

WOOTZ DECARBURISATION DURING THE FORGING OF A SWORD

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From historical and archaeological point of view, it is important to propose reconstructions of ancient techniques. In this way, theoretical simulation of decarburisation of small ingots of high carbon steel such as those of ancient Wootz, is proposed as a tool to provide information on techniques used by ancient Indian blacksmiths. The relation between the information accessible at the end of the forging process, which is the mean carbon content of the blade, can be related to the initial ingot, after reconstruction of tentative forging process. Such backward calculation cannot give absolute answer to the question of the initial carbon content of the ingot, that is on the capacities of the ancient furnaces to reach high temperatures, but it can help the description of different good or poor results. Criteria concerning “strong decarburisation” that is the layer thickness to be filed out on the forged blade, and the decarburised ductile layer necessary to avoid cracking during forging are discussed.

Keywords. Damascene sword, Decarburisation, Forging, Theoretical simulation, Wootz.

INTRODUCTION

Several methods of Wootz preparation and of other crucible steels have been described in the last 10 centuries, in which the metal was totally or partially melted. Sometimes, mixtures of cast iron and wrought iron were used as base products; sometimes carburisation of the metal was made in the solid state until the melting temperature was reached, etc.^{1,2,3}. The question discussed here concerns nature of the melting process, more precisely the temperature accessible in the crucible furnace, which is directly related to the minimum carbon content of the steel, that is the liquidus of the alloy. Verhoeven in 1987¹, discussed the relation

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between carbon content in the ingot and in the forged blade but his calculation was very rough and has to be precised.

It must be noticed that the treatment of Wootz steel and its transformation into objects like swords or other tools is very uncommon for the modern steel metallurgy, because of the high carbon content and the small size of the ingot (1 to 2 kg), that is a high surface/volume ratio. However, from historical and archaeological point of view, it is essential to reconstruct as far as possible the ancient metallurgical and forging techniques, to better understand from where our present knowledge is coming.

Whatever be the supposed melting process, we propose here a relatively simple numerical model of decarburisation during forging a Wootz ingot into a blade and, knowing the final carbon concentration in the blade, to go back to the initial carbon concentration of the ingot and possibly to find indications on the preparation process.

For presentation of the numerical model, we describe first the decarburisation of a sheet considered as an image of a blade, at one temperature and during several forging steps. Secondly, the simulation of a forging process is shown, with modification of geometry as a function of temperature and time.

1. PROBLEM

The carbon content of Damascus blades, at least those prepared after the 17th century, varies⁴ from 1.9% C to 1.5% C, but sometimes it is as low as 0.6% C. Such analyses of C content give a mean value on a section of the finished blade, after forging and induced decarburisation.

The *first question* is: what was the C content of the initial Wootz ingot and consequently its melting temperature? The answer will be the result of a backward calculation, with reference to the Fe-C phase diagram.

The *second question* concerns possible cracking during forging: what could be the influence of a pre-treatment at high temperature to decarburise the outer part of the ingot and avoid cracks development during forging? This point will be discussed on the basis of carbon profile calculation inside the blade.

The *third question* is: from the carbon profile, how can we estimate the decarburised layer thickness, that means for instance the thickness of metal to be filed away before finding a nice Damascene microstructure.

2. CALCULATION ON AN IDEAL SHEET

Here we describe our numerical model on the simple geometry of an “infinite sheet”.

2.1. Structure of the Model

2.1.1 Hypothesis

To treat the behaviour of a sheet, we consider a piece of metal with a given thickness E_0 , and using the Fick’s diffusion laws, we calculate the carbon profile inside the metal during hot forging, i.e. *a combination of heat treatment and variation of thickness* from E_0 to a final thickness E_f . The specimen is considered as an infinite sheet in which diffusion is unidirectional, perpendicularly to the surface. The model considers that the carbon profile inside the sheet changes as a function of time at temperature, and is deformed, while the thickness of the sheet decreases by forging. The carbon content at the surface of the sheet is imposed equal to zero, a condition corresponding to the situation of an oxidised surface.

A strong simplification is made: we consider that carbon is in solid solution in the metal whose structure is austenitic, which means that decarburisation takes place without carbide dissolution, but this hypothesis seems sufficient to give an order of magnitude of the decarburisation level. In the same manner, we do not consider the existence of a ferritic layer at the surface when the temperature is about 900°C-800°C.

2.1.2. Definitions

Forging step: Heating for Δt , for instance 3 min, at a given temperature T ; forging with a thickness reduction $\Delta E/E$, for instance 0.1, (that is 10% per step); re-heating for Δt at T .

The forging time is considered as very short as compared with the re-heating time Δt , at T and the T variations during forging are not taken into account.

Forging sequence: It represents a series of forging steps with the same parameters (Δt , T , $\Delta E/E$, ...). The number of steps in the sequence is named N . In a given sequence, the thickness E varies as a function of the number of forging steps (N), but the reduction ratio $\Delta E/E$ is taken constant. This choice corresponds to the fact that, when the sheet is thick you have more time to forge than when the thickness is low. In this case, the metal cools down rapidly and you must re-heat frequently.

The relation between the thickness E_N after N forging steps in a sequence and E_0 the initial thickness is given by the classical “geometrical progression” formula :

$$E_N / E_0 = [1 - (\Delta E/E)]^N$$

For a complete process from the initial thickness E_0 to the final thickness E_f , the ratio E_f/E_0 is the total reduction ratio, *for instance* 1:10.

2.1.3. Simulations

We can simulate different situations by varying the values of the parameters to obtain :

- * the carbon profile in the sheet as a function of time and temperature: $C(x,t)$;

- * the mean carbon concentration in the sheet, the value given by chemical analysis: (C_m/C_0) as a function of time; or of the number of forging steps N ; or as the cumulated reducing ratio : $E(t)/E_0$ or $E(N)/E_0$. The value C_0 is the carbon concentration at $t = 0$.

Further, it is possible to apply the calculation for different geometries: an infinite sheet, a long cylinder or a sphere.

The structure of the model is detailed in Appendix A.

2.1.4. Problem of the Carbon diffusion coefficient in austenite

This is a complicate problem because the diffusion coefficient D of carbon in austenite varies strongly with temperature, which is not surprising, but also with the carbon content of the alloy, which is less habitual. For instance at 800°C , D varies between 10^{-11} to $10^{-10} \text{ m}^2.\text{s}^{-1}$ when $C\%$ varies from 1% to 2%. However, for simplification, we do not take this variation

into account in our present calculation whereas the metal is in a situation of important carbon concentration gradients.

As a first approximation, we will use the following values⁵:

$D = 5 \cdot 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$, for temperatures about	1100°C
$D = 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$	” ” 900°C
$D = 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$	” ” 800°C

On the question of diffusion coefficients in austenite, see also Verhoeven⁶.

2.2. Decarburisation of a Sheet

2.2.1. Comparison of decarburisation with and without forging

The aim is to find an order of magnitude of the influence of the forging process on the decarburisation of the metal, by combining the effects of temperature and thickness reduction. The chosen situation is shown in Fig.1a, which gives the variation of the relative mean concentration C_m/C_o as a function of time, for short forging steps of 1 min at temperatures close to 900°C. The diffusion coefficient D is $10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$. The initial thickness was 20 mm.

It is not surprising to find that the curve without forging (No) is clearly above those with reduction of thickness (10% to 1%). It is clear also, that rapid forging with a strong reduction per step (e.g.10%), which leads to final thickness faster than with a smaller one (e.g.5%), shows less decarburisation. Here, for a final thickness of 2 mm, starting from a 20 mm initial thickness (ratio 1:10), the heat treatment and forging times are : 22 min for 10% reduction per step against 75 min for 3%. The curve $E_f = 2 \text{ mm}$ is drawn in Fig.1a.

NB: this theoretical forging sequence used for illustration of the model is not realistic in the first steps, because you cannot reheat a 10 mm thick piece of metal in 1 min!

Fig.1b illustrates the relation between the mean carbon content and the thickness of the sheet, in the same conditions as Fig.1a. If we need a final thickness of 4 mm, that is $E_f/E_o = 1:5$ (or 0.2), it appears that the strongest reduction ratio (10%) gives a final relative carbon content $C_m/C_o = 0.9$, largely higher than for a 1% reduction ratio, for which the final C_m/C_o is much lower, about 0.7. This is due to the difference in total time of heat treatment: 160 min

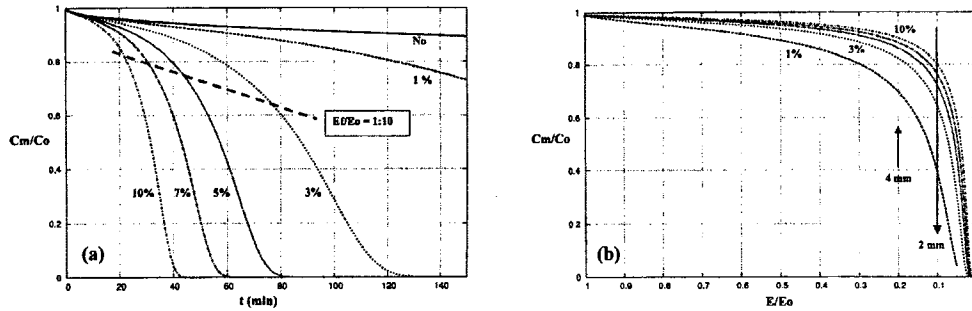


Fig. 1. Decarburisation in a sheet. Initial thickness 20 mm; forging steps $\Delta t = 1$ min with different reduction ratios $\Delta E/E$ from 0% to 10%. Diffusion coefficient $D = 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$ (T about 900°C). (a) Decarburisation C_m/C_o as a function of time for different $\Delta E/E$. The curve “No” is for “no forging”. The curve $E_f/E_o = 1/10$ corresponds here to $E_f = 2$ mm. (b) Decarburisation C_m/C_o versus total thickness reduction E/E_o for different reduction ratios $\Delta E/E$.

against 15 min. This question is not academic because forging by hand hammer needs a much longer time with smaller thickness reduction by step than with modern mechanical hammers.

If we choose a final thickness of $E_f = 2$ mm, in the process described above, an important variability of C_m/C_o can be possible. In the rapidly decreasing part of the curves of Fig.1b, for $E/E_o < 0.2$, it appears clearly that *small variations of the process will lead to very different decarburisation levels*, between 0.8 and 0.4, for instance around $E/E_o = 0.1$.

2.2.2. Addition of a decarburisation pre-treatment

The aim is to compare the effect of a preliminary heat treatment at high temperature (about 1100°C), as discussed by Verhoeven¹, with decarburisation during a long forging process at lower temperatures such as 900°C and 800°C . This new process is described in Table 1 and Fig.2a.

Fig.2b shows the decarburisation evolution as a function of the number of forging steps: $C_m/C_o = f(N)$, and Fig.2c gives the carbon profile $C(x)$ inside the sheet, at the end of each sequence S_0 to S_4 , when the total thickness of the sheet is decreasing from 50 mm to 3 mm. Only the right half sheet is represented, $x = 0$ being at the centre.

Table 1. Forging process of a sheet (initial thickness $E_0 = 50$ mm), with a preliminary decarburisation sequence.

	Sequence	Steps	Time Δt	D ($m^2 \cdot s^{-1}$)	Approximate T
Initial decarburisation	S0	No forging	5h	$5 \cdot 10^{-10}$	1100°C
Preliminary forging	S1	5 steps 10%	5 min	10^{-10}	900°C
	S2	5 steps 10%	3min	10^{-10}	900°C
Finishing period	S3	25 steps 5%	3min	10^{-11}	800°C
	S4	50steps 1%	1min	10^{-11}	800°C

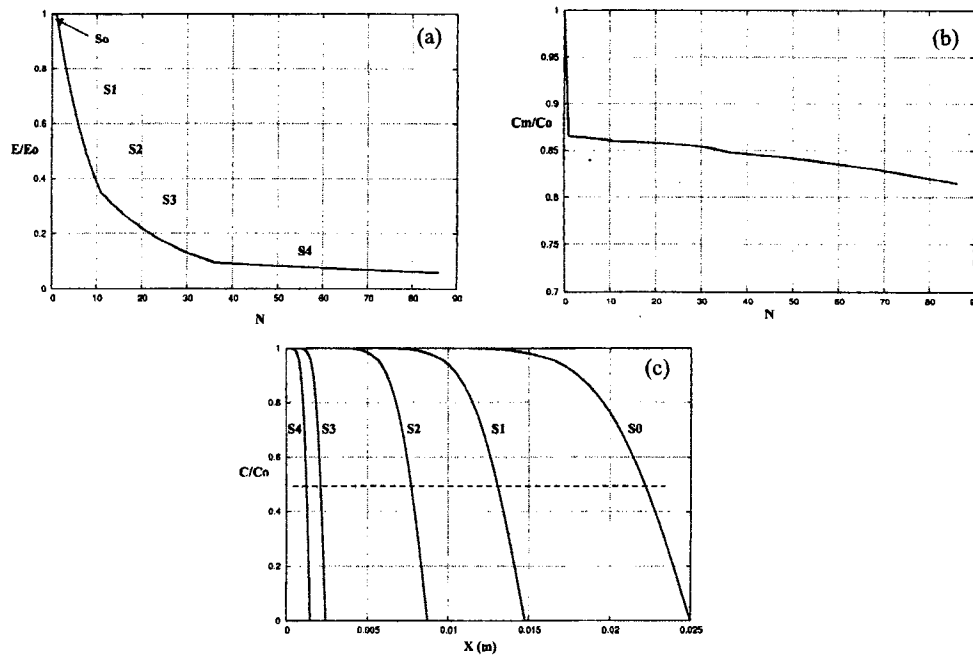


Fig. 2. Decarburisation in a sheet with different forging sequences (S1-S4) including a preliminary decarburisation S0. (Table 1). Initial thickness $E_0 = 50$ mm. (a) Reduction ratio E/E_0 versus N , the number of forging steps (Table 1). (b) Decarburisation C_m/C_0 versus N , the number of forging steps (Table 1). (c). Carbon profiles at the end of each sequence So-S4 (Table 1) taking into account the thickness reduction from 50 mm to 3 mm. Here is only shown a half sheet. The 0.5 C/C_0 line is shown.

In the present case, $C_m/C_0 = 0.86$ after decarburisation (S0) and $C_m/C_0 = 0.82$ at the end of the process. Decarburisation of the sheet (about: $0.82/0.86 = 0.95$) is negligible during forging at 900°C and 800°C. In this situation, the thickness decreases, at forging temperature, (S1-S4) has little effect on decarburisation as compared with a pre-treatment at a higher temperature (S0).

If the carbon content measured in the blade was 1.6%C, the initial value in the sheet would have been 1.9%C, in the conditions fixed in Table 1.

From Fig.2c, the “strongly decarburised” thickness of the blade can be approximate, by using the limit of solubility of carbon in austenite, for instance at 800°C, the last forging temperature in the above sequence. This value is 1%C, which gives $C(x)/C_0 = 1/1.9 = 0.5$, (the initial thickness was 20 mm). The corresponding thickness ratio in Fig. 2c is about 10%. At the end of the forging process the “strongly decarburised” layer is a few tenth of mm on each face of the blade, when the blade reaches a final total thickness of 3 mm.

After these first results on a simple one-directional diffusion geometry, a more complex forging process is described below.

3. SIMULATION OF A FORGING PROCESS

3.1. Definition

Here, a more practical application is proposed to simulate a forging process combining different temperatures, thickness reduction ratios, and modifications of geometries.

The shape of the metal will be changed from a compact one corresponding to the initial ingot, into a bar and finally into a sheet. The choice is motivated by the fact that decarburisation in a sphere of a given diameter is much faster than in a long cylinder of the same diameter and still more than in an infinite sheet of the same thickness⁷. During the first forging period, in which the process would be the transformation of the ingot into a disk, then into a cube or a short parallelepiped⁸, we consider that the shape remains a compact one and diffusion is approximated by the 3 dimension model of a sphere.

All along the process, the forging temperature will be lowered as the thickness of the blade decreases, and the time for each reheating step will be shorten (Table 2).

3.2. Modification of Geometry during the Process

For modelling purpose, we use the following approximations (Table 2 and Fig. 3a):

Table 2. Forging process of a small ingot (initial diameter 60 mm) into a cylinder and a sheet, with a preliminary decarburisation sequence

	Shape	Sequence	Steps	Δt / (total time)	D ($m^2.s^{-1}$) / Approx T ($^{\circ}C$)
A Initial decarburisation 2h	Sphere diam $2R_0$ = 6cm (initial and final)	So	No global shape change	- / 2h	5.10^{-10} / $\sim 1100^{\circ}C$
B Preliminary forging 43 min	Cylinder initial diam $2R_0$ = 6cm; final diam : $2R_1$ = 2.2 cm	S1	5 steps 10%	5min / (25min)	10^{-10} / $\sim 900^{\circ}C$
		S2	3 steps 10%	3min / (9min)	
		S3	3 steps 5%	3min / (9min)	
C Finishing period 110 min	Sheet initial thickness E_0 = $2R_1$ (2.2cm); final thickness E_f = 4.8mm	S4	20 steps 5%	3min / (60min)	10^{-11} / $\sim 800^{\circ}C$
		S5	50 steps 1%	1min / (50min)	

Period A (sequence So): The as-solidified ingot considered as a sphere, is heat-treated for decarburisation, without shape modification. The calculation is made by 3 dimension diffusion in a sphere.

Period B (sequences S1 to S3): In this preliminary forming to obtain a bar, the calculation is made by 2 dimension diffusion in a cylinder.

Period C (sequences S4-S5): Finishing into a sheet to obtain the final thickness of the blade, is treated as in the above example of 1 dimension diffusion (§ 2.2).

These three theoretical geometries are used for simplicity of calculation. However, the change from one geometry to another must be approximated by an artefact. At the end of a period, the concentration is “homogenised” in the following way: the mean carbon content calculated at the end of the periods A or B is taken as the uniform initial concentration for the next one.

The overall process-flow described in Table 2 and Fig.3 is inspired from those used by Verhoeven⁹ or by Massalski³.

A more realistic model would have to take into account the continuity of shape modification, temperature, etc. It would be a heavy finite element calculation, but we hope that our first approximation is sufficient to give an order of magnitude of the results.

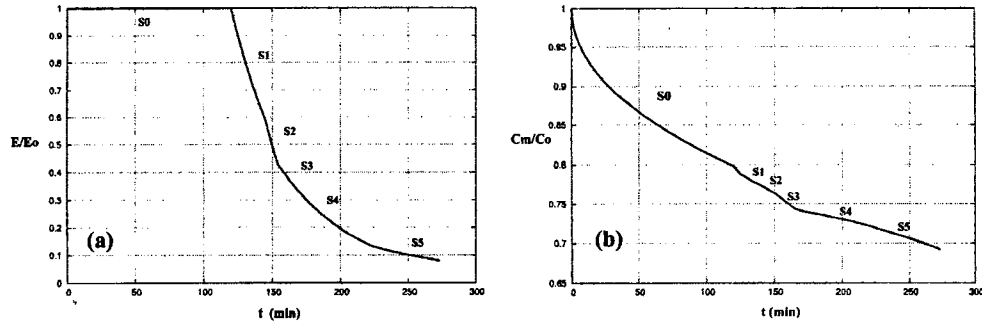


Fig. 3. Simulation of a forging process from a sphere (diam 60 mm) to a cylindrical bar and then a sheet as described in the different sequences S0 to S5 in Table 2.
 (a) Reduction ratio E/E_0 versus time. (b) Decarburisation C_m/C_o versus time.

3.3. Results

Fig 3b shows the evolution of C_m/C_o as a function of time. At the end of the first period (S0), the decrease in carbon content is $C_m/C_o = 0.80$ and at the end of the complete process, $C_m/C_o = 0.69$. That means: if C_m measured on the blade was 1.2%C to 1.9%C, the initial carbon content of the ingot would have been 1.7%C to 2.7 %.

If there were no stage S0, C_m/C_o would be $0.69/0.8 = 0.86$. That means: in our conditions, the effect the forging periods (S1-S5) is of the same order of magnitude as the treatment S0 at higher temperature.

3.4. Comments

For simplification, we concentrate now on a 1.6%C blade. In the above defined conditions, the carbon content of the corresponding ingot would have been : 2.3%C. At the end of the pre-treatment S0, the carbon content would have been 1.84%C, a situation apparently comfortable to forge a blade.

The preceding results do not take into account, the loss of metal by filing away the decarburised layer. The thickness of this decarburised layer can be obtained as above (§ 2.2.3), by using Fig.4. Two different points of view must be considered as a function of the technical objective.

(a) Decarburised layer necessary to avoid cracking during the first forging steps: The limit of the surface layer with a carbon content lower than 2%C at the end of S0 could be selected because this value (i) is the maximum

of the austenite carbon solubility at 1150°C, at the limit of the cast-iron domain, and (ii) was also found as a maximum carbon content for easy forging without cracking at 850°C, by Sherby^{10,11}. In the present situation (Fig.4), the corresponding thickness is 4.3 mm on the radius of the sphere. However, it can also be observed (Fig.4) that inside the metal, the carbon content remains at its maximum and that only after 20 h at high temperature does the carbon content in the centre of the sphere begin to decrease.

The questions is now : which thickness of the decarburised layer is sufficient to prevent cracking of the inside metal whose carbon content is still higher than 2.3% C? The mechanical properties of hypereutectoid steels have been studied by Sherby¹¹ but the answer is not easy, because no simple criterion is available. At a maximum, the heat treatment would be asked to decrease the carbon content inside the metal to about 2%C, the limit of the brittle cast-iron domain. The calculation of the time at 1100°C to decrease the carbon content from 2.3% to 2%C in the centre of the sheet ($x = 0$, in Fig.4) gives about 30 h.

(b) *“Strongly decarburised” layer that must be filed out at the end of the process*: We can take the carbon content corresponding to the solubility limit at the temperature of final forging. The metallurgical reason is that for lower carbon contents a nice Damascene microstructure will not be produced during forging cycles at this temperature, because carbide particles would be dissolved

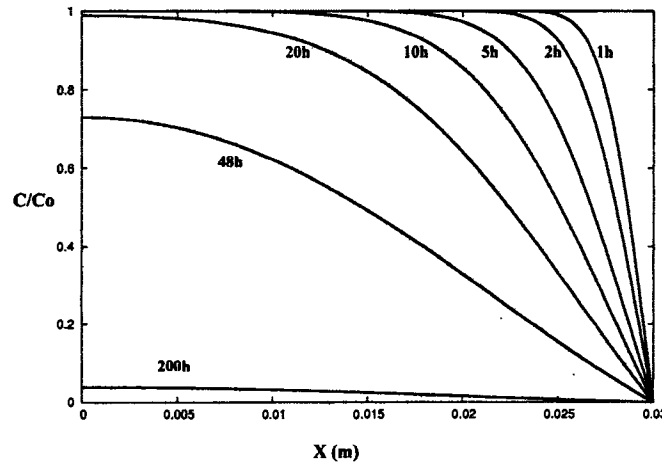


Fig. 4. Decarburisation profiles at different times (1 h to 200 h) without forging, on a 60 mm diameter sphere for $D = 5.10^{-10} \text{ m}^2.\text{s}^{-1}$ (T about 1100 °C).

at each re-heating from the pearlite domain. Here the temperature of last forging is 800°C and the carbon limit, given by the Fe-C phase diagram, is 1%C, which corresponds to $C/Co = 1/2.3 = 0.43$. The corresponding layer thickness (Fig.4) is $\Delta R = 2$ mm, on the radius (R_o) of the sphere, and the “strongly decarburised” volume at the surface of the sphere is $3\Delta R/R_o = 20\%$. The increase of this layer during the subsequent forging sequences (S1 to S5) cannot be seriously calculated, without oversimplified approximations, but we see from the preceding curves, that the carbon content will decrease a few per cent more.

The question is now : how can we use this new information to make a correction on the initial carbon content of the ingot? An approximate answer would be: the decarburised layer has a carbon content between 1% and zero, and we can take an intermediate value of 0.5%C in the layer (20% in volume), with 1.6%C in the blade (80% in volume). The carbon balance gives 1.38%C and for the value $C_m/Co = 0.69$, the Co level is found at 2.0%C in the ingot.

4. CONCLUSIONS

The present numerical model associating diffusion and dimension modifications during forging, gives an order of magnitude of the effects of temperature and thickness reduction during the transformation of a small ingot into a thin sword blade, such as in the Wootz forging process. Many different situations can be simulated, taking into account the initial carbon content of the ingot, which depends on the heating power of the melting furnace. The backward calculation from the carbon content of the blade to the initial carbon content of the ingot is possible when a realistic forging process is introduced in the computer. We can also take into account a decarburisation pre-treatment in the global process. However, this model cannot decide if a 1.6%C blade was directly forged from a 1.9%C ingot or from a 2.5%C ingot after a few hours decarburisation at 1100°C.

The situation of transformation of a Wootz ingot into a Damascus blade is not current in the modern steel industry, because (i) the ingot size was small and the final product was thin, (ii) hot forging was performed by hand hammer on a high carbon steel, and (iii) a special distribution of carbide particles was needed for mechanical and cutting properties and specially for beautiful aspect. For forging a small ingot, the following conclusions can be proposed:

Importance of different geometries: In Fig.2, we have found that decarburisation of austenite during forging a **sheet** at 900°C-800°C, is low as compared with a preliminary heat treatment at higher temperatures (about 1100°C). This result is not surprising but it was not evident *a priori* that the importance of the thickness decrease during forging, which accelerates decarburisation, was not predominant. In forging processes different from Fig.2, this conclusion would certainly be different. When the shape of the metal varies from compact ingot to thin blade, through that of elongated bar, it is clear (Fig.3) that decarburisation is more effective, even during forging, due to 3 or 2 dimension diffusion.

Initial C content of the ingot: As we know from the Fe-C phase diagram, two kinds of situations are possible for Wootz preparation, depending on the heating power of the furnace. Yater (cited by¹) thinks that charcoal fire would not allow a crucible temperature in excess of 2400°F (1315 °C). Mott (cited by¹) has obtained a mean temperature of 1425°C in a 30 cm bed, but this small bed does not seem to correspond to the furnace described by Buchanan¹² or other authors, where the number of crucibles was as high as 20. It is sure that many different configurations were used in different places and different times. The furnace described by Massalski³ in 1841 is close to those used by Huntsman in Sheffield.

If melting at temperatures of 1400-1450°C was possible, no 1st sequence of decarburisation was necessary. This could be the case of finished blades with carbon contents between 1.2-2%. Forging a compact ingot like a 60 mm diameter sphere, into a 5 mm thick blade, at 900°C-800°C (Fig.3) or at lower temperature, produces a decarburisation ratio $C_m/C_o \sim 0.86$ (S1 to S5), which corresponds to a final carbon content of the blade between 1 and 1.7%.

If melting temperatures lower than 1400°C were only accessible, for instance 1300-1350°C, a carbon content of 3 to 2.5 %C was needed for complete melting. Then a preliminary S₀ sequence is necessary before forging, to avoid cracking under the hammer, as explained above.

Decarburised thickness

Criteria of decarburisation efficiency have been chosen from the Fe-C phase diagram in two complementary directions:

First, the layer of metal that must be filed away to discover a nice Damascene microstructure was calculated at about 20% of the total weight of the

ingot, in the process described above. The criterion is selected from the solubility of carbon in austenite in the last forging temperature sequence. In the present case this temperature was 800°C and the limit of “strong decarburisation” is 1%C, for the surface layer which will not have a Damascene structure. *The 20% metal loss corresponds to a decarburisation treatment of 2 h at 1100°C on a 60 mm diameter ingot.*

Secondly, the thickness of the ductile layer working as a container against metal cracking during forging has been discussed, when the ingot has a carbon content in the cast-iron range. The criterion proposed is: the carbon content in the centre of the metal must be lower than 2%C, following the results of Sherby¹⁰. In this case, *a heat treatment of 30 h about 1100°C* would be needed on a 2.3%C ingot. This time is of the same order of magnitude as those given by several authors (Voysey, Heath, Schwarz)¹.

We must remember that decarburisation takes place mainly in the first hours at temperature and that its rate decreases strongly with time, for two cumulative reasons: the decarburised layer makes a longer path for migration of carbon atoms to the surface and the diffusion coefficient of carbon in austenite decreases when C% decreases in the decarburised outer layers. Massalski³ has described a convenient way that consists, after a certain working time, to file out the surface of the bar before forging a blade. This process would certainly accelerate a further decarburisation stage but with an important loss of metal, depending on temperature.

Finally, the present numerical model can be used as a tool for simulating many different forging situations of small ingots such as Wootz, or bars and thin blades, to give reasonable order of magnitude for decarburisation during a given process, and help to reconstruct ancient techniques.

REFERENCES

1. J. D. Verhoeven, “Damascus Steel, Part I, Indian Wootz Steel”, *Metallography*, 20 (1987) 145-151.
2. B. Bronson, “The making and selling of Wootz, a crucible steel of India”, *Archeomaterials* 1.1 (1986) 13-51.
3. Massalski, “Préparation de l’acier Damassé en Perse”, *Ann. du Journal des Mines de Russie* (1841) 297-308.
4. B. Zschokke, “Du Damassé et des lames de Damas”, *Rev. Met.*, 21 (1924) 635-669.

5. H.K. Badeshia, "Diffusion of carbon in austenite", *Metal Science*, 15 (1981) 477-479.
6. J. D. Verhoeven, "An Analysis of Decarburization of Hypereutectoid Fe-C Alloys and its Application in Evaluating the Processing Steps Used to Make Damascus Steel Swords", *Materials Characterization*, 25 (1990) 221-239.
7. G.H. Geiger, D.R. Poirier. *Transport phenomena in metallurgy*, pp 486-490, Addison-Wesley (1973).
8. J.D. Verhoeven, "Wunder der Schmiedekunst Damaszenerklingen", *Spektrum der Wissenschaft*, (2001) 62-67. (Scientific American, January 2001).
9. J.D. Verhoeven, A.H. Pendray, "Experiments to Reproduce the Pattern of Damascus Blades", *Materials Characterization*, 29 (1992) 195-212.
10. J. Wadsworth, O.D. Sherby, "On the Bulat-Damascus steel revisited", *Prog. Mat. Sci.*, 25 (1980) 35-68.
11. B. Walser, O.D. Sherby, "Mechanical behaviour of superplastic ultrahigh carbon steels at elevated temperature", *Metall. Trans. A*, 10A (1979) 1461-1471.
12. F. Buchanan, *A Journey from Madras through the countries of Mysore, Canara and Malabar*, London 1807.

APPENDIX A

STRUCTURE OF THE COMPUTATION

As defined in § 2.1, each forging step has a length time Δt and a reduction factor $\Delta E/E$. A sequence is defined over $[k\Delta t; (k+1)\Delta t[$, with $k = 1, N$, where N is the number of steps of the sequence. E_k is the thickness after k steps.

Using Fick's Law, the governing equation is: $\frac{\partial C(x,t)}{\partial t} = D \frac{\partial^2 C(x,t)}{\partial x^2}$, $\forall x \in]0, E_k[$.

During each step the domain length is reduced by a constant factor $r \in [0,1]$, with $r = 1 - \Delta E/E$. As a consequence $E_k = Er^k$.

The initial condition is: $\forall x \in [0, E] : C(x,0) = 1$ and the boundary condition: $C(0,t) = 0 = C(E_k, t) \quad \forall t \in [k\Delta t; (k+1)\Delta t[$.

For, $k \geq 1$ a numerical discretization of the linear diffusion equation over a moving domain is required in order to obtain the concentration distribution over time and space as the domain size is reduced geometrically. A finite-difference technique is used which needs a temporal grid and a spatial grid. For simplicity we will present here the discretization with a uniform spatial grid of the domain

$[0, E_k]$ with constant step size $\Delta x = \frac{E_k}{I}$, and time step Δt . There are $I+1$ spatial grid-points, $(x_i = i.\Delta x)_{0 \leq i \leq I}$. The approximation of concentration $C(x,t)$ at $(x_i, t_n) = (i.\Delta x, n.\Delta t)$ is denoted C_i^{n+1} .

A second order accurate Crank-Nicholson approximation, is applied for the temporal discretization. The finite difference approximation of the diffusion equation in a Cartesian coordinate system uses a three-points stencil and is written at each grid-point:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = \frac{1}{2} \left(\frac{C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}}{\Delta x^2} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{\Delta x^2} \right), \quad \forall i \in]0, I[.$$

The discretization of the boundary conditions are: $C_0^{n+1} = C_I^{n+1} = 0$.

The spatial discretization of the diffusion equation and the boundary conditions is second order accurate in space. It can be proved that the Crank-Nicholson scheme is unconditionally stable, and that the computed solution converges to the solution of the diffusion equation when both $\Delta x, \Delta t$ tend to zero.

Let us define the sparse tridiagonal matrix A , the vector b^n and the unknown vector C^n as:

$$A = TriDiag \left((0,1,0), \dots, \left(-\frac{1}{2\Delta x^2}, \frac{1}{\Delta t} + \frac{1}{\Delta x^2}, -\frac{1}{2\Delta x^2} \right), \dots, (0,1,0) \right),$$

$$b^n = \left(1, \dots, \frac{C_i^n}{\Delta t} + \frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{2\Delta x^2}, \dots, 1 \right)^T,$$

$C^n = (C_0^n, C_1^n, L, C_i^n, L, C_{I-1}^n, C_I^n)^T$. Superscript T represents the transposed vector.

The numerical scheme can then be written as: $A.C^{n+1} = b^n$.

The linear system of unknown C^{n+1} has a unique solution since the matrix A is invertible. C^{n+1} is computed by the classical Gauss i.e. LU decomposition which splits the matrix into a product of a lower triangular matrix and an upper triangular matrix.

The computational algorithm implemented into our numerical software is the following:

- 1. Initialization: $\Delta x = \frac{E_0}{I}, \forall i \in [0, I]: C_i^0 = 1$
- 2. Computation of solution: Loop over time-step
 - Compute $C^{n+1} = A^{-1}b_n$
 - $E_k = rE_{k-1}, \Delta x = r\Delta x$
 - End Loop.

