# INTRODUCTION

## BHĀSKARĀCĀRYA II - AUTHOR OF KARAṆAKUTŪHALAM

येषां सुजातिगुणवर्ग विभूषिताङ्गी
शुद्धाखिलव्यवहृतिः खलु कण्ठसक्ता ।
लीलावतीहसरसोक्तिमुदाहरन्ती
तेषां सदैवसुखसम्पदुपैतिवृद्धिम् ॥

"There is always happiness, wealth and prosperity to those around whose neck a chaste and pure lady, Līlāvatī, belonging to a respectable family, endowed with good virtues, throws her arms" - Līlāvatī of Bhāskara II.

This is the śloka with which Bhāskarācārya, the most popular among Indian mathematicians and astronomers, concludes his popular text Līlāvatī. But then anyone wonders how this stanza, glorifying pleasure and fortune of one who is blessed with the grace of a beautiful and virtuous lady, be related to mathematics. This śloka is a double entendre (śleṣa) having another meaning : "Joy and happiness are indeed ever increasing in this world for those who have the text of Līlāvatī clasped to their throats (i.e., mastered by them) decorated as the members are with (the mathematical topics) of neat reduction of factors, multiplication and involution, pure and perfect as are the solutions (of the problems), and tasteful as is the speech which is with examples".

Bhāskara's greatness lies in making mathematics highly irresistible and attractive. Our celebrated mathematician Bhāskarācārya, of the twelfth century, is generally referred to as Bhāskara II to distinguish him from his namesake of the sixth century.

**1. Bhāskara's time and works**

According to Bhāskara's own statement, he belonged to Vijjaḍa Viḍa (or Bijjaḍa Biḍa) near the line of Sahyādri mountains. He was born in 1114 AD. Bhākara's father was Maheśvara, a saintly and scholarly person belonging to the Śāṇḍilya gotra.

The place Vijjaḍa Viḍa is identified with modern Bijapur belonging to Karnataka. However, some scholars have identified the name with other places in Maharashtra.

Bhāskara's celebrated work, Siddhānta Śiromaṇi consists of four parts namely, Līlāvatī, Bījagaṇitam, Grahagaṇitam and Golādhyāya. The first two, generally treated as independent texts, deal exclusively with mathematics and the last two with astronomy.

In 1183 AD, when he was 69 years old, Bhāskara composed another smaller astronomical text called Karaṇa Kutūhalam.

Bhāskara has given the ayanāṃśa (the amount of the precession of the equinoxes) as $11^\circ$ for the śaka year 1105 (i.e., 1183 AD) when he composed his karaṇa work (handbook).

A stone inscription was discovered at a place called Pāṭan, about 10 miles southwest of Chalisgaon in Maharashtra. According to that inscription (see Epigraphia Indica, Vol. I, pp. 340), Changadeva, a grandson of Bhāskara II, was an astronomer at the court of King Singhana of the Yadava dynasty. King Singhana ruled at Devagiri from śaka 1132 to 1159 (i.e., 1210 to 1237 AD). Changadeva built a monastery at Pāṭan for propagating the works of Bhāskarācārya and his descendents. King Saideva of Nikhumbha dynasty made an endowment for the maintenance of the monastery in śaka 1129 (i.e., 1207 AD).

Changadeva, in his inscription, states that King Jaitrapāla invited Lakṣmīdhara, son of Bhāskarācārya from the town Pāṭan.

Līlāvatī is an extremely popular text dealing with arithmetic, elementary algebra, geometry and mensuration. Bījagaṇitam is a treatise on advanced algebra.

Grahagaṇitam and Golādhyāya are completely devoted to computations of planetary motions, eclipses etc., and rationales of spherical astronomy.

Bhāskara has condensed in his mathematical texts the remarkable contributions of his predecessors, Āryabhaṭa, Brahmagupta, Śrīdhara and Padmanābha. Although Bhāskara does not mention the ninth century Karnataka mathematician Mahāvīra's name, he seems to be greatly influenced by the latter's work. Many examples given by Bhāskara greatly resemble similar examples given in the Gaṇita Sāra Saṅgraha of Mahāvīra. The coincidence cannot be just accidental especially since Mahāvīra preceded Bhāskara by nearly three centuries and hailed from almost the same region.

Bhāskara has written a detailed commentary on the Siddhānta Śiromaṇi and it is called Vāsanā Bhāṣya. In this commentary very interesting and illustrative examples are worked out.

Bhāskara mentions that his birth took place in the śaka year 1036 (i.e., 1114 AD) and that he composed his Śiddhānta Śiromaṇi when he was 36 years old (i.e., in 1150 AD)

In the Grahagaṇitam part, Bhāskara has extensively dealt with the determination of mean and true positions of planets, the three problems ("tripraśna") relating to time, direction and place, the lunar and solar eclipses, risings and settings and conjunctions of the planets.

The chapter on spherical astronomy, Golādhyāya, is very important from the point of view of theoretical astronomy. Rationales for the formulae used are provided. The eccentric and epicyclic theories for the motions of planets, as theoretical bases, are clearly developed.

An account of the large number of astronomical instruments is given in Yantrādhyāya. Bhāskara greatly improved upon the formulae and methods adopted by earlier Indian astronomers.

In the Karaṇa Kutūhalam, Bhāskara has adopted as epoch, for computations, the sunrise of February 24, 1183 AD (Julian), Thursday. This tract is also well-known as Grahāgama Kutūhalam. Some almanac-makers are using this text even now for their computations. In fact a voluminous work called Jagaccandrikā Sāraṇī consists of ready-to-use tables based on Bhāskara's tract. The text of Karaṇa Kutūhalam consists of 139 ślokas.

The work Siddhānta Śiromaṇi of Bhāskara has attained the highest degree of excellence, among the Indian astronomical treatises, mainly because of various simplified methods and rationales for the underlying theories. This lucid but detailed treatment extends starting from the computation of ahargaṇa upto abstruse questions like those of parallax and the sine tables. Among all the siddhāntic texts, Bhāskara's Siddhānta Śiromaṇi merits as the best and exhausive text for understanding Indian astronomy.

**2. Corrections to mean positions of planets**

In finding the true positions of planets the earlier astronomers had recognised the following important corrections to be applied to the mean positions :

(i) Deśāntara samskāra due to the difference in longitudes of the given place and the central meridian (Ujjayinī); there is difference in the timings of the sunrise on the same day at places with different longitudes.

(ii) Cara samskāra due to the difference between the latitude of the given place and the latitude of Laṅkā (i.e., the equator).

(iii) Bhujāntara samskāra - When the above two corrections are effected, we get the mean position of a planet at the mean midnight of the given place. But the true midnight at the place differs from the mean midnight by what is called "equation of time". This equation of time is made up of two constituents - one due to the eccentricity of the earth's orbit and the other due to the obliquity of the ecliptic with the celestial equator.

(iv) Udayāntara samskāra - As pointed out earlier, this is the correction to get the positions of planets at the true midnight or sunrise caused by the inclination (obliquity) of the ecliptic with the celestial equator.

Although it was astronomer Śrīpati (1025 AD) who gave this udayāntara correction for the first time and actually called it "yātāsava", Bhāskara later provided the rationale for this additional correction.

**3. Moon's equations**

After obtaining the mean longitude of the Moon, some important equations have to be applied for securing the true position. Among hundreds of such corrections to be applied to the mean position, the following are the three most important equations. Their approximate coefficients are also given according to modern astronomy (see Brown's Lunar Theory).

(i) Equation of Centre (Mandaphala) :

$$\text{Equation of centre} = \left(2e - \frac{1}{4}e^3\right) \sin \varphi$$

$$= (377' \ 19.06'') \sin \varphi$$

where e is the eccentricity of the Moon's elliptic orbit and $\varphi$ is the Moon's mean anomaly given by

$$\varphi = (\text{Moon's mean long.} - \text{perigee})$$

Note : In Indian astronomy, instead of perigee, its opposite point, apogee is considered.

(ii) Evection :

$$\text{Evection} = \frac{15}{4}\, me \sin\, (2\xi - \varphi)$$

$$= (76'\ 26'')\sin\,(2\xi - \varphi)$$

where $\xi$ is the elongation of the Moon from the Sun

i.e., $\xi$ = (Mean long. of the Moon – Mean long. of the Sun) and m is the ratio of the mean daily motions of the Sun and the Moon.

(iii) Variation :

$$\text{Variation} = \left[\frac{11}{8}\, m^2 + \frac{59}{12}\, m^3 + \frac{893}{72}\, m^4\right]\sin\,(2\xi)$$

$$= (39'\ 30'')\sin\,(2\xi)$$

Thus, considering only these three important equations of the Moon, the (approximate) true longitude ($\lambda$) of the Moon is given by

$$\lambda = L + 377' \sin\varphi + 76' \sin\,(2\xi - \varphi) + 39'.5 \sin(2\xi)$$

where L is the mean longitude of the Moon. In particular, at the syzygies (i.e., the newmoon and fullmoon), $\xi = 0^\circ$ or $\xi = 180^\circ$ in which case the variation term vanishes and the evection term reduces to $-\ 76' \sin\,\varphi$. In that case, the (approximate) true longitude of the Moon at the new or full moon is given by

$$\lambda = L + 377' \sin\,\varphi - 76' \sin\,\varphi$$

$$= L + 301' \sin\,\varphi$$

The equation of centre (mandaphala) was known since even before Āryabhaṭa I (476 AD). In fact, Āryabhaṭa himself gave the coefficient in the correct term as $300' \ 15''$. Brahmagupta in his Uttara Khaṇḍa Khādyaka gives it as $301'.7$.

Actually, the second equation of the Moon viz., evection (combined with a part of the equation of centre) was first given, among the Indian astronomers, by Muñjala (or Mañjula, 932 AD) in his Laghumānasa. Sengupta points out, "In form the equation is most perfect, it is far superior to Ptolemy's; it is above all praise."

Bhāskara II gets the credit of being the first among the Hindu astronomers in introducing the Moon's equation which is now called evection into a siddhāntic text. It is remarkable that Bhāskara's discovery preceded that in the west (by Tycho Brahe) by nearly four centuries.

Apart from the three major equations of the Moon, there is another important fourth equation called, the annual equation. The credit of the discovery of this lunar correction, among Indian astronomers, goes to the Orissa astronomer, Candraśekhara Simha Sāmanta (19th Century). It is noteworthy that Candraśekhara discovered this important correction independently since he was trained in the orthodox Sanskrit style and totally ignorant of English education or the western development of astronomy. In fact, Candraśekhara's fourth equation works out to be

Annual correction = $(11' \ 27''.6)$ sin (Sun's anomaly).

Tycho Brahe took the coefficient wrongly as $4' \ 30''$ while Horrock's (1639 AD) value is $11' \ 51''$.

### 4. Cakravāla method to solve $Nx^2 + 1 = y^2$

Brahmagupta (628 AD) has the unique honour, in the history of world mathematics, of discovering the general method of solving a second-order indeterminate equation, Varga Prakṛti of the form $Nx^2 + 1 = y^2$ by his Bhāvana method.

Bhāskara II improved upon Brahmagupta's method in his Cakravāla (cyclic) method. Bhāskara's method dispenses with the necessity of seeking a trial solution, to start with, for the equation. However, an earlier author Ācārya Jayadeva is known to have discussed the Cakravāla method.

The Cakravāla method is essentially as follows : Suppose

$Nx^2 + K = y^2$ when $K = \pm 1, \ \pm 2$ or $\pm 4$

We can find a and b such that $Na^2 + K = b^2$ for any suitable K. We also have N. $1^2 + (m^2 - N) = m^2$. Applying the Samāsa Bhāvana of Brahmagupta, we readily obtain

$$N\left[\frac{am + b}{K}\right]^2 + \frac{m^2 - N}{K} = \left[\frac{bm + Na}{K}\right]^2 \quad \text{..... (*)}$$

By the kuṭṭaka method, choose m such that $am + b$ is divisible by K, where m is suitably chosen so as to make $(m^2 - N)$ numerically small. Let

$$\frac{am + b}{K} = a_1, \quad \frac{m^2 - N}{K} = K_1 \text{ and } \frac{bm + Na}{K} = b_1.$$

Then, we have

| Bhāskara's theorem 1 : When $a_1$ is an integer, then $b_1$ and $K_1$ are also integers. |
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Equation (*) takes the form $Na_1^2 + K_1 = b_1^2$ where $a_1, K_1, b_1$ are integers. Now using $a_1, b_1, K_1$ instead of a, b, K, the process is repeated. Let the new set of integers thus obtained be $a_2, b_2, K_2$ so that $Na_2^2 + K_2 = b_2^2$. The process is repeated successively.

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| Bhāskara's theorem 2 : After a finite number of repetitions, two integers a and b can be obtained such that $Na^2 + \lambda = b^2$ where $\lambda = \pm 1$ or $\pm 2$ or $\pm 4$ |

Thus, starting with $Na^2 + K = b^2$, where K is any convenient integer, we can arrive at a solution $(\alpha, \beta)$ of the equation

$$Nx^2 + \lambda = y^2$$

where $\lambda$ takes the value 1 or 2 or 4 with either the positive or the negative sign.

Once this solution is obtained, Brahmagupta's usual method will lead to an integral solution of the given equation, $Nx^2 + \lambda = y^2$.

While Bhāskara's first theorem has been proved by Datta and Singh and also by the famous German mathematician Hankel, the proof of Bhāskara's second theorem has been given by A.A. Krishnaswami Ayyangar (see Jour. Ind. Math. Soc. Vol. 18 (First Series), Second part, 232-245).

Krishnaswami Ayyangar has also shown that Bhāskara's Cakravāla method requires less number of steps than the modern Euler-Lagrange method of solving a Varga Prakṛti equation.

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| "It (Bhāskara's **Cakravāla** method) is beyond all praise : It is certainly the finest thing achieved in the theory of numbers before Lagrange".<br>- Hankel, the famous German mathematician. |

Considering the equation $61x^2 + 1 = y^2$, as an example, Bhāskara II obtains the solution :

$$x = 226\ 153\ 980, \quad y = 1766\ 319\ 049$$

In fact, these are the least non-trivial integral values of x and y (having 9 and 10 digits respectively) satisfying the equation $61x^2 + 1 = y^2$.

Note : There is an interesting history behind this very particular equation, $61x^2 + 1 = y^2$. The famous French mathematician, Fermat, in 1657 AD, proposed the above equation for solution, as a challenge, to Frenicle and other fellow-mathematicians. None of them succeeded in solving the equation in integers. But the very same equation, though coincidentally, was completely solved by Bhāskara II about five hundred years earlier.

**5. Bhāskara II on differentials**

Bhāskara II introduces the concept of instantaneous motion (tātkālika gati) of a planet in the chapter on true positions of planets (Spaṣṭādhikāra) of his Siddhānta śiromaṇi. He clearly distinguishes between sthūla gati (gross or average velocity) and sūkṣma gati (accurate velocity) in terms of differentials.

If y and $y'$ are the mean anomalies of a planet at the ends of consecutive intervals, then according to Bhāskara,

$$\sin y' - \sin y = (y' - y) \cos y$$

which is equivalent to the result (in our modern notation) :

$$d\,(\sin y) = \cos y \; dy$$

In Bhāskara's own words :

बिम्बार्धस्य कोटिज्यागुणस्त्रिज्याहरः फलं दोर्ज्यायोरन्तरम्

"The product of cosine of the semi-diameter by the element of the radius gives the difference of the two sines."

However, much before Bhāskara, nearly two centuries earlier, Mañjula (932 AD) has given the same idea in his Laghumānasam.

Mañjula uses the fact that the tabular difference of sines for an arc are proportional to the cosines.

Bhāskara II goes further to state that the derivative (taken as a ratio of differentials) vanishes at a maxima. He says :

यत्र ग्रहस्य परमंफलं तत्रैवगतिफलाभावेन भवितव्यम्

"Where the planet's motion is maximum, there the fruit of the motion is absent (i.e., stationary)."

**6. Cubic and biquadratic equations**

The solution of cubic and higher order equations was a favourite topic in algebra dealt with by the medieval Indian mathematicians.

Bhāskara II gives the solutions of cubic and biquadratic equations in his Bījagaṇitam :

1. Solve the cubic equation

$$x^3 + 12x = 6x^2 + 35$$

**Solution :** The equation can be written as

$$x^3 - 6x^2 + 12x - 8 = 27$$

or $(x - 2)^3 = 3^3$

so that $x - 2 = 3$

or $x = 5$

This is the only real root.

2. Solve the biquadratic (i.e., fourth degree) equation

$$x^4 - 2x^2 - 400x = 9999$$

**Solution :** Adding $4x^2 + 400x + 1$ to both sides, we get

$$x^4 + 2x^2 + 1 = 4x^2 + 400x + 10{,}000$$

or $(x^2 + 1)^2 = (2x + 100)^2$

or $x^2 + 1 = 2x + 100$

i.e., $x^2 - 2x + 1 = 100$ or $(x - 1)^2 = 100$

so that we get

$$x - 1 = 10$$

or $x = 11$

The other roots, having complex values, are not considered since the idea of complex numbers was introduced many centuries later by the European mathematicians.

### 7. Contents of Karaṇakutūhalam (KK)

This handbook (karaṇa) of Indian astronomy consists of ten chapters. The contents are more or less similar to those of Gaṇeśa Daivajña's *Grahalāghavam* (*GL*, English exposition by S.Balachandra Rao and S.K.Uma, IJHS, Vol. 41, No. 1-4, New Delhi, 2006). The algorithms are briefer. However, in this text unlike GL, the trigonometric ratios jyā and koṭijyā are retained. The present English translation and exposition is based on the printed text of *Karaṇakutūhalam*, with Sumatiharṣa's com., *Gaṇaka kumuda kaumudī* and Sudhakara Dvivedi's com., *Vāsanā vibhūṣaṇa*, ed. Dr. Satyendra Mishra, Published by Krishnadas Academy, Varanasi, 1991.

Chapter 1 (Madhyamādhikāra) explains the procedure of finding the ahargaṇa (the heap of elapsed civil days since the epoch) for a given day according to the lunar calendar. The epoch chosen is the mean sunrise (at Ujjayinī) on Thursday, February 24, 1183 AD (Julian).

Obtaining the mean positions of all the heavenly bodies is explained in this chapter. In finding the deśāntara correction for a planet, its true daily motion is multiplied by the distance of the place (in yojanas) from the central meridian (madhyarekhā) and divided by 80. In obtaining the divisor, it is significant that Bhāskara II takes the circumference of the earth as 4800 yojanas (approxly. 24000 miles). This means that Bhāskara II took the radius of the earth a little less than 3900 miles which is correct as per modern knowledge.

In the Spaṣṭādhikāra (Chapter 2), the method of finding the true positions of the Sun and the Moon by applying the manda samskāra and those of the five planets by the manda and the śīghra samskāras is explained. For this purpose the text has provided tables of mandoccas (apogees), parākhyas and maximum śīghraphalas and mandaphalas. Since the model of epicycles is adopted for the true positions of planets, Bhāskara II has used the peripheries (paridhis) for the manda and the śīghra epicycles as given in his Siddhānta Śiromaṇi.

Towards the end of this chapter, the computations of tithi, nakṣatra, yoga and karaṇa are explained.

In Chapter 3 (Tripraśnādhikāra), problems connected with direction (dik), place (deśa) and time (kāla) are discussed. Related to these major issues, the concepts and computations of lagna (ascendant), krānti (declination), akṣa (latitude of a place), nata (zenith distance) and unnata (altitude) are explained.

Chapters 4 and 5 deal with lunar and solar eclipses respectively. Expressions for śara (latitude of the Moon) and the angular diameters of the Sun, the Moon and the earth's shadow cone are given. Methods of determining the half-durations, khagrāsa, totality etc. are explained.

The methods given in the text are applied to an example and it is shown how the results are close to those given in the modern ephemerides.

The akṣa and āyana valanams are explained and their algebraic sum called spaṣṭa valanam is calculated. The valanam is used in drawing the diagram (parilekha) of the eclipse.

The effects of parallax on the longitude and the latitude of the Moon, respectively called lambana and nati are considered at length. The methods given in the text are applied to the example of the solar eclipse which occurred on August 11, 1999. The beginning and the middle instants of the eclipse differ from the ephemeris values by just about 5 minutes.

Chapter 6 deals with the rising and setting of the heavenly bodies both heliacally and diurnally. In the chapter on Śṛṅgonnati (elevation of the Moon's cusp) the method of finding of the valanam is explained.

In Chapter 8, on Grahayuti, determination of the instant of conjunction (yuti) of two planets is explained. Depending on the true positions of planets, which are likely to be in conjunction, the method of finding the duration from the given instant for the gata (elapsed) or the gamya (to be covered) is discussed in detail.

The 'parallel' aspects of the Sun and the Moon (when their declinations are numerically equal) are called Vyatīpāta and Vaidhṛti. These are of significance in Indian astrology. Computations of these two pāta yogas are explained in detail in Chapter 9.

Possibility of an eclipse and calculation of related parametes at the end of a bright or dark fortnight using the elapsed māsagaṇa (lunar months) are discussed in Chapter 10. Actually this is the concluding chapter. However, there is one more chapter after this, considered as Chapter 11, called Nīradādhyāya. In this chapter method is prescribed for computing the position of the Sun for which rain bearing clouds are supposed to form. However, this topic is considered as a later interpolation.

At the end of the present text, Appendices and a detailed Bibliography are included.

In the course of our exposition of the texts, mathematical derivations, comparison with modern results and illustrative examples, diagrams and tables are provided.

**8. Commentators of Karaṇakutūhalam**

The well-known commentators of Bhāskara's Karaṇakutūhalam are Ekanātha (Brahmatulya bhāṣya, 1370 AD), Padmanābha (Nārmadī, 1400), Viśvanātha (Brahmatulya udāharaṇa), Soḍhala (Karaṇakutūhala ṭīkā, before 1462), Tables Brahmatulya Sāriṇī, Sumatiharṣa gaṇi (Gaṇaka kumuda kaumudī, 1621) and Caṇḍīdāsa (before 1658).

In our present work we have mostly used Sumatiharṣa's commentary especially for examples related to dates in the 17th century.

# CHAPTER 1

# MADHYAMĀDHIKĀRAḤ

## (Mean Positions of Planets)

**Śloka 1 : I, Bhāskara, prostrating before Gaṇeśa, Sarasvatī, Brahmā (Padmajanma), Śiva (Īśa) and the planets led by Sūrya (Bhāskara), enunciate the methods (of computing) planets, in this short work (handbook), which are (giving) results equivalent to the Brahma siddhānta.**

**Ślokas 2 and 3 : In the (desired) *Śaka* (year) subtract 1105, multiply (the remainder) by 12 (giving solar months) and add the elapsed (number of) months (counted) from *Madhu* (*Caitra*); multiply (the sum) by 2 and add 66 (to the product); (keeping the result in two places subtract the (integer) quotient, obtained by dividing the first result by 900, from the second (result); dividing (the remainder) by 65, (thus) obtained *adhi(ka)māsas* (intercalary months) are added to the upper result (i.e. solar months). Multiply (the thus obtained lunar months) by 30 and add (the elapsed) *tithis* and 3. This becomes the *ahargaṇa* starting from Thursday.**

The ahargaṇa (the heap of civil days elapsed) since the chosen epoch is determined for a given date as follows :

(i) Subtract 1105 from the given Śālivāhana śaka (Śā. Śa.) elapsed year.

(ii) Multiply the result of (i) by 12 to obtain the number of elapsed solar months (saura māsas).

(iii) Add the elapsed number of lunar months counting Caitra as 1.

(iv) Multiply the result of (iii) by 2 and add 66 to it.

(v) Divide the result of (iv) by 900.

(vi) Subtract the integer quotient obtained in (v) from the result of (iv).

(vii) Divide the result of (vi) by 65. The integer quotient gives the number of elapsed mean adhikamāsas (intercalary months) since the epoch. If an adhikamāsa is lost or gained in the process then that has to be added or subtracted accordingly to get the true number of adhikamāsas.

(viii) Add the number of true adhikamāsas (vii) to the number of saura māsas (solar months) obtained in (ii) to get the number of elapsed lunar months (cāndra māsas).

(ix) Multiply the result of (viii) by 30 to obtain the number of lunar days (tithis) and add the elapsed tithis, in the given lunar month, counting from pratipat (the first lunar day) of the śukla pakṣa (bright fortnight).

(x) Add 3 to the total elapsed tithis obtained in (ix). This gives the number of Cāndra dinas.

(xi) Divide the result by 703. Add the integer quotient thus obtained to the result of (x).

(xii) Divide the result of (xi) by 64. This gives the kṣaya dinas. Subtract the resulting integer quotient from the result of (ix). This gives the mean ahargaṇa for the given date.

(xiii) Dividing the ahargaṇa obtained in (xii) by 7, if the remainder is 0, the given date is a Thursday; if 1, Friday etc. However, depending on the actual week day (known beforehand), 1 day may have to be added to or subtracted from the result of (xii), so that the obtained weekday coincides with the actual one, to get the true ahargaṇa.

**Note :** (i) The epoch chosen in the Karaṇa kutūhalam is the mean sunrise on February 24, 1183 A.D. (Julian), Thursday at Laṅkā and Ujjayinī on the same meridian.

(ii) Tables for finding the ahargaṇa, from the Karaṇa kutūhalam (KK) epoch, for any date of the Christian era are given in the Appendix.

**Example :** Vikrama Samvat 1676, Śā. Śa. 1541 Jyeṣṭha Kṛṣṇa 14 (caturdaśī), Sunday, corresponding to 12-5-1619 A.D. (G).

We shall determine the ahargaṇa, the number of civil days elapsed since the epoch of KK as explained in steps (i) to (xiii) above.

(i) The number of elapsed solar years, 1541 – 1105 = 436.

(ii) The number of elapsed solar months = $436 \times 12 = 5232$.

(iii) $5232 + 1 = 5233$.

(iv) $5233 \times 2 = 10466$ ; $10466 + 66 = 10532$

(v) Dividing 10532 by 900, the integer quotient = 11.

(vi) $10532 - 11 = 10521$

(vii) Dividing 10521 by 65, the integer quotient is 161.

(viii) The number of lunar months elapsed = 5233 + 161 = 5394.

(ix) $5394 \times 30 = 1{,}61{,}820$ gives the number of elapsed tithis till the beginning of the corrected lunar month.

In the current lunar month of Jyeṣṭhā, the number of elapsed tithis = 15 + 13 = 28, till the beginning of the running Kṛṣṇa Caturdarśī (14).

Adding this to the result of (ix) we get the number of tithis elapsed (since the epoch) = 1,61,820 + 28 = 1,61,848.

(x) 1,61,848 + 3 = 1,61,851.

(xi) Dividing 1,61,848 by 703, we get the integer quotient = 230. Adding this to 1,61,851 [obtained in (x)], we get 1,62,081.

(xii) Dividing 1,62,081 by 64 we get the integer quotient = 2532 (kṣayadina). Subtracting this from the result of (ix), we get 1,61,848 – 2,532 = 1,59,316.

(xiii) Dividing 1,59,316 by 7, the remainder is 3. Counting from Thursday (the weekday of the epoch) as 0, the given date falls on a Sunday (at sunrise) which coincides with the actual weekday. Hence the true ahargaṇa (at the mean sunrise of the given date) is 1,59,316.

**Note :** Ahargaṇa is the basic parameter using which the mean positions of planets etc. are obtained as explained in what follows.

**Ślokas 4, 5, 6 : 10 (*rāśis*), 29(*aṃśas*), 13(*kalās*) form (the *kṣepaka*) of the sun; 10, 29, 5, 50 of the moon; 4, 15, 12, 59 of the moon's *ucca* (i.e. *mandocca*); 9, 17, 25, 9 of the moon's *pāta* (Rāhu); 7, 21, 24, 21 of Kuja; 2, 21, 14, 30 of Budha (*śīghrocca*); 2,4,0,51 of Guru; 8, 18, 5, 55 of *Śukra* (*śīghrocca*); 4, 3,43,17 of Śani. The *kṣepaka,* starting with *rāśī*, when combined with the planet obtained from the *ahargaṇa* becomes the *madhya* (mean) planet at the sunrise in Laṅkā city.**

The mean positions of the Sun, the Moon and the planets at the epoch are as given in **Table 1.1** at the mean sunrise. These are called kṣepakas.

**Śloka 7 : The *ahargaṇa* is multiplied by 13 and divided by 903; this (result considered) as degrees etc. is subtracted from the *dyugaṇa***

**Table 1.1 Mean positions at the epoch**

| | Ravi | Candra | Moon's Mandocca | Moon's Node | Kuja | Budha śīghrocca | Guru | Śukra śīghrocca | Śani |
|---|---|---|---|---|---|---|---|---|---|
| Rāśi | 10 | 10 | 04 | 09 | 07 | 02 | 02 | 08 | 04 |
| Aṃśa (°) | 29 | 29 | 15 | 17 | 21 | 21 | 04 | 18 | 03 |
| Kalā (′) | 13 | 05 | 12 | 25 | 24 | 14 | 00 | 05 | 43 |
| Vikalā (″) | 00 | 50 | 59 | 09 | 21 | 30 | 51 | 55 | 17 |

**(*ahargaṇa*). (The result) reduced by the *abdavṛnda* (the number of years elapsed since *śaka* 1105) divided by 64 in *kalās* (minutes of arc) gives (the mean) Ravi (sun), Jña (Budha, Mercury) and Śukra (Venus).**

The ahargaṇa must be kept in two places. One of them must be multiplied by 13 and divided by 903. The result, considered as amśas etc., must be subtracted from the ahargaṇa kept in the other place taken as amśas. The result gives the ahargaṇa related mean motions of Ravi, Budha and Śukra. This is subject to a correction called abda bīja samskāra : Divide the number of years elapsed since the epochal year (Śā. śa. 1105) by 64. The result obtained is taken as kalās (minutes of arc) etc. and subtracted from the above obtained mean motions to get the corrected ones for Ravi, Budha and Śukra. Add the kṣepaka of the Sun to get the mean positions. If A is the ahargaṇa for a given date, then the mean positions of the Sun, Mercury and Venus (taken as the same) are given by

Mean longitude = $(A - 13A/903)\,\text{deg.} + K$

$$= (1 - 13/903\,)\,A^{\circ} + K = \frac{890}{903}\,A^{\circ} + K$$

where the kṣepaka $K = 10^{R}\,29^{\circ}\,13'$. This means that KK has adopted the mean daily motion of the Sun as $890/903$ deg. i.e., $0^{\circ}\,59'\,08''\,10'''.365432$ .

Further, if Y is the Śā. śa. year then the abda bīja correction is to subtract $(Y-1105)/64$ minutes of arc from the above obtained mean positions. Thus, we have corrected mean position of Sun = $(A-13A/903)^{\circ}+K-(Y-1105)/64'$.

**Example :** In the example under Ślokas 2 and 3, the ahargaṇa $A=1{,}59{,}316$. Now, ahargaṇa related motion is

(i) $A^{\circ}-\dfrac{13A^{\circ}}{903}$ = $1{,}59{,}316^{\circ}-2{,}293^{\circ}\,35'\,09''=$

(ii) Abda bīja correction = $(1541-1105)/64'\approx 06'49''$ noting that the given Śā. śa. year $Y=1541$.

Now, subtracting the result of (ii) from that of (i), we get the bīja corrected motion of the Sun :

(iii) $1,57,022^{\circ}\,24'\,51''-06'\,49''$ = $1,57,022^{\circ}\,18'\,02''$. Dividing by $30^{\circ}$ the value in the degrees place, we get $5234^{R}\;2^{\circ}\,18'\,04''$. Dividing the number in the rāśis ($^{R}$) place viz. $5234^{R}$ by $12^{R}$ we get 436 as the completed number of revolutions (bhagaṇas) and the balance is $2^{R}\;2^{\circ}\,18'\,04''$.

(iv) Adding the Sun's kṣepaka = $10^{R}\;29^{\circ}\,13'$ to the ahargaṇa related corrected motion of the Sun obtained in (iii), we get

Mean Sun = $2^{R}\;2^{\circ}\,18'\,04''+10^{R}\;29^{\circ}\,13'$ = $1^{R}\;01^{\circ}\,31'\,04''$ at the mean sunrise in Laṅkā. This is taken as the mean longitudes of Budha and Śukra.

**Note** : In step (iii) above, if the value in the degrees place of the ahargaṇa related bīja corrected motion of the Sun is greater than (or equal to) $360^{\circ}$, then completed revolutions (bhagaṇas) can be obtained directly also by dividing the value in the degrees place by $360^{\circ}$ and retaining the balance i.e., by removing the nearest multiples of $360^{\circ}$.

**Śloka 8 : The *ahargaṇa* multiplied by 14 and reduced by the 17th part (of the product) and (when) subtracted the division of the *ahargaṇa* by 8600 is the (mean) moon (Candra) in degrees.**

The mean position of the Moon is obtained as follows :

(i) Multiply the ahargaṇa by 14 and consider the result as amśas (degrees).

(ii) Divide the result of (i) by 17 and subtract the quotient (in degrees etc.) from the result of (i).

(iii) Divide the ahargaṇa by 8600 and subtract the quotient (in degrees etc.) from the result of (ii).

(iv) Remove the nearest integral multiples of $360^\circ$ (i.e., completed revolutions) from the result of (iii).

(v) Add the kṣepaka of the Moon, $10^R\ 29^\circ\ 05'\ 50''$ to the result of (iv).

This gives the mean position of the Moon at the mean sunrise in Laṅkā for the given date (having the ahargaṇa A).

If A is the ahargaṇa, then

$$\text{Mean Moon} = 14A^\circ - \frac{14A^\circ}{17} - \frac{A^\circ}{8600} + K$$

where the kṣepaka, $K = 10^R\ 29^\circ\ 05'\ 50''$ for the Moon. This means that KK has taken the mean daily motion of the Moon as $13^\circ\ 10'\ 34''\ 52'''.534$.

**Example :** For A = 1,59,316, we get

Mean Moon = $13^0 09' 37''.23$

**Śloka 9 : The *ahargaṇa* is kept in two places. The sum of (the results of) the one (in the first place) divided by 9 and the other (in the second place) divided by 4012 is the (*manda*-) *ucca* of the moon.**

**The *ahargaṇa* is kept in two places. The sum of (the results) of the one (in the first place) divided by 19 and the other (in the second place) divided by 2700 becomes the moon's node (*indupāta*).**

(i) The mandocca (apogee) of the Moon is obtained as follows :

Keep the ahargaṇa in two places. Divide one of them by 9 and the other by 4012. Consider both the results as amśas (degrees) etc. and add them. Removing the completed revolutions (i.e., integral multiples of $360^\circ$) and then adding the kṣepaka $4^R\ 15^\circ\ 12'\ 59''$ to the result, the mean position of the Moon's mandocca is obtained.

Thus, for the ahargaṇa A, we have

Mandocca of the Moon = $\dfrac{A^\circ}{9} + \dfrac{A^\circ}{4012} + K$

where $K = 4^R\ 15^\circ\ 12'\ 59''$. Here, the KK has taken the mean daily motion of the Moon's mandocca as $0^\circ\ 6'\ 40''\ 53'''.838$.

(ii) The Moon's pāta (i.e., node) is obtained as follows :

Keep the ahargaṇa in two places. Divide one of them by 19 and the other by 2700 and then consider both results as amśas (degrees) etc. To the thus obtained sum adding the kṣepaka of the pāta, we obtain the mean position of the pāta. That is, for the ahargaṇa A, we have

Moon's Pāta = $\dfrac{A^\circ}{19} + \dfrac{A^\circ}{2700} + K$

where $K = 9^R\ 17^\circ\ 25'\ 09''$. The KK has taken the mean daily motion of Moon's Pāta (node) as $0^\circ\ 03'\ 10''\ 48'''.421$.

**Example** : For ahargaṇa $A = 1{,}59{,}316$, we get

(i) Moon's mandocca = $\dfrac{159316^\circ}{9} + \dfrac{159316^\circ}{4012} + 4^R\ 15^\circ\ 12'\ 59''$

$= 17741^\circ\ 29'\ 15'' + 135^\circ\ 12'\ 59'' = 7^R\ 26^\circ\ 42'\ 14''$

(removing the completed revolutions viz. 49).

(ii) Moon's Pāta = $\dfrac{159316^\circ}{19} + \dfrac{159316^\circ}{2700} + 287^\circ\ 25'\ 09''$

$= 24^{rev.}\ 3^R\ 01^\circ\ 18'\ 39''$

Removing the completed revolutions 24, we get Pāta = $3^R\ 01^\circ\ 18'\ 39''$.

**Śloka 10 : The *ahargaṇa* is kept in two places. The sum of (the results of) the one (in the first place) multiplied by 11 and divided by 21 and the other (in the second place) divided by 52444 is the mean Kuja (the son of the Earth, Mars).**

**The *ahargaṇa* multiplied by 4 added with 43rd part (of the product) and reduced by 1421st part of the *ahargaṇa* becomes the *śīghrocca* of Budha.**

(i) The mean Kuja (Bhauma) is obtained as follows :

Consider the ahargaṇa in two places. Multiply one of them by 11 and divide by 21. That in the second place is divided by 52444. The two results, considered as amśas (degrees) etc. are added. By adding Kuja's kṣepaka $7^R\ 21^\circ\ 14'\ 21''$ to this sum we obtain the mean position of Kuja.

Thus, for ahargaṇa A, we have

Mean Kuja = $\dfrac{11A^\circ}{21} + \dfrac{A^\circ}{52444} + K$

where $K = 7^R\ 21^\circ\ 14'\ 21''$. The mean daily motion of Kuja is taken as $0^\circ\ 31'\ 25''\ 42'''.857$.

(ii) The position of Budha's śīghrocca is obtained as follows :

Keep 4 times the ahargaṇa in two places. Divide one of them by 43 and add the results, considered as amśas (degrees) etc., From the result thus obtained again subtract the quotient got by dividing the ahargaṇa by 1421 in amśas etc. Adding to the result the kṣepaka we get Budha's śīghrocca.

For ahargaṇa A, we have

Bhudha's śīghrocca = $4A^{\circ} + \dfrac{4A^{\circ}}{43} - \dfrac{A^{\circ}}{1421} + K$

where kṣepaka $K = 2^{R}\ 21^{\circ}\ 14'\ 30''$ for Budha śīghrocca. The mean daily motion of Budha's śīghrocca is taken as $245'\ 32''\ 21'''$

**Example :** For $A = 1{,}59{,}316$ we have

(i) Mean Kuja = $\dfrac{11}{21}(159316)^{\circ} + \dfrac{159316^{\circ}}{52444} + 231^{\circ}\ 14'\ 21''$

$= 232^{rev.}\ 6^{R}\ 16^{\circ}\ 03'\ 36''$

Removing the completed revolutions 232, we have

Mean Kuja =

(ii) Budha's śīghrocca

$= 4(159316)^{\circ} + \dfrac{4}{43}(159316)^{\circ} - \dfrac{159316^{\circ}}{1421} + 81^{\circ}\ 14'\ 30'' = 3^{R}\ 03^{\circ}\ 13'\ 09''$

(removing the completed revolutions 1811).

**Ślokas 11 and 12 (first half) : The *ahargaṇa* is kept in two places. The one (in the first place) is divided by 12 and the other (in the second place) by 4227. The difference in their results (the latter subtracted from the former) is the (mean) Guru (the minister of Indra).**

**The *ahargaṇa,* multiplied by 16, is kept in two places. The one (in the first pace) divided by 7451 which is added to the other (in the second place) divided by 10, is *aṃśas* (degrees) etc. gives the (śī*ghra*) *ucca* of Śukra (the preceptor of demons).**

(i) Guru's (Indra mantrī) mean position is determined as follows :

Keep the ahargaṇa in two places. Divide one of them by 12 and the other one by 4227. Considering them as amśas (degrees) etc., subtract the latter from the former. Adding to this difference the kṣepaka of Guru (and removing the completed revolutions), we get the mean position of Guru.

Thus, for ahargaṇa A, we have

Mean Guru = $\frac{A^\circ}{12} - \frac{A^\circ}{4227} + K$

where kṣepaka $K = 64^\circ\ 0'\ 51''$ Here, Guru's mean daily motion is taken as $0^\circ\ 5'\ 0''\ 51'''.1$.

(ii) Śukra's śīghrocca is obtained as follows :

Multiply the ahargaṇa by 16 and keep the product in two places. Divide one of them by 7451 and the other one by 10. Add the two quotients taking them as amśas (degrees) etc. Add to this sum the kṣepaka $8^R\ 18^\circ\ 5'\ 55''$. Removing the completed revolutions, we get Śukra's śīghrocca.

Thus, for ahargaṇa A, we have

Śukra's śīghrocca = $\frac{16A^\circ}{7451} + \frac{16A^\circ}{10} + K$ where K = $258^\circ\ 5'\ 55''$

The mean daily motion of Śukra śīghrocca is taken as $96'\ 7''\ 43'''.83$.

**Example :** For ahargaṇa A = 159316,

(i) Mean Guru = $\frac{159316}{12} - \frac{159316^\circ}{4227} + 64^\circ\ 0'\ 51''$ = $11^R\ 12^\circ\ 39'\ 27''$

removing the completed revolutions 36.

(ii) Śukra's śīghrocca = $\frac{16}{7451}(159316)^\circ + \frac{16}{10}(159316)^\circ + 258^\circ\ 5'\ 55''$

= $8^R\ 25^\circ\ 48'\ 28''$

removing the completed revolutions 709.

**Śloka 12 (second half) : The *ahargaṇa,* kept in two places, divided (respectively) by 30 and 9367 and added together is the (mean) *Śani* (the son of *Arka,* the Sun).**

The mean position of Śani is obtained as follows :

Keep the ahargaṇa in two places. Divide one of them by 30 and the other by 9367. Add the two results as amśas (degrees). Expressing this sum as revolutions, rāśis, amśas etc. and adding Śani's kṣepaka, we get the mean position of Śani (for the given day at the mean sunrise).

For ahargaṇa A, we have

$$\text{Mean Śani} = \frac{A}{30}^{\circ} + \frac{A}{9367}^{\circ} + K .$$

Here, kṣepaka K = $4^{R}\ 3^{\circ}\ 43'\ 17'' = 123^{\circ}\ 43'\ 17''$.

Śani's mean daily motion is taken as $0^{\circ}\ 2'\ 0''\ 23'''.06$.

**Example :** For $A = 159316$, we have

$$\text{Mean Śani} = \frac{159316^{\circ}}{30} + \frac{159316^{\circ}}{9367} + 123^{\circ}\ 43'\ 17'' = 1^{R}\ 21^{\circ}\ 15'\ 46''$$

removing the completed revolutions 15.

**Śloka 13 : The (mean daily) rate of motion of Ravi is 59|8 *kalās*, of Śaśī (Candra) 790|35 *kalās,* of Candrocca (moon's apogee) 6|41 *kalās,*of Candrapāta (moon's node, Rāhu) 3|11 *kalās*, of Kuja 31|26 *kalās,* of Budha *śīghra* 245|32 *kalās,* of Guru 5|0 *kalās*, of Śukra *śīghrocca* 96|8 *kalās,* (and) of Śani 2 *kalās.***

The mean daily motions of Ravi, Candra etc. are as shown in **Table 1.2.**

**Table 1.2 Mean daily motions**

| | Ravi | Candra | Moon's Mandocca | Moon's Node | Kuja | Budha śīghrocca | Guru | Śukra śīghrocca | Śani |
|---|---|---|---|---|---|---|---|---|---|
| Kalā (') | 59 | 790 | 6 | 3 | 31 | 245 | 5 | 96 | 2 |
| Vikalā ('') | 08 | 35 | 41 | 11 | 26 | 32 | 0 | 08 | 0 |

**Śloka 14 : Madhyarekhā (terrestrial meridian) is the line joining Laṅkā (the city of demons) and the Meru (north pole). This line passes through Kanyākumārī, Kāñcī, Sita parvata, Vatsa, Ujjayinī, Gargarāṭa and Kurukṣetra in between.**

**Śloka 15 : [To get the deśāntara samskāra (correction)] the distance of the given place from the madhyarekhā (central meridian) in yojanas is multiplied by the daily motion of a planet and divided by 80. The result is in vikalās (seconds of arc) and it is additive (dhana) or subtractive (ṛṇa) according as the place is to the west or east of the madhyarekhā.**

**Example :** The distance of Śivapurī is 30 yojanas from Gargarāṭa (lying on the madhyarekhā). The Sun's (mean) daily motion is $59'\,08''$. Multiply these and dividing by 80, we get

$$\frac{30\times 59'\,08''}{80} = 22.175 \text{ vikalās} \approx 22''$$

Since Śivapurī lies to the west of the madhyarekhā, the deśāntara correction $22''$ is positive.

**Remark :** The radius of the earth $r \approx 4000$ miles $\approx 800$ yojanas

(taking 1 yojana $\approx 5$ miles).

$\therefore$ Earth's circumference = $2\pi r \approx 4800$ *yojanas* (as taken by Bhāskara II)

To cover 4800 yojanas, the earth takes 60 ghaṭīs (in 1 rotation). Therefore, for 1 yojana, the

earth takes $\frac{60}{4800} = \frac{1}{80}$ g<u>h</u>.

**Śloka 16 : The *abdas* divided by 78 and 63 are subtractive (*bījas,* corrections) in *vikalās* (respectively) of Candra and Guru. Then, (the *abdas* divided respectively) by 13,30,22 and 9 are additive in *vikalās* (corrections) of (Candra's) *pāta* (node), *ucca* (apogee), Budha (*śighrocca*) and Śukra (*śighrocca*).**

To the above obtained mean positions of planets a correction called abda bīja (samskāra) is applied as follows :

From the given Śā.śa. year subtract 1105. This gives the elapsed years (gatābda) since the epochal year.

For Candra, Guru, Candra pāta (node), Candra ucca (Moon's mandocca),

Budha śīghrocca, Śukra śīghrocca, the abda corrections are given by dividing the elapsed years (gatābda) respectively by 78, 63, 13, 30, 22 and 9. These are in vikalās (seconds of arc) among which the first two are negative and the rest positive.

There is no abda corrections for other bodies viz., Sun, Kuja and Śani.

**Example :** In the example considered earlier,

Gatābda (elapsed years) = $1541 - 1105 = 436$.

The corrections for the different bodies are as follows :

(i) Candra : $-436/78 \approx -5''$

(ii) Guru : $-436/63 \approx -6''$

(iii) Pāta (Rāhu) : $436/13 \approx 33''$

(iv) Candrocca : $436/30 \approx 14''$

(v) Budha śīghrocca : $436/22 \approx 19''$

(vi) Śukra śīghrocca : $436/9 \approx 48''$

# CHAPTER 2

# SPAṢṬĀDHIKĀRAH

### (True Positions of Planets)

For finding the true positions of planets from their mean positions, obtained in Chapter 1, we have to apply two important corrections viz. mandaphala and śīghraphala.

**Śloka 1 : The *mandocca* (apogee) of the sun is 78$^0$. (Those of) of Kuja etc. are respectively 128½, 225, 172½, 81 (and) 261 (degrees).**

Mandoccas of the planets are given in **Table 2.1.**

| | **Table 2.1 Mandoccas of planets** | | | | | |
|---|---|---|---|---|---|---|
| | **Sun Ravi** | **Mars Kuja** | **Mercury Budha** | **Jupiter Guru** | **Venus Śukra** | **Saturn Śani** |
| Rāśi | 2 | 4 | 7 | 5 | 2 | 8 |
| Aṃśa | 18 | 8 | 15 | 22 | 21 | 21 |
| Kalā | 0 | 30 | 0 | 30 | 0 | 0 |

**Śloka 2 : The *parākhyās* of Kuja etc. are 81, 44, 23, 87 (and) 13. Then the mean sun is the *śīghrocca* of Guru, Kuja and Śani.**

The parasaṅkhyā (or parākhya) of each planet is given in the following table, which will be used in finding śīghraphala of the planet.

The śīghrocca of the (superior) five planets viz. Kuja, Guru and Śani is the mean Sun. The śīghroccas of Budha and Śukra are determined as given in Chapter 1, ślokas 10 and 12.

| | Table 2.2 Parākhyas of planets | | | | |
|---|---|---|---|---|---|
| **Planet** | **Kuja** | **Budha** | **Guru** | **Śukra** | **Śani** |
| Rāśi | 2 | 1 | 0 | 2 | 0 |
| Amśa | 21 | 14 | 23 | 27 | 13 |

The parama (maximum) śīghraphalas are given in **Table 2.3.**

| Table 2.3 Maximum Śīghraphalas | | | | |
|---|---|---|---|---|
| **Kuja** | **Budha** | **Guru** | **Śukra** | **Śani** |
| $42^\circ 27'14''$ | $21^\circ 30' 36''$ | $11^\circ 3' 0.23''$ | $46^\circ 28' 7.8''$ | $6^\circ 13' 9.29''$ |

The values for the maximum śīghraphalas in the printed version of Sumatiharṣa's commentary are wrong and hence not shown here. The śīghra equations for all the five planets, as per the Sūrya siddhānta are tabulated for every degree of the śīghra anomaly in the Appendix.

**Śloka 3 : The planet subtracted from the *ucca* of *mṛdu* (*manda*) and the *cañcala* (*śīghra*) are respectively the *mṛdu* (*manda*) and the *cañcala* (*śīghra*) *kendras*. Considering (each group of) three *rāśis* (zodiacal signs) as a quadrant, the *phala* (result) is positive or negative (according as) the quadrant is from *Meṣa or Tulā* ($0^0$ or $180^0$).**

This śloka tells about the mandakendra and the śīghrakendra of planets and positivity and negativity of manda and śīghra phalas.

(i) Mandakendra = Mandocca – Mean planet

(ii) Śīghrakendra = Śīghrocca – Mean planet

If the kendra (manda or śīghra) is within six rāśis from Meṣa (i.e., $0^\circ < Kendra < 180^\circ$), then the phala is positive. If the Kendra is within six rāśis from Tulā (i.e., $180^\circ < Kendra < 360^\circ$), the phala is negative.

**Śloka 4 : If the *kendra* is less than three *rāśis* ($90^0$), that is (itself) the *bhuja*; if greater than three *rāśīs* (i.e. in II quadrant) then the *kendra* reduced from the half-zodiac ($180^0$) is the *bhuja*; if the (*kendra*) exceeds half the zodiac (greater than $180^0$ i.e. in III quadrant)then reduce (the *kendra*) by the half-zodiac ($180^0$). If (the *kendra*) is greater than nine *rāśis* (in IV quadrant), then, the *kendra* reduced from twelve *rāśis* ($360^0$) is the *bhuja*. The *bhuja* reduced from three *rāśis* is the *koṭi*.**

This śloka gives the Bhuja and Koṭi of Kendra as follows :

(i) Bhuja = kendra if $kendra < 90^\circ$

(ii) Bhuja = $180^\circ - kendra$ if $90^\circ < kendra < 180^\circ$

(iii) Bhuja = $kendra - 180^\circ$ if $180^\circ < kendra < 270^\circ$

(iv) Bhuja = $360^\circ - kendra$ if $270^\circ < kendra < 360^\circ$

(v) Koṭi = $90^\circ - Bhuja$ (in all cases).

**Śloka 5 and 6 (first half) : The lesser of the elapsed (*bhukta*) and the balance (to covered, *bhogya*) in *liptas* (minutes of arc) in the quadrant of Kuja's *śīghakendra* is divided by 400. This, taken as *amśas*, is subtracted from or added to (the *mandocca*), according as the (*śīghra*) *kendra* is in the six *rāśis* from Karka (i.e. II and III quadrants) or *Makara* (i.e. IV and I quadrants) is the *spaṣṭa* (corrected) *mandocca* (apogee) of Kuja. One third of the obtained *amśas* (degrees) reduced (from the *parākhya*) is the *spaṣṭa* (corrected) *parākhya* of the earth's son (i.e. Kuja).**

Obtaining the corrected (spaṣṭa) mandocca of Kuja is explained.

Find the quadrant (pāda) in which the śīghrakendra of Kuja lies. In that quadrant, subtract the bhukta (elapsed) rāśi etc. (of the śīghrakendra) from 90°. This gives the bhogya (to be covered) rāśi etc. in that quadrant. Between the bhukta (elapsed) and bhogya rāśis etc., find the lesser one (alpa) and convert it into kalās (minutes of arc). Divide these kalās by 400. The result will be in amśa etc. This has to added to or subtracted from the mandocca of Kuja according as the śīghrakendra is six rāśis from Makara (in IV or I quadrant) or in the six rāśis from Karka (i.e., II or III quadrant). One third of the amśas etc. obtained earlier must be subtracted from the parākhya.

**Ślokas 6 (second half) and 7** : **The blocks of *R* sine values (*jyākhaṇḍas*) are 21, 20, 19, 17, 15, 12, 9, 5 and 2. The *degrees* (*amśas* of the *kendra bhuja*) divided by 10 is the (integer quotient) giving the elapsed block (*bhukta khaṇḍa*). The remainder (in degrees etc.) is multiplied by the block to be covered (*bhogya khaṇḍa*) is divided by 10 and what is obtained is added to the sum of the elapsed blocks (*khaṇḍa*) to get the *jyā* (*R* sine).**

For finding jyā of a planet's kendra the jyākhaṇḍas are given in these ślokas as listed below.

**Table 2.4 Jyākhaṇḍas** (R sine tables; R = 120)

| Aṅkas | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Differences | 21 | 20 | 19 | 17 | 15 | 12 | 9 | 5 | 2 |
| Jyākhaṇḍa | 21 | 41 | 60 | 77 | 92 | 104 | 113 | 118 | 120 |

The bhuja of the kendra is divided by 10. The quotient gives the elapsed number of khaṇḍas. The remainder (in amśas etc.) is multipled by the bhogya khaṇḍa (given in the second row of Table 2.4) and divided by 10. The result is added to the sum of the elapsed (gata) khaṇḍas. This gives the jyā of the given kendra.

**Śloka 8** : **(From the given *R* sine value) by subtracting (successively) the (*R* sine) blocks, the balance is multiplied by 10 and divided by the *aśuddha* (that which cannot be subtracted) block. (The result) is added to 10 times the (serial) number of the (last) *śuddha* (subtractable) block to get the *dhanu* (bow, inverse of *R* sine).**

Finding **dhanu** (the inverse of jyā) is explained.

From the jyā value, subtract the successive jyākhaṇḍas (in the middle row of Table 2.4) until it is possible to do so (these are called śuddha khaṇḍas). The last khaṇḍa which cannot be subtracted is called aśuddha khaṇḍa. The balance (śeṣa) jyā saṅkhyā is multiplied by 10 and divided by the aśuddha khaṇḍa. The result is added to the product of 10 and the śuddha saṅkhyā (above the last śuddha khaṇḍa) to get the dhanu.

**Example :** Consider jyā value = 100.

The nearest sum of the śuddha khaṇḍas to the given jyā value 100 is that of the first five entries in the middle row. i.e., 21 + 20 + 19 + 17 + 15 = 92. Subtracting this from the given value, we have 100 – 92 = 8. Multiplying the above remainder (śeṣa) by 10 and dividing by the aśuddha khaṇḍa 12, we get $8 \times \frac{10}{12} = 6°40'$. Adding this result to the product of 10 and the śuddha saṅkhyā 5 (lying above 15), we get dhanu = $(10 \times 5) + 6°40' = 56°40'$.

**Śloka 9 and 10** : **The *R* sines (*jyā*) of the *manda* of Sun etc. are multiplied by 10 and divided (respectively) by 550, 238, 107, 198, 228, 784 and 157 in degrees are the (*manda phalas*) which are positive or negative based on the (*manda*) anomaly (whether less or greater than $180^0$). This operation (gives) the (fully) corrected Sun and Moon and (gives) the *manda*-corrected other planets.**

Finding the mandaphala of the Sun etc. is explained.

The jyā of the mandakendra of the required planet is found out from Table 2.4. This is multiplied by 10. The result is divided by 550, 238, 107, 198, 228, 784 and 157 respectively for Ravi, Candra, Kuja, Budha, Guru, Śukra and Śani. This gives the mandaphala which is positive or negative according as the mandakendra (MK) is within six rāśis from Meṣa (*i.e.,* $0^\circ < MK < 180^\circ$) or from Tuta (*i.e.,* $180^\circ < MK < 360^\circ$).

In the cases of Ravi and Candra, the **true** positions are obtained thus by applying the mandaphala to the mean positions. For other planets, we get the **manda-sphuṭa** positions.

The maximum (parama) mandaphalas of the planets are as given in **Table 2.5.**

| | **Table 2.5 Parama mandaphalas** | | | | | | |
|---|---|---|---|---|---|---|---|
| | Ravi | Candra | Kuja | Budha | Guru | Śukra | Śani |
| Kalās | 130 | 302 | 672 | 362 | 315 | 110 | 458 |
| Vikalās | 50 | 31 | 54 | 10 | 43 | 0 | 33 |

According to the Siddhānta Śiromaṇi of Bhāskara II, the peripheries of the manda epicycles (taken as fixed), the corresponding maximum (parama) mandaphalas and the denominators (like 550, 238 etc.) are given in **Table 2.6.**

**Explanation :** The mandaphala (MP) of a body is given by

$$MP = \frac{a}{R} \sin m$$

where m = mandakendra (MK), *a* is periphery of the manda epicycle and $R = 360^\circ$, the periphery of the deferent circle.

The peripheries p of the different bodies are listed in **Table 2.7.**

**Table 2.6 Manda paridhis and parama mandaphalas**

| Planets | Manda paridhis | Maximum mandaphala | Denominator |
|---|---|---|---|
| Ravi | 13° 40′ | 2° 10′ 30″.4 | 551.69 |
| Candra | 31° 36′ | 5° 01′ 45″.47 | 238.60 |
| Kuja | 70° | 11° 08′ 27″ | 107.71 |
| Budha | 38° | 6° 02′ 52″.4 | 198.41 |
| Guru | 33° | 5° 15′ 7″.61 | 228.47 |
| Śukra | 11° | 1° 45′ 2″.54 | 685.44 |
| Śani | 50° | 7° 57′ 27″.8 | 150.80 |

**Table 2.7 Peripheries of manda epicycles**

| Bodies | At the end of odd quadrant ( $p_o$ ) | At the end of even quadrants ( $p_e$ ) |
|---|---|---|
| Ravi | 13° 40′ | 14° |
| Candra | 31° 40′ | 32° |
| Kuja | 72° | 75° |
| Budha | 28° | 30° |
| Guru | 32° | 33° |
| Śukra | 11° | 12° |
| Śani | 48° | 49° |

**Note :** The mandaphalas of all heavenly bodies, according to the Śūrya siddhānta for every degree of the manda anomaly are tabulated in the Appendix. The peripheries in **Table 2.7** are according to the Sūrya siddhānta.

Here, the periphery of each body varies with the mandakendra (MK). For example, in the case of Ravi, if m is at the end of an odd quadrant (i.e., $m = 90^\circ$ or $m = 270^\circ$), the periphery (p) is $13^\circ\ 40'$ and at the end of an even quadrant (i.e., $m = 180^\circ$ or $0^\circ$), $p = 14^\circ$. When m has other values, the periphery p is given by

$$p = (p_e - p_o)\ |\sin m|$$

where $p_e$ and $p_o$ are the peripheries at the ends of the even and odd quadrants respectively.

The maximum (parama) mandaphala for the Sun is when $m = 90^\circ$, $a = p_0$

i.e, $(MP)_{\max} = \dfrac{a}{R} = \dfrac{13^\circ\ 40'}{360^\circ} = 0.0379629$ radians $= 2^\circ\ 10'\ 30''$

For the Moon,

$$(MP)_{\max} = \frac{a}{R} = \frac{31^\circ\ 40'}{360^\circ} \text{ rads.} = 5^\circ\ 2'\ 23''.$$

Similarly, the maximum mandaphala for other planets is calculated.

**Example** : Mean Sun = $1^R\ 1^\circ\ 37'\ 51''$, Mandocca of Sun = $2^R\ 18^\circ$

$\therefore$ Mandakendra $MK = 2^R\ 18^\circ - 1^R\ 1^\circ\ 37'\ 51''$

$= 1^R\ 16^\circ\ 22'\ 09'' = 46^\circ\ 22'\ 09''$

Bhuja of $MK = 46^\circ\ 22'\ 09''$ (since $MK < 90^\circ$)

Jyā ($46^\circ\ 22'\ 09''$) = 86.55375

$\therefore$ Mandaphala, $MP = \dfrac{86.55375 \times 10}{550} = 1^\circ\ 34'\ 25''$

Since $MK < 180^\circ$, MP is additive.

$\therefore$ True Sun = Mean Sun + MP

$= 1^R\ 1^\circ\ 37'\ 51'' + 1^\circ\ 34'\ 25'' = 1^R\ 3^\circ\ 12'16''$

The cara correction = $-86'' = -1'\,26''$

$\therefore$ Cara corrected Sun = $1^R\ 3^\circ\ 12'16'' - 1'\,26'' = 1^R\ 3^\circ\ 10'\,50''$.

(The cara samskāra is explained in Ślokas 19 and 20).

**Śloka 11 (first half) : The (*manda*) *phala* of Sun divided by 27 is applied to the mean Moon, additive or subtractive as is the case of the (*phala*) of Sun.**

The bhujāntara correction for the Moon is given by $\frac{1}{27}^{th}$ of the Sun's equation of centre and is additive or subtractive according as the latter is. This is combined with the mean Moon.

**Explanation :** The (true midnight and hence) the true sunrise differs from the mean by an amount of time called the 'equation of time'. This is caused by

(i) the eccentricity of the earth's orbit and

(ii) the obliquity of the ecliptic with the celestial equator.

The correction to the longitude of a planet due to the part (i) of the equation of time caused by the eccentricity of the earth's orbit is called the bhujāntara correction. The correction due to (ii) is called the udayāntara correction. We have for the Moon,

Bhujāntara correction = (Equation of centre of the Sun in deg.)

$\times$ Daily motion of the Moon in minutes $/21600$

= (Eqn. of centre of the Sun in deg.) $\times\ 790'.58112/21600$

$\approx Eqn.\,of\ centre/27$ (in degrees)

**Ślokas 11 (second half) and 12 : The *bhogya khaṇḍa* (to be covered block of *R* sine values) of Sun divided by 9, (that) of the Moon multiplied by 13 and divided by 4, (those) of Kuja and Budha multiplied by 2 and divided by 7, (those) of Guru etc. divided (respectively) by 50, 12 and 120 are the *gatiphalams* (motion results to be applied to the mean rates of motion), it (each of the motion results) is positive or negative according as the *manda kendra* (apsidal anomaly) is from *Karka* or *Makara* (i.e. from $90^0$ or $270^0$).**

The true daily motions of the Sun and the Moon and the manda corrected daily motions of other planets are explained.

For the planet whose manda correced daily motion is required, the bhuja of its mandakendra (MK) is consdered. Find the bhogya (to be covered) khaṇḍa for that bhuja from the jyā table (Table 2.4).

In the case of the Sun, divide this bhogya khaṇḍa by 9 which gives its gatiphalam. This has to be added to or subtracted from the mean daily motion of the Sun to get the true daily motion. The gatiphalam is additive or subtractive according to MK is in IV and I quadrants or in II and III quadrants.

For other heavenly bodies, the gatiphalam is obtained as follows :

| | | |
|---|---|---|
| Candra | : | Bhogyakhaṇḍa × 13/4 |
| Kuja and Budha | : | Bhogyakhaṇḍa × 2/7 |
| Guru | : | Bhogyakhaṇḍa / 50 |
| Śukra | : | Bhogyakhaṇḍa / 12 |
| Śani | : | Bhogyakhaṇḍa / 120 |

These are in kalās (minutes of arc).

**Remark :** We provide the rationale for the method of obtaining the true daily motions of the Sun and the Moon given in the above śloka.

**(i) True daily motion of the Sun**

If n is the mean daily motion and $\Delta n$ be the correction to the same to get the true daily motion, then we have

$$\Delta n = \frac{a}{R} \cos m \, (\Delta m)$$

where $a = 13^\circ \, 40' = 13 \frac{2^\circ}{3} = \frac{41^\circ}{3}$, $R = 360^\circ$ and $\Delta m$ = mean daily motion of the Sun $\approx 60'$

$$\therefore \quad \Delta n = \frac{41}{3 \times 360} \times 60' \times \cos m \text{ radian} = \frac{41}{3 \times 360} \times \frac{180}{\pi} \times 60 \times \cos m$$

degrees

$$= \frac{41}{6\pi} \times 60 \cos m = \frac{10 \times 41 \times 60}{10 \times 6\pi} \frac{(120 \cos m)}{120} \text{ degrees}$$

$$\approx \frac{10}{9} \text{ koṭijyā (m)} = \frac{1}{9} \text{ Bhogyakhaṇḍa}$$

(since in obtaining bhogyakhaṇḍas, the multiplier 10 is already used).

**(ii) True daily motion of the Moon**

The correction to the mean daily motion to get the true daily motion is

$$\Delta n = \frac{a}{R} \cos m \, (\Delta m)$$

where for the Moon, $a = 31^\circ \, 36'$, $R = 360$, $\Delta m$ = mean daily motion = $790' \, 35''$.

$$\therefore \quad \Delta n = \frac{(31.6)}{360} (790' \, 35'') \cos m$$

$$= \frac{(31.6)(790' \, 35'')}{360 \times 120} \times (120 \cos m) \text{ radian}$$

$$= \frac{(31.6)\ (790'\ 35'') \times 180 \times \textit{koṭijyā}\ \ (m)}{360 \times 120 \times \pi} \text{ degrees}$$

$$= 33.13 \text{ koṭijyā (m)} = 10 \times 3.313 \text{ koṭijyā (m)}$$

$$\approx \frac{13}{4}\ [10 \text{ koṭijyā (m)}]\ = = \frac{13}{4} \times \text{(Bhogyakhaṇḍas)}$$

(since in the bhogyakhaṇḍas the multiplier 10 is already used).

**Śloka 13 : The *śīghrakendra* generated *koṭijyā* (*R* cosine) is multiplied by the *parākhya* and by 2 and subtracted from or added to the square of the *parākhya* according as the (*śīghra*) *kendra* is from *Karka* or *Makara* (i.e. from $90^0$ or $270^0$), to this is added 14400; the square-root of this (result) is the (*śīghra*) *karṇa*. The *parākhya* multiplied by the *bhujajyā* (*R* sine of *śīghra kendra bhuja*) and divided by (*śīghra*) *karṇa*, the *cāpa* (bow, inverse of *R* sine) of this result is the *śīghra phala* (result) to be added to or subtracted from, (according as the *śīghra kendra* is from $90^0$ or $270^0$), the *manda* corrected (body) is the *true* (body).**

Finding the śīghraphala of Kuja etc. is explained.

(i) By subtracting (the already obtained) manda corrected planet from its śīghrocca we get the śīghrakendra of the (manda corrected) planet.

(ii) Find the bhuja jyā and koṭi jyā of the sīghra kendra.

(iii) Consider the product $2 \times$ koṭijyā $\times$ parākhya.

where parākhya is given in Table 2.2.

(iv) The result of (iii) is added to or subtracted from the $(parākhya)^2$ according as the śīghrakendra is in I and IV quadrants or in II and III quadrants.

(v) Add $(120)^2$ i.e., 14400 to the result of (iv) and take its square-root. This is called śīghrakarṇa.

(vi) Then parākhya $\times$ bhuja jyā / śīghrakarṇa gives the jyā of the śīghraphala.

(vii) The dhanu (or cāpa) i.e., the inverse of jyā of the result of (vi) gives the required śīghraphala.

The śīghraphala is added to or subtracted from the manda sphuṭa planet according as the śīghrakendra is in I and II quadrants or in III and IV quadrants.

Thus, we have

(i) Śīghrakarṇa = $\sqrt{(Parakhya)^2 \pm 2\ parakhya \times ko\d{t}ijy\bar{a}\ + 14400}$

(ii) Jyā (Śīghraphala) = Parākhya $\times$ bhujajyā / Śīghrakarṇa

The śīghraphala is the cāpa (dhanu) of the above.

**Remark :** Following Brahmagupta's Khaṇḍakhādyaka, P.C. Sengupta has derived the following expression for the śīghraphala SP of a planet :

$$SP = \frac{m}{2} - \tan^{-1}\left[k \tan \frac{m}{2}\right] \qquad .... (1)$$

where m is the śīghra anomaly of the planet and k is a constant for a planet based on the periphery of its śīghra epicycle.

Let us denote $(m/2)$ by $x$ in (1) so that we have

$$S \equiv SP = x - \tan^{-1}(k \tan x)$$

Differentiating w.r.t. $x$, we get

$$\frac{dS}{dx} = (1 - k)\,(1 - k\,\tan^2 x)/(1 + k^2\,\tan^2 x)$$

For SP to be maximum, $dS/dx = 0$ so that $1 - k\tan^2 x = 0$ or $\tan x = \frac{1}{\sqrt{k}}$

$$\text{or } x = \tan^{-1}\left(\frac{1}{\sqrt{k}}\right) \text{ i.e., } m = 2\tan^{-1}\left(\frac{1}{\sqrt{k}}\right) \qquad .... (2)$$

Substituting (2) in (1), we get

$$SP_{\max} = \tan^{-1}\frac{1}{\sqrt{k}} - \tan^{-1}\sqrt{k} = \tan^{-1}\left[\frac{1-k}{2\sqrt{k}}\right]$$

The values of k for the planets and the corresponding maximum śīghraphala (SP) are listed in **Table 2.8**.

| | **Table 2.8 Maximum mandaphala and value of k** | |
|---|---|---|
| Planet | Value of k | Maximum śīghraphala |
| Kuja | 0.21212 | 40° 32′ 30″ |
| Budha | 0.46341 | 21° 30′ 37″ |
| Guru | 0.6666 | 11° 32′ 23″ |
| Śukra | 0.16129 | 46° 14′ 17″ |
| Śani | 0.8 | 6° 22′ 45″.7 |

**Note :** The śīghraphalas of the five tārāgrahas, according to the Śūryasiddhānta, are tabulated in the Appendix for every degree of the śīghra anomaly.

**Śloka 14 : To the thus obtained *manda* corrected planet (treating it, as) *madhya*, the *śīghra* correction is applied (treating this corrected as *madhyama* the second) *manda* correction is (obtained and) applied to the (original) *madhyama* planet. (Again) the *śīghra* correction is applied to the (latest) *manda* corrected planets. (Specially, in the case) of Kuja, the first corrections are halved and then (the later corrections are taken) in full. This procedure is applied repeatedly till the result is stable.**

The method of applying the manda and the śīghra corrections successively to get the true planets is explained. A still more accurate correction specially for Kuja is also explained.

First by applying the mandaphala to the mean planet, the manda corrected planet is obtained. Second step : by considering the manda corrected planet as the mean, obtain the śīghraphala and apply it to the same. Third step : Find the mandaphala for the corrected planet obtained in the second step and apply it to the original mean planet. Apply the śīghra correction to the latest manda corrected planet.

Thus, if the mean planet is P, the first manda corrected planet $P_1 = P + ME_1$ where $ME_1$ = manda equation for P.

The first śīghra corrected planet $P_2 = P_1 + SE_1$ where $SE_1$ = śīghra equation for $P_1$.

The second manda corrected planet $P_3 = P + ME_2$ where $ME_2$ = manda equation for $P_2$ and P = Mean planet

The second śīghra corrected planet $P_4 = P_3 + SE_2$ where $SE_2$ = śīghra equation for $P_3$.

In the case of Kuja the manda and śīghra corrections are applied slightly differently as follows :

With the notations $P$, $P_1$, $ME$ etc. as explained earlier, now for Kuja we have

$$P_1 = P + \frac{1}{2} ME_1$$

$$P_2 = P_1 + \frac{1}{2} SE_1$$

$P_3 = P + ME_2$ and

$$P_4 = P_3 + SE_2$$

**Ślokas 15 and 16 : The mean motion (of a planet) corrected with *gatiphalam* becomes *manda* (corrected) *gati*. The (daily rate of) mo-**

**tion of the *śīghrocca* reduced by the *manda* corrected motion becomes the motion of *śīghra kendra*.**

**This *drāk* (*śīghra*) *kendra* multiplied by the *aśuddha khaṇḍa* (non-subtractable block) of the *cāpa* (bow, inverse of *R* sine) of the *śīghraphalajyā*, multiplied by 40 and divided by 7 times the *śīghrakarṇa*; the result in *kalās* subtracted from the (rate of) motion of the *śīghrocca* becomes the true (rate of) motion of a planet.**

Obtaining of true daily motions of planets is explained.

(i) The gatiphala of a planet obtained earlier (Ślokas 11 and 12) is applied to the mean daily motion to get the manda sphuṭa (corrected) gati (motion).

(ii) The daily rate of motion of the śīghrocca of the planet is combined with the manda corrected motion [obtained in (i)].

This gives the motion of the śīghra kendra.

This must be multiplied with the aśuddha khaṇḍa obtained in finding the cāpa (inverse of jyā) of the śīghraphala jyā. The result is multiplied by 40 and divided by 7 times the śīghra karṇa. The result, which is in kalās, must be subtracted from the rate of motion of the śīghrocca. This is the true daily motion of the planet.

In case the kalās obtained in the above step is greater than the śīghrocca motion (in kalās), then subtract the śīghroccagati from the said kalās to get the true daily motion. In this case the planet is in retrograde motion (vakragati).

**Śloka 17 : The *karaṇābda* (elapsed years since epoch) in minutes of arc added to $11^0$ gives the *aynāmśa* (precession of equinox) in degrees.**

Obtaining of ayanāmśa is explained:

Subtract 1105 from the given (śālivāhana śaka) year and divide the result by 60 to get aṃśas etc. Add this to $11^\circ$ to obtain the ayanāṃśa at the beginning of the given lunar year. For the elapsed lunar months (counting from Caitra), add at the rate of 5 vikalās (seconds of arc) per completed lunar month to get the total ayanāṃśa.

**Note :** The ayanāṃśa, the accumulated amount of precession of equinoxes, is calculated in this text at the rate of $1'$ (or $60''$) per year. Since Bhāskara II takes the ayanāṃśa for his epochal year 1105 (Śā. śa) as $11^\circ$, it means that he assumes the year of zero-ayanāmśa as Śā. śa $(1105 - 11 \times 60) = 445$ Śā. śa (i.e., about $522 - 23$ A.D.) .

However, from modern astronomy we know that the mean annual rate of precession of the equinoxes is $50''.2$. The Govt. of India has adopted, 285 A.D. as the year of zero-ayanāṃśa based on the report of the Calendar Reform Committee.

**Śloka 18 : The *bhujajyā* of twice the (mean *sāyana*) Ravi divided respectively by 5 and by 21 are *udayāntara* (corrections) of Ravi and Candra in *vikalās* (are seconds) and *kalās* (are minutes); these are additive and subtractive respectively as (*sāyana* mean) Ravi is in even or odd quadrant.**

The correction called udayāntara samskāra is explained.

(i) The bhujajyā of twice the mean Sāyana Ravi is determined..

(ii) Divide the result by 5 to get udayāntara in vikalās for the Sun.

(iii) Divide the result of (i) by 21 to get the udayāntara correction for the Moon in kalās.

The correction is positive or negative according as the sāyana Sun is in the even or odd quadrants.

**Example :** We have

Mean Sun = $1^R\ 1^\circ\ 33'\ 26''$

Ayanāṃśa = $18^\circ\ 16'\ 10''$

Sāyana mean Sun = $49^\circ\ 49'\ 36''$

Bhuja jyā of (2 × sāyana mean Sun) = $118.30084$

∴ Udayāntara of the Sun $= \dfrac{118.30084}{5} = 23''.660168 \approx 24''$

Udayāntara of the Moon $= \dfrac{118.30084}{21} = 5'\ 38$

**Note :** If S = Bhuja of sāyana Sun, then

$$\text{Udayāntara of } \begin{cases} \text{Sun is } 120\,\dfrac{\sin(2S)}{5} = 24 \sin(2S) \quad \textit{vikalās} \\ \text{Moon is } 120\,\dfrac{\sin(2S)}{21} = \dfrac{40}{7} \sin(2S)\ \textit{kalās} \end{cases}$$

**Śloka 19 : As many days as the *ayanāṃśas* (precession of equinoxes in degrees) prior to the *Meṣa sankrānti* (*nirayaṇa* solar ingress into Meṣa) the shadow of the *śaṅku* (gnomon) at the midday is the *akṣaprabhā*. This *akṣabhā* multiplied by 10, 8 and 10/3 are the *carakhaṇḍas* of three *rāśis* of *sāyana* Ravi.**

**Śloka 20 : The (elapsed) degrees of the *bhuja rāśi* (of the *sāyana* Ravi) multiplied by the *bhogya khaṇḍa* (the *cara* block of the *bhuja rāśi*) and divided by 30 and added (to the *cara khaṇḍas* of the elapsed *bhuja rāśis*) is the *cara* in *palas*; this (*carapala*)is negative or positive as the *sāyana* Ravi is in the northern or southern hemisphere.**

**The *carapala* multiplied by the (true rate of) motion and divided by 60, in *vikalās* is additive or subtractive (as is the *carapala*) at the time of rising and the reverse for setting.**

Finding the **cara** corrections for Sun, the Moon etc., is explained. Place a cone (śaṅku) of 12 aṅgulas length in any place at the mid-day of the sāyana Meśa saṅkrānti (solar ingress of Aries). Consider the length of the shadow of the śaṅku. (This is called palabhā or akṣabhā or akṣaprabhā). The palabhā (in aṅgulas) is multiplied separately by 10, 8 and $10/3$ resulting in three carakhaṇḍas. Arranging these 3 in the above order and its opposite order alternately we get the sthānīya cara khaṇḍas of the 12 rāśis.

Now after obtaining the carakhaṇḍas, we shall find the carapala as follows :

(i) Consider the bhuja of sāyana true Sun.

(ii) Find the carakhaṇḍa corresponding to the elapsed rāśi (bhukta rāsi).

(iii) Consider the bhuktāṃśa of the running rāśi and multiply it by the corresponding carakhaṇḍa and divide by 30.

(iv) Add the results of steps (ii) and (iii). The sum gives the carapala or carakarmapala, which is additive or subtactive according as the sāyana Ravi is in the uttaragola (i.e., $0^\circ <$ sāyana Ravi $< 180^\circ$) or dakṣiṇa gola (i.e., $180^\circ < \textit{sāyana}$ Ravi $< 360^\circ$).

(v) Now multiply the above carapala by the true daily motion of the Sun and divide by 60. The result which will be in vikalās is added to or subtracted from the Sun to get the cara corrected Sun.

Similarly we can find the cara correction for the other planets by considering the corresponding sāyana planets and their true daily motions.

**Finding the day (Ayana dina) on which the shadow is to be considered :**

**Method (i) :** Ayanadina = Ayanāṃśa $- \frac{Ayanāṃśa}{42}$

where the ayanāṃśa is taken in degrees.

The result will be in days etc. Another method is as follows.

**Method (ii) :** Multiply the true daily motion of the Sun by ayanāṃśa and divide by 60. The result will be in days etc.

**Example :** We have, ayanāṃśa = $18^\circ\ 16'\ 10''$, true motion of the Sun = $58'\ 35''$

$$\text{Ayanadina} = \frac{58'\ 35'' \times 18^\circ\ 16'\ 10''}{60} = 17^{d}\ 50^{gh}\ 17^{vig}$$

By method (i) we have

$$\text{Ayanadina} = \text{Ayanāṃśa} - \frac{Ayanāṃśa}{42}$$

$$= 18^\circ\ 16'\ 10'' - \frac{18^\circ\ 16'\ 10''}{42} = 17^{d}\ 50^{gh}\ 4^{vig}$$

(with a small difference between the values from the two methods).

On the above day at a place called **Śivapurī,** we have the shadow of the cone = $5|30$ aṅgulas.

The three carakhaṇḍas are

| | | | |
|---|---|---|---|
| | $10 \times 5\|30$ , | $8 \times 5\|30$ , | $\frac{10}{3} \times 5\|30$ |
| i.e., | 55 | 44 | 18 |

Now, the sāyana Sun = $1^{R}\ 21^\circ\ 23'\ 44''$

Bhuja of sāyana Sun $= 51^\circ\ 23'\ 44''$

From this we understand that the first rāśi namely Meṣa is over and the elapsed part of the Vṛṣabha rāśi is $21^\circ\ 23'\ 44''$. Therefore the carakhaṇḍa corresponding to the bhukta rāśi (Meṣa) is 55 and that of bhogya rāśi (Vṛṣabha) is 44.

The elapsed part of the bhogya khaṇḍa = $\frac{21^\circ\ 23'\ 44'' \times 44}{30} \approx 31|22$

Now carapala = Bhuktakhaṇḍa + elapsed part of the Bhogyakhaṇḍa

$$= 55 + 31|22 \approx 86 \text{ palas}$$

Since the sāyana Sun is in the uttara gola the above carapala is subtractive.

Now, taking the true daily motion of the Sun as $57'\ 9''$, we get

cara correction = $\frac{carapala \times 57'\ 9''}{60} = \frac{86 \times 57'\ 9''}{60} \approx 82''$ (in seconds of arc).

Since carapala is negative, subtract $82''$ from the true Sun to get the cara corrected true Sun.

Similarly, the cara correction in vikalās for the Moon, Kuja, Budha, Guru, Śukra, Śani and Rāhu are 1203, 7, 148, 16, 87, 10, 4 obtained respectively. Here except in the case of Rāhu, all are subtractive.

**Śloka 21 : (The longitude of) Candra less (that of) Ravi in degrees divided separately by 12 and 6 are (respectively) the *tithis* and the *karaṇas*. Subtracting 1 (from the number of elapsed *karaṇas*), there are *cara* (moving *karaṇas*) *Bava* etc. (start from the latter half of *śukla pratipat*) and the four (*sthira*, fixed, *karaṇaś*) *Śakuni* etc. from the latter half of *kṛṣṇa caturdaśī*.**

**Śloka 22 : (The longitude of) a planet, in *kalās* (minutes of arc) and (the longitudes of) the Moon added with (that of the) Sun, in *kalās* (arc minutes) divided by 800 are respectively the *nakṣatra* and the *yoga*.**

**(The remainder, the elapsed part) *gata* and (its complement from the divisor 800) *gamya* (to be covered portion) in *viliptas* (arc seconds) divided by the respective (rates of) motion are the elapsed and the balance (*nakṣatra* and *yoga*) *nāḍikās* (*ghaṭīs* each of 24 minutes of time).**

Obtaining of the (pañcāṅgas viz.) tithi, etc. is explained below :

**1. Finding the tithi :**

(i) Subtract the position of the true Sun from that of the Moon (in degrees) and divide by 12. The integer quotient q gives the number of elapsed tithis. Let r be the remainder (in degrees).

(ii) In the running tithi (i.e., $q+1$), the remainder (in kalās) divided by the difference of the true daily motions in kalās of the Moon and the Sun and multiplied by 60 gives the elapsed part (gata) of the $(q+1)^{th}$ tithi.

(iii) Similarly, the gamya (balance) part of the current $(q+1)^{th}$ tithi is given as follows :

Subtract the remainder r (deg.) from $12^\circ$ and convert the difference into kalās. Divide this difference (kalās) by the difference between the true daily motions of the Moon and the Sun (in kalās) and multiply by 60. This gives the balance (gamya) part of the current tithi in ghaṭīs.

**Example :** On a given day,

True Sun = $1^R\ 3^\circ\ 6'\ 12'' \equiv S$

True Moon = $0^R\ 8^\circ\ 47'\ 21'' \equiv M$

$\therefore\quad M - S = 11^R\ 5^\circ\ 41'\ 09'' = 335^\circ\ 41'\ 09''$.

Dividing by $12^\circ$, we get $q = 27$ and $r = 11^\circ\ 41'\ 09''$.

The elapsed number of tithis 27 means that 12 tithis are elapsed in the kṛṣna pakṣa (dark fortnight) and the 13th tithi (trayodaśī) is running.

In the 13th tithi, the elapsed portion (gata) is $r = 11^\circ\ 41'\ 09''$ = $701'\ 09''$. The difference between true daily motions of the Moon and the Sun = $783'\ 22''$.

(True daily motions of the Sun and the Moon are respectively $57'\ 28''$ and $840'\ 50''$).

$\therefore$ the gata portion = $\dfrac{701'\ 09''}{783'\ 22''} \times 60\ gh.$ = $53|42\ gh.$

Now, $12^\circ - r$ = $12^\circ - 11^\circ\ 41'\ 09''$ = $18'\ 51''$

$\therefore$ the gamya portion = $\dfrac{18'\ 51''}{783'\ 22''} \times 60\ gh.$ = $1|36\ gh.$

Therefore, the duration of the current 13th tithi (trayodaśī) = gata + gamya portions = $53|42\ \ gh. + 1|36\ \ gh.$ = $55|18\ \ gh.$

**2. Finding the karaṇa :**

Each tithi is divided into two halves called karaṇas. Thus, a lunar month has 60 karaṇas. These are divided into two groups viz. cara (movable) and sthira (fixed).

The cara karaṇas : Bava, Bālava, Kaulava, Taitila, Gara, Vaṇij, Viṣṭi.

The sthira karaṇas : Śakuni, Catuṣpāda, Nāga and Kimstughna.

The second half of kṛṣṇa pakṣa caturdaśī, both halves of amāvāsyā and the first half of the following śukla pratipat are successively the four sthira karaṇas named earlier.

The 7 cara karaṇas start with the latter half the śukla pratipat and end with the first half of kṛṣṇa caturdasī repeating themselves cyclically. These are determined as explained below :

Let M and S be the true longitudes of the Moon and the Sun.

(i) Find $(M - S)$: If M is less than S, then add $360^\circ$ to $(M - S)$ to get its revised positive value.

(ii) When $(M - S)$ is divided by $6^\circ$ let K be the integer quotient and R be the remainder. Subtract 1 from the quotient so that $(K - 1)$ karaṇas have elapsed. This means that the $K^{th}$ karaṇa is currently running.

(iii) If K is 57, 58, 59, 60 (or 0), then the running karaṇa is correspondingly Śakuni, Catuṣpāda, Nāga, and Kimstughna.

(iii) If $(M - S)/6$ is less than 7 the integer quotient is taken as K. If $(M - S)/6$ is greater than 7, then subtract the nearest integral multiple of 7 and K be the balance quotient and R be the remainder. Subtracting 1 from the quotient, $(K - 1)$ represents the elapsed karaṇa counting from Bava in the list of cara karaṇas and K is the running karaṇa.

**Example :** True Ravi S = $1^R\ 3^\circ\ 06'\ 12''$, True Candra M = $0^R\ 8^\circ\ 47'\ 21''$.

$$\therefore \quad M - S = 11^R\ 5^\circ\ 41'\ 09'' = 335^\circ\ 41'\ 09''$$

Dividing $(M - S)$ by 6, we have quotient = 55 and remainder $5^\circ\ 41'\ 09''$. i.e., K = 55 and $R = 5^\circ\ 41'\ 09''$.

This means 54 karaṇas are completed and the $55^{th}$ one is running. This is not in the list of the four sthira karaṇas. Dividing 55 by 7 (i.e., removing the multiples of 7) we get the $6^{th}$ karaṇa viz. Vaṇij, the currently running karaṇa.

**Note :** When $(M - S)$ is divided by 6, if K is the integer quotient, we subtract 1 from K to get the elapsed karaṇa since the list of Bava karaṇa etc. starts with the second half (and not the first half) of the śukla pratipat.

Now, in the running karaṇa of Vaṇij, the elapsed portion is given by the remainder $R = 5^\circ\ 41'\ 09''$ (the gata part). The balance (gamya) portion is given by $6^\circ - (5^\circ\ 41'\ 09'') = 18'\ 51''$.

Converting into kalās, we have gata part = $341'\,09''$, gamya part = $18'\,51''$. Differnce between the true daily motions of the Moon and the Sun is $781'\,52''$. Therefore we have

$$\text{gata ghaṭīs} = \frac{341'\,09''\times 60}{781'\,52''} = 26|10\ gh.$$

$$\text{gamya ghaṭīs} = \frac{18'\,51''}{781'\,52''}\times 60 = 1|26\ gh.$$

$\therefore$ Total duration of Vaṇij karaṇa = $26|10\ gh. + 1|26\ gh. = 27|36$ gh.

**3. Finding the yoga :**

The sum of the true Sun and the true Moon (in kalās) is divided by 800. The integer quotient q indicates the number of yogas completed and the $(q+1)^{th}$ yoga is currently running and r be the remainder.

The remainder r is converted into kalās and divided by the sum (in kalās) of the true daily motions of the Sun and the Moon. This multiplied by 60 gives the gata (elapsed) portion of the running yoga in ghaṭīs. The remainder r in kalās is subtracted from 800 and the result (in kalās) is divided by the sum (in kalās) of the true daily motions of the Sun and the Moon and multiplied by 60. This gives the gamya (elapsed) portion of the current yoga.

The sum of the gata and the gamya portions in ghatīs gives the total duration of the running yoga in gh.

**Example :** True Sun, $S = 1^R\ 3°\ 6'\,12''$ and true Moon, $M = 0^R\ 8°\ 47'\,21''$. Their sum, $S+M = 1^R\ 3°\ 6'\,12'' + 0^R\ 8°\ 47'\,21'' = 1^R\ 11°\ 53'\,33'' = 2513'\,33''$.

The sum of the true daily motions of the Sun and Moon = $898'\,18''$.

Dividing $(S+M)$ by 800, we get $q=3$ and $r = 113'\,33''$.

This means that 3 yogas are completed and the 4th yoga viz. Saubhāgya is running (see Appendix for the list of yogas).

The gata (elapsed) portion of the Saubhāgya yoga is $113'33''$ and the gamya (balance) portion of that is $800' - 113'33'' = 686'27''$. Therefore, we have

(i) gata ghaṭīs = $\dfrac{113'33''}{898'18''} \times 60 = 7|35\ gh.$

(ii) gamya ghaṭīs = $\dfrac{686'27''}{898'18''} \times 60 = 45|50\ gh.$

Duration of the yoga = $7|35 + 45|50 = 53|25\ gh.$

**4. Candra and Sūrya nakṣatras :**

The (nirayaṇa) positions of the Sun and the Moon in kalās are divided separately by 800. Let $q_1$ and $q_2$ be the quotients and $r_1$ and $r_2$ be the remainders.

The quotients $q_1$ and $q_2$ represent respectively the number of nakṣatras completed by the Sun and the Moon. The next nakṣatras are the currently running ones.

The remainders $r_1$ and $r_2$ in kalās divided respectively by the true daily motions of the Sun and the Moon (in kalās) and multiplied by 60 give the gata (elapsed) portion of the running nakṣatras in ghaṭīs. Similarly, $(800 - r_1)$ and $(800 - r_2)$, in kalās, divided respectively by the true daily motions (in kalās) of the Sun and the Moon and multiplied by 60 give the gamya (balance) portions of the nakṣatras in ghaṭīs.

**Example** : True Sun, $S = 1^R\ 3°\ 6'12'' = 1986'12''$; True Moon, $M = 0^R\ 8°\ 47'21'' = 527'21''$. Dividing S and M (in kalās) by 800, we get the quotients and the remainders :

$q_1 = 2$, $r_1 = 386'\,12''$ for the Sun

$q_2 = 0$, $r_2 = 527'\,21''$ for the Moon

Therefore, Sūrya nakṣatra is the 3rd nakṣatra viz., Kṛttikā. The Moon is in the 1st nakṣatra i.e., Aśvinī. That is, Aśvinī is the Candra nakṣatra.

In the case of the Moon, on the given day, the true daily motion is $840'\,50''$. Therefore

$$\text{gata ghaṭīs of Aśvinī} = \frac{527'\,21''}{840'\,50''} \times 60 = 37|37\ gh.$$

$$\text{gamya ghaṭīs of Aśvinī} = \frac{800' - 527'\,21''}{840'\,50''} \times 60 = 19|27\ gh.$$

Total duration of the Āśvinī nakṣatra = $37|37\ gh. + 19|27\ gh. = 57|04\ gh.$

For the Sun, on the given day, the true daily motion is $57'\,28''$.

The true Sun = $1986'\,12''$ which is in the Kṛttikā nakṣatra. Each nakṣatra has 4 equal parts each of $200'$ angular range called a pāda (quarter). Thus, in Kṛttikā $386'\,12''$ has elapsed. This means that the Sun is in the 2nd pāda.
In this 2 pāda, the gata (elapsed) part is $186'\,12''$ and the gamya (balance) part is $200' - 186'\,12'' = 13'\,48''$. Since the true daily motion of the Sun is $57'\,28''$ we have

(i) gata part = $\frac{186'\,12''}{57'\,28''}$ days = $3^d\ 14^{gh}\ 24^{vig}$

(ii) gamya part = $\frac{13'\,48''}{57'\,28''}$ days = $0^d\ 14^{gh}\ 24^{vig}$

both of the 2nd pāda of Kṛttikā. In fact, the duration of this pāda is

$$3^d\ 14^{gh}\ 24^{vig} = 0^d\ 14^{gh}\ 24^{vig} = 3^d\ 28^{gh}\ 48^{vig}.$$