	218.60	224
α-Taurus	0.132	47.40	49.5	α-Scorpii	0.625	225.04	229
λ-Orionis	0.174	62.79	63	λ-Scorpii	0.662	238.48	241
α-Orionis	0.189	68.07	67.33	δ-Sagittarii	0.698	251.29	254
β-Gemini	0.256	92.30	93	σ-Sagittarii	0.723	260.35	260
δ-Cancri	0.302	108.62	106	α -Lyrae	0.740	266.43	266.66
α-Cancri	0.314	112.96	109	α-Aquilae	0.775	279.14	280
α-Leonis	0.363	130.86	129	β-Delphini	0.810	291.53	290
δ -Leonis	0.409	147.06	144	λ-Aquarii	0.897	322.90	320
β-Leonis	0.436	156.85	155	α-Pegasi	0.909	327.33	326
δ-Corvi	0.465	167.52	170	α-Andromeda	0.953	342.99	337
α-Virginis	0.506	182.02	180	ζ-Piscium	0.996	358.71	359.83

An error of 4 minutes in the fixing up of the moment would have led to an error of one degree. For Agastya the method gives the value of 87.08, which is the same as of *Paitāmahasiddhānata*. In the same manner as above using Sine & Cosine tables of Altitude, the ancient astronomers could have determined the zenith distance, declination and the *vikṣepa* or polar latitude with some approximations. Under no circumstances determination of these values of λ* and β* demanded adaptation of a Greek catalogue. Any such adaptation would have been more difficult than the determination as above.

A corollary of the above is that if we have as reference an epoch where the mean sun is zero at noon, the transit time of the star shall be the same as the right ascension and the polar longitude. 21st March 522 AD, JD (UT) =1911797.78958333 UT, Ujjain local apparent noon, in fact was such an epoch and with the solar theory available in *Sūryasiddhānta* the polar longitudes could be determined with reference to the above epoch. Appendix-III gives a comparison of the polar longitudes of *Sūryasiddhānta* with those computed using the transit time interval. Values are in good agreement despite the fact that the values of column 6 have not taken into account the oblique ascensions of the signs and are therefore approximate only. Data casts sufficient light on the origin of the polar longitudes of *Sūryasiddhānta* and sets at rest all speculations as the measurement of the longitudes have taken place across a span of 300 years from 285 AD to 585 AD.

HIPPARCHUS' LONGITUDES - POLAR ?

No discussion on polar longitudes can be complete without a reference to the Greek origin and to the predecessors of Ptolemy who had been using it - the greatest name of course is of Hipparchus of Nicaea. Controversy surrounds the polar coordinates as well as on the issue of its use by the doyen of Greek astronomy almost three centuries before Claudius Ptolemy and in the seventh century before the 500 AD epoch of the *Sūryasiddhānta*.

1. Controversy in the Western World of History of Science

Controversy is still (after 2000 years) raging in the western world as to whether Hipparchus did use the polar coordinates in his star catalog that got lost in antiquity or in his sole surviving work *Commentary to Aratus.* According to O. Neugebauer, it is quite obvious from the *Commentary to Aratus* that at Hipparchus' time a definite system of spherical coordinates for stellar positions did not yet exist[16]. But this view has met with criticism of those who have been pursuing the line of thinking of Delambre[17], who in 1817 had proposed that Hipparchus was the original author of the *ASC* and he had a definite system of celestial spherical coordinates, namely, the right ascension and declination system that we use today. Debate is still continuing with many controversial propositions as:

1. Hipparchus or some other predecessor of Ptolemy measured a fairly complete star catalog in *equatorial* coordinates.

2. Hipparchus' *Commentary to Aratus* had as its data basis in the above catalog.

3. The catalog in equatorial coordinates was converted into ecliptic coordinates

- analog methods like the use of Celestial Globe by Hipparchus and Ptolemy added 2°40' in adapting them for the *Almagest*.

4. O. Neugebauer[16] has denied all the above propositions and maintains that Hipparchus had used the polar co-ordinates as it was a more convenient form for readings on a globe and for graphic construction/trigonometric computation based on stereographic projections.[17]

5. Duke Dennis in a latest paper [18] as argued in favour of the first three propositions notwithstanding the discussion of Neugebauer and the evidences quoted such as the works of H. Vogt. His arguments are:

a) The star coordinates in Hipparchus' *Commentary to Aratus* are clearly equatorial right ascensions and declinations.[19]

b) Ecliptical stellar coordinates are conspicuous in their absence

c) Error correlations between the *Almagest* data and the *Commentary* data show that those two data sets are associated in some way. Several stars substantiate this with large common errors in each data set, detailed statistical analysis of the error correlations between the two sets of data, and similar systematic errors in the two data sets.

d) A historian of no mean repute like Neugebauer is being contradicted by Duke claiming support from the facts:

- In the *Commentary* Hipparchus quotes the positions of numerous stars directly in right ascension or declination (or its complement, polar distance),
- Polar longitudes are not directly measurable, since the measurement of any longitude is always with respect to some other previously measured longitude, and there is no way to measure one polar longitude with respect to another polar longitude.
- Polar longitudes are in fact never quoted directly for a single star in the *Commentary.*
- In an explicit example on the computation of rising, setting and culmination numbers, Hipparchus has used right ascension and declination as the initial input data.

6. In contrast to the claims of Dennis Duke, we may place the inferences of Keith Pickering [20] that advocate an ecliptic coordinate basis for the Commentary of Hipparchos.

- According to Pickering, Hipparchus gives in his commentary about 200 simultaneous culminations of stars and ecliptic points referred to as "mid-heaven" phenomena.
- Pickering asserts that the *ASC* is the plagiarized Hipparchus' catalogue with the addition of 2°40' as precession correction.
- To prove the above proposition, Pickering has attempted to demonstrate that Hipparchus' had been using an armillary astrolabe with which the ecliptic position was measured and the same was converted to the polar longitudes seen in the commentary. To quote Pickering:

> "Not much is known about how Hipparchus obtained the data in the Commentary. It has been frequently assumed, on the basis of comments by Ptolemy, that Hipparchus used a celestial globe to chart his star positions, and may also have used the globe to perform spherical coordinate transformations. However, if Hipparchus used an armillary astrolabe to obtain star positions (and even this has been disputed by Neugebauer), a much simpler method is available"

- As regards the conversion of the ecliptic longitude to the polar one, Pickering says:

> "Obviously, a polar longitude is simply a different way of expressing the right ascension α of a star. Converting ecliptically-observed coordinates into a polar longitude is a two-step process: first Hipparchus computes the star's α from its ecliptical latitude and longitude. Then he finds the degree of the ecliptic that has the same α. This second step was probably done tabularly, and need not have been done at the same time as the first step"

2. Reconciliation of Differing Views on Hipparchus' Coordinate

Method suggested by Pickering for hundreds of stars shall be really cumbersome and time consuming. We have a far easier method as discussed under Section IV that involve only a single step in measuring both the right ascension and the polar longitude. The epoch of Hipparchus for the star catalog according to *Almagest* 7.4 and as verified by D. Rawlins[21] is 24th September 128 BC, local apparent noon at Rhodes. Rawlins has given the mean longitude of sun to be 180°.5 and thus the epoch was well suited to derive the right ascension and the polar longitude by observing the meridian transit of the star and fixing the interval from apparent noon.

Epoch: Sunday, 12:00,24 September, 128 BC at 36°08N, 28°05E, JD (UT) = 1674937.92222. Sun α = 11-53-14. Upper Transit = 11-53-14, ω = 23°42'.5

Considering for example the controversial entry of the catalog 32 Cygni: λ =302°40', β = 64°30' which yields Hipparchus's value [22] λ = 300°. We are not sure as to whether Hipparchus did use the ecliptic longitude or not in finding the polar longitude used in the commentary. On the above date 32 Cygni had its transit at $19^h08^m35^s$ with a right ascension of 19-09-46.10 and the excess in RA α was only 1-minute in time and 15-minutes of arc. Altitude was 83-32-35, thus the zenith distance of 6.46 and δ = 42°.59. Above epoch could have thus yielded a value of α = 287.146 and λ^* = 285.96 (precisely 285.73) which meant λ of 300° precisely and β = 64.27. Hipparchus needed only a transit instrument in getting the polar λ and no trigonometric conversion as is made out to be.

Appendix - IV gives a number of λ^* values ranging from 0° to 360° computed from Hipparchus' λ in control to the λ^* determined from other methods - simple method of Section III and the use of right ascension α. In fact the method described in section III effectively means the use of right ascension as it computes the mid-heaven by use of the mean sun.

It's apparent from the contrast of columns IV, V and VI that the values of λ * derived from Hipparchus are not that accurate as to be from an astrolabe and the source of his values may be the transit time interval and his own solar theory. It must be noted here that:

1. No evidence has become available for the use of armillary astrolabe by Hipparchus.
2. As noted by Neugebauer a system of well defined spherical coordinates was not in prevalence during the time of Hipparchus.

3. Solar theory of Hipparchus and the oblique ascensions of signs did make their presence felt as indicated by the 0° norm of cardinal points introduced by Hipparchus.

4. In the aforesaid method, the right ascension α had its genesis with the choice of the epoch 24th September 128 BC, from the polar λ and in the same manner declination was born of the altitude measurement as ($\delta = \phi$ - zenith distance).

5. In fact, a coordinate system must be a product of the method of observing the sky and in this respect we can understand that the polar coordinates had their origin in the "mid-heaven" phenomena observations and with the evolution of solar theory the polar coordinates evolved into right ascension and declination.

6. Pickering's latest paper[23] speaks of a 'multi-ring' astrolabe that addresses the measurement of ecliptic longitudes, which means that Hipparchus had a coordinate system readymade at hand and an instrument suited for the same was invented. This is definitely impossible in view of (5) above and is not attested by any evidence.

7. Dukes Dennis [24] quoted Hipparchus in respect of the usefulness of the information given in the third part ot the Commentary:

> "This is useful for us both for determining with accuracy the hour of the night and for understanding the times of lunar eclipses and many other subjects contemplated in astronomy."

   Such use of the mid-heaven phenomena arose out of their mapping made in terms of the interval of time and such intervals of time were obviously right ascension in view of the epoch taking care of the mean sun as well as the derivation of the polar longitude using the oblique ascensions/rising times of signs.

8. Commentary contains hundreds of stellar positions in terms of the degrees of the ecliptic that rises or sets simultaneously with the star and these horizon phenomena could not have been well observed with the facilities or observatories of those days. On the contrary the meridian transit offered easy observations and the rising or setting points could be computed taking into account the rising times of signs and the solar theory.

9. Dennis Duke argument that the polar longitude is not measurable due to lack of reference point is not sustainable in view of the computation of λ * in terms of the time interval relative to local apparent noon.

10. Inaccuracy in the values of Hipparchus as may be noticed from Appendix-IV is due to the error involved in measuring the intervals of time.

11. Neugebauer [25] has mentioned the remark of Hipparchus about the difficulties encountered by astronomers in the matter of rising times of the zodiacal signs and observes:

> "How Hipparchus himself would have determined the rising times we do not know. If he had - as is quite likely - stereographic projection at his disposal, he could have found the correct solution in the same way as we know it from Ptolemy's Planisphaerium"

Here we have another source of error to the star catalog of Hipparchus and an idea of the discrepancy may be understood from Appendix-IV in which we have given λ* derived from *ASC*.

3. Polar Longitudes of *Sūryasiddhānta*

In the light of the above discussion, it is well evident that the polar longitudes of *Sūryasiddhānta* represent a very early phase of development of astronomy and may owe its origin to the Greek sources. Further, it becomes apparent that the Greek source of the data or method is pre-Ptolemaic, or the *Sūryasiddhānta* too had an evolution from a distant epoch such as that of Hipparchus when the use of the polar coordinates was the state of the art method in astronomy. In the context of S*iddhāntic* astronomy, we have two reassuring factors that tend to negate the existence of any astrolabe or ecliptic coordinate as the basis of the polar longitudes:

1. *Sūryasiddhānta* had no precise value of the obliquity such as the one 23°55′ or 23°40′ used by Hipparchus almost 600 years ago- in relation to 499AD- (Śaka 421)/AD 506 (Śaka 427 of Varāhamihira) or 522AD of *Siddhāntic* tradition (latter value extremely precise for the epoch of Hipparchus). In 500AD, ω = 23°38′ while *Sūryasiddhānta* had the value of 24 degree which in effect meant that the elevations of the sun at solstices could not be accurately measured by Indian astronomers as did by Ptolemy or Hipparchus and they took the difference to be

48° against the actual value of 47.26. A value of 24° for ω would have caused irreconcilable error in the value of solar λ at the solstices, as the ecliptic ring could not have been tilted properly with the equator.

2. Further, the *Siddhāntic* meridian was of Ujjain, the latitude of which was taken as 24° instead of 23.15. Combined with ω, the wrong value of ϕ would have played havoc with the measurements of longitude and declination of the stars. With such errors looming large on the face of *Siddhāntic* astronomy, it's possible to infer that no precise observations were possible when compared to the accurate $\omega = 23.66$ and $\phi = 36°$ of Hipparchus who lived 650 years ago.

Polar longitudes thus characterize a phase of astronomy where accurate observations using instruments were yet to come in prevalence. This deduction can be applied back to the times of Hipparchus to suggest that the accurate value of ω and coordinate systems established by Hipparchus in turn facilitated the development of instruments like astrolabe and it was not otherwise.

CONCLUSIONS

Polar coordinates have a history that antedates the extant version of *Sūryasiddhānta* by nearly 600 years and we are able to trace its origin to the sole completely surviving work of Hipparchus viz., Commentary *on Aratus and Eudoxus.*

It has been shown that the polar longitudes were derived by observing the meridian transit of stars and by use of the transit time interval relative to noon and the rising times of zodiacal signs.

In the light of Hipparchus' solar theory and the BC 128 epoch of fixed star observations, we can visualize the evolution of polar longitude and polar distance into the present day used parameters of right ascension and declination.

Modern studies on polar longitudes by their conversion to modern α and δ had bred confusion in analyzing and fixing the epoch of the polar longitudes of *Sūryasiddhānta.* The method suggested here removes all ambiguity in the matter of the epoch of *Sūryasiddhānta* vis-a-vis the polar longitudes of stars given. The analysis of polar longitudes of Stars and the conclusions reached by the Calendar Reform Committee are shown to be wrong in the light of the possibility of derivation of those longitudes at the epoch of 522 AD - the traditional zero *ayanāṃśa* year of *Siddhāntic* astronomy.

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3. W. D. Whitney, Notes to the *Sūryasiddhānta,* Tr. by Rev. E. Burgess, *Journal of the American Oriental Society*, VI, 1860; reprint Calcutta 1935, pp.204-54.
4. M. N. Saha and N. C. Lahiri, *Indian Calendar,* reprint 1992, CSIR, New Delhi, p.263.
5. D. Pingree and Patrick Morissey, "On the Identification of the Yogatārās of the Indian Nakṣatras", JHA 20 (1989) 100-119.
6. D. Pingree, 'History of Mathematical Astronomy in India' *Dictionary of Scientific Biography*, New York, 15 (1978) 533-633.
7. K A Pickering, "Evidence of an Ecliptical Coordinate Basis in the Commentary of Hipparchos", DIO 9.1 (1999), June, p.26.
8. Background of the study by Pickering may be understood from the following observations that he makes in the paper [7]. "There is now ample evidence that the *Ancient Star Catalog* (*ASC*) preserved in the *Almagest* of Claudius Ptolemy (2nd centry AD) was in fact plagiarized from Hipparchus, after adding 2°40′ to the longitudes for precession. But contra these evidences, some believe that because the Commentary (unlike the *ASC*) contains no star positions in the standard ecliptical reference frame, that Hipparchus did not take ecliptical positions - and therefore could not have been the true author of the *ASC*. So if it can shown that Hipparchus did in fact take ecliptical positions of stars, the case for his authorship of the ASC becomes even stronger".
9. Pickering has added the notable remark: Since the ecliptic ring is not directly adjacent to the horizon ring, however, the result necessarily will be rougher than the star's ecliptical position and indeed, the positions in the Commentary are recorded only to the nearest half-degree, about three times less precise than the positions in the *ASC*.
10. Polar longitude and latidude shall be given by the notation λ^* and β^*
11. M. N. Saha and N. C. Lahiri, *History of the Calendar*, CSIR, New Delhi (1992), p.263.

12. S.B. Dikshit, *Bhāratīya Jotiḥs'āstra-II*, Controller of Publication, Civil Lines, Delhi, 1981, p.349

13. Paramesvara, *Grahanyāyadīpikā*, ed. K. V. Sarma, VVRI, Hoshiarpur, (1966), verse 15

14. Rising times of Zodiacal signs for 0° latitude or equator.

15. Precise values require the rising times at equator, which for the signs are (with $\omega = 24°$): 1.856, 1.994, 2.15, 2.15, 1.994, 1.856, 1.856, 1.994, 2.15, 2.15, 1.994, 1.856 hours respectively for the 12 signs from Aries.

16. O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Springer Verlag, New York, 1975, p.277.

17. J. B. J. Delambre, *Histoire de l'Astronomie Ancienne*, (1817, reprinted New York, 1965) Vol. 1, pp.117,172,184.

18. Duke Dennis, "The Measurement Method of the Almagest Stars", DIO, September 2002, p.35

19. Duke has observed further that - "we have no surviving hint how those coordinates were measured, or even who measured them"

20. K.A. Pickering, "Evidence of an Ecliptical Coordinate Basis in the Commentary of Hipparchus", DIO, 9.1, June 1999, p.26.

21. D. Rawlins, Hipparchus' Ultimate Solar Orbit and the Babylonian Tropical Year, DIO 1. 1#6, p.61, '91.

22. By subtracting 02°40′ following the suggestion of Tycho Brahe, which has received added support in recent times. According to Brahe, Hipparchus had made the *Catalog* in the second century BC; and Ptolemy had plagiarized the *Catalog* by precessing all of the longitudes by 2 2/3 degrees while leaving the latitudes unchanged. This would explain the 1-degree longitude error, because the actual precession between Hipparchus' time and Ptolemy's was closer to 3 2/3 degrees; so the one-degree difference nicely accounted for the systematic longitude error. (Pickering at the 4th biennial History of Astronomy Workshop, Notre Dame University, 1999 July 3)

23. K. A. Pickering, "The Instruments used by Hipparchus", DIO, 12, September 2002, pp.51-58.

24. W. Duke Dennis, "Hipparchus' Coordinate System", *Archive for the History of Exact Sciences*, 56 (2002) 427-433.

25. O. Neugebauer, *History of Mathematical Astronomy*, Vol. 2, 1975, p.715.

Appendix - I
Polar Longitudes of *Sūryasiddhānta* in Contrast to Modern Values

Yogatāras of Sūrya	Sūryasiddhānta					Transmit time	Right Ascension	δ	λ	Computed Values	
	λ		β		N/S	hh-mm-ss	hh-mm-ss			Dhruvaka λ*	Vikṣepa β*
Col 1	2		3			4	5	6	7	8	9
β-Arietis	8	0	10	0	N	12-32-47	00-34-34	12.95	13.14	9.45	9.12
41-Arietis	20	0	12	0	N	13-23-29	01-25-25	20.22	27.38	23.17	11.17
η-Taurus	37	30	5	0	N	14-19-22	02-21-26	12.22	35.40	37.84	3.89
α-Taurus	49	30	5	0	S	15-09-37	03-11-50	7.58	48.86	50.52	-6.07
λ-Orionis	63	0	10	0	S	16-11-10	04-13-33	5.78	62.78	65.40	-14.13
α-Orionis	67	20	9	0	S	16-32-16	04-34-42	30 17	67.83	70.37	-16.75
β-Gemini	93	0	6	0	N	18-09-11	06-11-53	22.46	92.59	92.72	6.2
δ-Cancri	106	0	0	0		19-14-29	07-17-22	16.56	107.83	107.78	-0.33
α-Cancri	109	0	7	0	S	19-31-51	07-34-46	18.53	112.77	111.85	-5.62
α-Leonis	129	0	0	0		20-43-25	08-24-33	28.32	129.05	129.08	0.12
δ-Leonis	144	0	12	0	N	21-48-15	09-51-34	22.76	140.24	145.51	15.01
β-Leonis	155	0	13	0	N	22-27-24	10-30-49	-8.22	150.79	155.83	13.17
δ-Corvi	170	0	11	0	S	23-10-04	11-13-36	-3.00	172.73	167.34	-13.34
α-Virginis	180	0	2	0	S	00-08-05	12-07-49	27.53	183.01	182.14	-2.13
α-Bootis	199	0	37	0	N	1-27-27	13-07-21	-9.00	183.15	198.33	34.87
α-Librae	213	0	1	30	S	1-30-29	13-30-27	-17.17	204.24	204.51	0.71
δ-Scorpii	224	0	3	0	S	02-34-24	14-34-33	-21.78	221.71	221.19	-1.63
α-Scorpii	229	0	4	0	S	03-00-10	15-00-23	-34.36	228.91	227.68	-4.28
λ-Scorpii	241	0	9	0	S	03-53-55	15-54-17	-28.85	243.76	240 83	-13.55
δ-Sagittarii	254	0	5	30	S	04-45-10	16-45-40	-26 59	253.71	252 93	-5 97
σ-Sagittarii	260	0	5	0	S	5-21-24	17-21-60	6.07	261 51	261.31	-2 89
α-Lyrae	266	40	60	0	N	05-45-43	17-46-23	38.24	285.3	266 88	-62.2
α-Aquilae	280	0	30	0	N	06-36-33	18-37-21	10.37	295 61	278.55	29.78
β-Delphini	290	0	36	0	N	7-26-08	19-27-05	15.03	331.35	290 04	32 84
λ-Aquarii	320	0	0	30	S	09-31-37	21-32-55	7.54	332.77	320.71	29.96
α-Pegasi	326	0	24	0	N	09-49-18	21-50-38	20 34	348.4	325.27	20.94
α-Andromeda	337	0	26	0	N	10-51-57	22-53-27	20.9	353.68	341 89	28 16
ζ-Piscium	359	50	0	0		11-54-50	23-56-31	0.64	359.47	359 05	0.26

Epoch: 21 March 500 AD, Ujjain, 23°09N, 75°43E

Appendix II

Vikṣepa using Zenith Distance

[Ujjain: 23° 09N[1], 75° 43E, 18 March 500AD: Meridian Transit of Stars]

Star	Right Ascension hh mm ss	Declination ° ′ ″	Altitude ° ′ ″	Zenith Distance	d = f- ZD	S-1 (Sλ*.Sω) S = sine	Viksepa β*	SS β*
β Arietis	00-34-34	12 56 55	79 48 7	10 2	12.95	3.25	9.78	10
35 Arietis	1-19-14	20 30 33	87 21 36	2 64	20.51	8 00	12 51	12
η-Tauri	2-21-27	18 20 05	85 11 10	4.81	18.34	14.34	4.00	5
α-Tauri	3-11-50	12 13 14	79 4 26	10 93	12 22	18.02	-5.79	-5
35 Orionis	04 09 50	11 51 33	78 42 45	11.29	11.86	21.25	-9.39	↓[2]
134 Tauri	04 26 11	10 46 25	77 37 38	12.37	10.78	22.04	-11.27	↓
54 Orionis	04 26 31	18 31 41	85 22 46	4.62	18.53	22.04	-3 52	↓
λ-Orionis	04 13 33	7 34 19	74 25 36	15.57	7.57	21.25	-13.67	-10
α-Orionis	04 34 43	5 46 05	72 37 24	17.38	5 77	22.04	-16 77	-9
104 Tauri	03 40 34	15 13 20	82 4 28	7.93	15.22	21.25	-6.02	
126 Tauri	04 15 44	14 19 28	81 10 37	8.82	14.34	21.25	-6.92	
β-Geminorum	06 11 53	30 10 28	82 58 39	7 02	30 17	23 97	6 21	6
δ—Cancri	07 17 22	22 27 20	89 18 20	0.69	22.46	23.02	-0.56	0
α-Cancri	07 34 47	16 33 18	83 24 25	6.59	16 56	22.62	-7	-7
α-Leonis	08 46 33	18 31 47	85 22 52	4 62	18 53	18.43	0.1	0
δ-Leonis	09 51 34	28 19 12	84 49 54	5.17	28.32	13.83	14.49	12
β-Leonis	10 30 49	22 45 49	89 36 49	0.386	22.76	9.9	12.86	13
δ-Corvi	11 13 36	8 13 35	58 38 2	31.37	-8.22	4.05	-12.27	-11
α-Virginis	12 07 49	-2 59 02	63 52 28	26.13	-2.98	0	-2.98	-2
α-Bootis	13 07 21	27 31 52	85 37 12	4.38	27.53	-7.61	35.14	37
γ-Librae	14 13 59	-8 45 27	58 6 11	31.9	-8.75	-12.8	4.05	-1.5
β-Librae[3]	13 58 16	-2 55 50	63 55 40	26.07	-2.92	-12.8	9.88	x
δ Scorpii	14 34 33	-17 10 05	49 41 47	40.3	-17 15	-16.41	-0.74	-3
α-Scorpii	15 00 23	-2146 47	45 5 14	44.91	-21.76	-17.88	-3.88	-4
λ-Scorpii	15 54 17	-34 21 53	32 30 43	57.49	-34.34	-20 84	-13 5	-9
λ-Sagittarii[4]	16 55 53	-24 41 15	42 10 52	47.82	-24 67	-23 16	-1.51	↓
δ-Sagittarii	16 45 40	-28 50 36	38 1 42	51.97	-28.82	-23.02	-5.8	-5.5
σ-Sagittarii	17 22 00	-26 35 01	40 17 11	49.71	-26 56	-23.61	-2.95	-5
α-Lyrae	17 46 23	38 14 34	74 18 57	15.68	38.83	-23.96	62.79	60
α-Aquilae	18 37 21	6 4 11	72 55 30	17.08	6.08	-23.61	29.69	30
β-Delphini	19 27 04	10 22 08	77 13 22	12.78	10 37	-22.47	32 84	36
λ-Aquarii	21 32 55	-15 2 19	51 49 29	38.18	15.03	-15.16	0.13	0
α-Pegasi	21 50 38	7 32 59	74 24 16	15.6	7.55	-13.15	20.7	24
β-Pegasi[5]	21 52 37	20 20 44	87 9 32	2.84	20.3	-13.18	33.48	↓
α-Andromeda	22 53 27	20 53 12	87 44 14	2.26	20.89	- 9.14	30.03	26
ζ-Piscium	23 56 31	- 38 49	66 12 38	23.79	-0.64	-0.068	-0.57	0

1. Siddhantic astronomers had $\phi = 24$ instead of $23°09'$.
2. Arrow has been shown to look for a matching value of *Sūryasiddhānta* below
3. $\lambda^* = 211.77$ and $\beta^* = 9.02$ by modern computation. $\beta^* = 9.88$ in the table of $\lambda^* = 213$
4. $\lambda^* = 255.23$ and $\beta^* = -1.51$ by modern computation but ω is taken as 24°
5. $\lambda^* = 325.9$ and $\beta^* = -1.51$ by modern computation, $\omega = 24°$, not modern value of 23.45

Appendix-III
Polar Longitudes of *Sūryasiddhānta*
[Computed with Transit time Data of 21 March 522]

Yogatāras		*Sūryasiddhānta*				Transit	Polar λ from
of Sūrya...	λ*		β*		N/S	hh-mm-ss	day fraction
Col.1	2		3			4	6
β-Arietis	8	0	10	0	N	12-35-13	8.8
41-Arietis	20	0	12	0	N	13-25-59	21.5
η-Taurus	37	30	5	0	N	14-21-53	35.47
α-Taurus	49	30	5	0	S	15-12-07	48.03
λ-Orionis	63	0	10	0	S	16-13-39	63.41
α-Orionis	67	20	9	0	S	16-34-44	68.68
β-Gemini	93	0	6	0	N	18-11-52	92.97
δ-Cancri	106	0	0	0	--	19-17-05	109.27
α-Cancri	109	0	7	0	S	19-34-24	113.6
α-Leonis	129	0	0	0	--	20-45-57	131.48
δ-Leonis	144	0	12	0	N	21-50-48	147.7
β-Leonis	155	0	13	0	N	22-29-52	157.47
δ-Corvi	170	0	11	0	S	23-12-28	168.12
α-Virginis	180	0	2	0	S	00-10-30	182.62
α-Bootis	199	0	37	0	N	01-09-45	197.44
α2-Librae	213	0	1	30	S	01-32-56	203.23
δ-Scorpii	224	0	3	0	S	02-36-55	219.23
α-Scorpii	229	0	4	0	S	03-02-44	225.68
λ-Scorpii	241	0	9	0	S	03-56-37	239.15
δ-Sagittarii	254	0	5	30	S	04-47-50	251.96
σ-Sagittarii	260	0	5	0	S	05-24-03	261
α-Lyrae	266	40	60	0	N	05-47-45	266.94
α-Aquilae	280	0	30	0	N	06-38-55	279.72
β-Delphini	290	0	36	0	N	07-28-28	292.12
λ-Aquarii	320	0	0	30	S	09-34-07	323.53
α-Pegasi	326	0	24	0	N	09-51-41	327.92
α-Andromeda	337	0	26	9	N	10-54-19	343.58
ζ-Piscium	359	50	0	0		11-57-15	359.31

Note: In deriving the polar longitudes of column 6, we have not taken care of the rising times of the *rāśis* and hence the difference with the *Sūryasiddhānta* values of col.2. The mean Sun is only about quarter of a degree at local apparent noon and as such 21 March 522 AD, 12:00 noon at Ujjayini could be the epoch of *Sūryasiddhānta* polar longitudes. It must be noted here that the most of the Siddhāntic works of the post-Aryabhaṭa period have taken 522AD as the zero *ayanāṃśa* year.

Appendix - IV

λ* of Hipparchus' λ Versus λ* from other Methods

[Table gives Hipparchus' longitudes derived from ASC and the computed polar longitudes]

Star	λ(H)	β(H)	λ*(H)	λ*(JD)	λ*(α)
I	II	III	IV	V	VI
α-Arietis	8	10.5	7.92	3.51	3.64
β-Arietis	5	8.33	4.94	0.79	0.65
41-Arietis	19	10.17	18.92	13.18	14.17
35-Arietis	17	11.17	16.92	11.67	12.54
β-Tauri	53	5	52.98	49.12	51.54
α-Tauri	40	-5.17	40.03	39.58	41.99
ζ(123)-Tauri	55	-2.5	55.01	53.46	55.78
o(1)-Tauri	21.67	-9.25	21.73	23.63	25.39
χ(2)-Tauri	22	-8.3	22.06	24.04	25.83
λ-Tauri	31	-8	31.05	31.83	34.02
δ-Tauri	37.67	-4.25	37.69	36.31	38.64
27-Tauri	31	3.67	30.98	27.34	29.33
α-Gemini	80.67	9.67	80.65	79	79.93
β-Gemini	84	6.25	83.99	83.04	83.66
ε-Gemini	70.33	1.5	70.33	68.4	70.06
χ-Gemini	84	2.67	84	83.34	83.93
δ-Gemini	79	-0.5	79	77.97	78.97
λ-Gemini	79	-6	79.01	78.72	79.68
γ-Gemini	69.33	-7.5	69.35	68.87	70.51
β- Cancri	97.17	-10.5	94.49	94.69	94.36
α Cancri	106.5	-5.5	103.82	104.59	103.5
I Cancri	98.33	11.83	95.68	97.9	97.31
α Leonis	122.5	0.17	119.83	122.56	120.46
η Leonis	120.67	4.5	118.02	121.38	119.32
δ Leonis	134.17	13.67	131.58	138.49	136.17
θ Leonis	136.33	9.67	133.72	139.24	136.93
β Leonis	144.5	30	142.07	148.89	146.84
β Virginis	149	0.17	146.33	149.38	147.35
δ Virginis	164.33	8.5	161.73	166.74	165.86
α Virginis	176.67	-2	173.98	173.71	173.45

Note: Hipparchus' Location was 36° 08′ N, 28° 05′ E

(Continued...)

Appendix - IV (Continued)

Star	λ(H)	β(H)	λ*(H)	λ*(JD)	λ*(a)
I	II	III	IV	V	VI
ζ Virginis	174.83	8.67	172.23	176.55	176.56
α 2 Librae	198	0.67	195.34	194.18	195.79
β Librae	202.17	8.83	199.56	201.22	203.35
σ Librae	203	-7.5	200.28	196.27	198.05
β 1 Scorpii	216.33	1.33	213.67	211.37	214.05
δ Scorpii	215.67	-1.67	212.99	209.72	212.34
π Scorpii	215.67	-5	212.97	208.83	211.4
σ Scorpii	220.67	-3.75	217.98	214.11	216.89
α Scorpii	222.67	-4	219.98	215.87	218.7
τ Scorpii	224.5	-5.5	221.8	217.04	219.9
ε Scorpii	228.5	-11	225.77	219.49	222.39
λ Scorpii	237.5	-13.33	234.77	228.3	231.21
ε Sagittarii	248	-10.83	245.3	240.84	243.37
σ Sagittarii	255.33	-3.17	252.66	250.36	252.36
α Aquilae	273.83	29.17	271.16	271.08	271.49
β Capricorni	277.35	5	274.66	274.12	274.29
α Aquarii	306.33	11	303.62	302.95	301.31
α Delphini	290.17	33.33	287.43	284.55	283.91
ε Pegasi	305.33	22.5	302.58	299.11	297.61
α Pegasi	326.67	19.67	323.88	319.22	317.41
β Pegasi	332.17	31	329.3	320.17	318.37
ι Ceti	334.67	-9.67	332.07	336.46	335.33
β Ceti	335.67	-20.33	333.16	342.53	341.84
γ Pegasi	342.17	12.5	339.41	335.73	334.56
α Andromedae	347.83	26	344.97	335.08	333.86
δ Andromedae	355.33	24.5	352.47	342.16	341.44
β Andromedae	3.83	16.33	1.04	348.94	348.78
α Piscium	2.5	-8.5	-0.1	3.67	3.81

- Column-II: λ (H) = λ (ASC) - 02° 40′
- Column-III: β (H) = β (ASC)
- Column-IV: λ^*(H) = Hipparchus' Polar longitude
  [Correction applied for λ to λ^* conversion: $\delta \lambda = \mathrm{Tan}^{-1}$ [Cos λ /(Sine λ - Cot β. Cot ω)], ω =23° 40′.
- Column-V: λ^* (JD) Polar longitude from fractional JD for meridian transit at 36°08N, 28°05E for 24 March 130 BC, as described in Section III. Mean sun for 12:00 noon is 358°.25′.
- Column-VI: λ^* Computed from Right Ascension as $\lambda^* = \mathrm{Tan}^{-1}$ (Tan 15α /Cos ω).