Indian Journal of History of Science, 46.1 (2011) 159-165

BOOK REVIEW

C.K. Raju : ***Cultural Foundations of Mathematics : The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. AD,*** Volume X, Part 4, PHISPC, Center for Studies in Civilizations, Pearson Longman, Delhi, India, 2007; 477 pages.

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This book deals with history of important and very significant Indian contributions to advanced mathematics during the 14th and 15th centuries originating in what is now Kerala, India and the direct transmission of this knowledge to Europe through Portuguese traders and Christian missionaries in the 16th century. The book's primary focus is on approximately the time period from when Vasco da Gama (1460-1524), the first European (Portuguese) explorer, came (1498) to India and the year the Queen Elizabeth-I (1533-1603) of Great Britain signed (31st December, 1600) the charter for the East India Company which was formed initially for exploratory trade but eventually came to conquer and rule India.

This 477 page book has nine chapters organized in four parts. In the first part containing two chapters, the author dealt with the nature of mathematical proof with references to Hilbert (1862-1943) and Euclid (ca. 325 BC – ca. 270 BC) of Europe contrasting with the differing but much deeper thoughts of the Hindu civilization in India that created the revolutionary concept of '*śūnya*' or zero, the pivotal concept in the decimal positional number system that conquered Europe. The second part containing three chapters dealt with research and development works on advanced mathematics in India and the underlying practical needs that motivated this creative advancement. This is the most important part of this book. Chapter 3 deals with the infinite power series expansion of trigonometric functions and their inverses that originated in India in the fourteenth century onwards along with the associated conceptual and philosophical underpinnings.

Chapter 4 dealt with the needs for accurate computations of trigonometric functions arising out of necessity for reliable and accurate determinations of various terrestrial parameters (time, latitudes, longitudes, radius of the earth etc.) and dynamics of celestial bodies. Chapter 5 dealt with applications of this advanced mathematics in the creation of accurate navigational techniques for commercial applications in voyages through high seas. A remarkable application of this was demonstrated to stunned Vasco da Gama in 1498 when he learnt it first hand from an expert Indian Navigator, who accurately guided him to cross the equator in a perfect voyage through the mighty Indian Ocean to arrive at Calicut (11.25^0 N, 75.77^0 E), on May 25, 1498 in Kerala, India from Malindi (3.223611^0 S, 40.129944^0 E), Kenya.

The third part containing three chapters dealt with the transmission of knowledge of advanced mathematics to Europe in the sixteenth century that originated in India during the fourteenth and fifteenth centuries. Chapter 6 dealt with the circumstances and evidentiary considerations surrounding this assertion of direct propagation of advanced mathematics from the East to the West. Chapter 7 discussed the very hungry practical needs for advanced mathematical calculations required in accurate navigation through high seas that existed in Europe in the fifteenth and sixteenth centuries and beyond. Chapter 8 dealt with advanced philosophical concepts that were behind the Hindu civilization's invention of the zero (*śūnya*) and the following development of advanced mathematics that formed the foundation of numerical and computational mathematics in this digital age. The fourth part contains a chapter and an appendix dealing with the author's perception of differences in understanding and practices of mathematics, roles of empirical, formal and intuitive approaches, characteristic of the two distant civilizations, Indian and European.

Glorious contributions of the Hindu civilization in India towards advancement of science and technology on the world stage need to be studied, appreciated and correctly narrated to the world by its able descendants for education and enrichment of the minds of all — present and future. The author has taken this solemn responsibility to study and candidly present in this book an important documented contribution from India in the field of advanced mathematics and its direct transmission to Europe during the sixteenth century AD. This later contribution has its logical foundation in the positional decimal number system with the zero — the epoch making contribution of the Hindu civilization to the world in the first millennium and recognized all over the world. Special appreciation must be accorded to the General Editor Prof. D. P. Chattopadhyaya for allowing Prof. Raju the freedom to express his thoughts freely. All citizens of India, present and future should be grateful to Professor Chandra Kant Raju for undertaking this solemn task.

Diffusion of knowledge is a fundamental natural process continuously in action through all times in human history. The speed at which this action gets transmitted in space from one place to another depends on many factors. These include the medium of transmission, distance involved, practical importance of the knowledge and the social, economic and political conditions in existence along the various paths. In the history of European civilizations there are two important dates which are very relevant to this discussion. The first one is the Fall of Rome on 14th September 476 AD when Odoacer (435-493), the barbarian, threw out Western Roman Emperor Romulus Augustus and became the German King of Italy plunging Europe into the Dark Age (Great Indian mathematician Āryabhaṭa was born in India, the same year, 476 AD). The other one is the Fall of Constantinople on 29th May 1453 designating the fall of the Eastern Roman Empire to the Ottoman Turks. There were no scientific and mathematical works worth mentioning that originated in Europe during this long 977 years. In fact, there were no contributions worth mentioning towards advancement of science and technology in Europe since Claudius Ptolemy (90 AD - 168 AD) until Nicholaus Copernicus (1473 AD – 1543 AD) and Leonardo da Vinci (1452-1519). In order to cover up this embarrassing situation, some unscrupulous western historians of science and technology have and remain engaged in deliberate omission, distortion and outright falsehood in portraying the imported contributions of other advanced civilizations to bridge this long thirteen hundred years gap between Ptolemy and Copernicus.

The Dark ages continued for many centuries since the fall of Rome through 1202 when Leonardo of Pisa (more commonly known as Fibonacci) published his famous book *Liber Abaci*, and went beyond into the fifteenth Century AD.

Leonardo of Pisa or Leonardo Pisano was born in Italy around 1175 AD. Son of Mr. Guglielmo Bonachi, Leonardo Pisano studied the vastly superior Indo-Arabic arithmatics used by Arab traders in international commerce in North Africa around the Mediterranean. Fibonacci wrote the book, *Liber Abaci*, in 1202 on Hindu arithmatics based on decimal positional number system with zero as the tenth digit, invented in India and showed its (‘*Modus Indorum*’) vast superiority over the Roman number system, then still in use in many parts of Europe. Publication of Fibonacci's book is the definitive event that made this immortal invention of India in computational mathematics to establish the foundation for later advancements in mathematics in Europe including the contributions of Isaac Newton (1642-1727) and Leibniz (1646-1716) in the development of infinitesimal calculus.

Soon after its publication, this definitive book *Liber Abaci* by Fibonacci was widely circulated and read in Europe. The book described in details the superiority of the Hindu system of arithmetic calculations, based on the positional decimal number system with zero as the tenth digit, for day to day personal and commercial applications. Originally written in Latin, *Liber Abaci* has recently been translated into English by Laurence Sigler (Springer-Verlag, 2002, ISBN 0-387-40737-5). Since Europe, in the early thirteenth century, was still passing through the long lasting Dark Ages, it had taken several centuries for the '*Modus Indorum'* to take a firm foothold there. It is through this definitive book of 1202 by Fibonacci that the greatest contribution of the Hindu civilization of India conquered Europe and the world. Prior to Fibonacci's book, Brahmagupta's (598-668) book on Hindu mathematics, the *Brāhmasphuṭasiddhānta* appeared in Europe to limited number of scholars in 1126 in Latin, translated from the Arabic version, which was first translated by Middle East scholar Muhammad (ibn Ibrahim) al-Fazari (ca.735 – ca.806) from the original Sanskrit. It is eminently puzzling that Professor Chandra Kant Raju who has studied history of mathematics for at least a decade and himself a professional of eminence in the area of mathematics and computer science in India has failed to mention Fibonacci and his contributions correctly in this book. It is this failure that prevented Professor Raju from assuming the mantle and the tone of a conqueror to narrate the rest of the story and this is the fundamental weakness of this book and significantly diminishes its scholastic value.

It followed naturally then, that European scholars would be interested in further advanced developments of mathematics in India, useful for commercial and scholarly applications. Later works of advanced mathematics originally done in the Sanskrit language got translated into Latin, either directly or via the Arabic language, diffused into Europe and came into the hands of scholars who in those days were simultaneously involved in theology as well as in natural and physical sciences. A subject matter scholar primarily deals with the acquisition, assimilation and propagation of new knowledge and must not be automatically assumed to be interested in or knowledgeable about the true history of the subject. This is especially true when the acquired knowledge is from a distant civilization. Once the superiority and usefulness of the Hindu number system had been appreciated and established in Europe by the end of the fifteenth century, it followed naturally that further developments of advance knowledge in mathematics and astronomy originating in India will diffuse into Europe.

Indian mathematics scholar Mādhava of Saṅgamagrāma (1350-1425) created power series expansion techniques for accurate computations of trigonometric functions and their inverses for applications in astronomy and navigation. That knowledge must have gotten transmitted into Europe to various scholars including Vincenzo Viviani (1622-1703) and Stefano degli Angeli (1623-1697). As Europe was passing through dark ages learning, absorbing and appreciating the usefulness of the vastly superior Hindu number system, further advances were being made in India in calculations of trigonometric functions and their inverses. The demands came from the needs for better navigation and astronomy requiring very accurate calculations of those functions. In Europe, Fibonacci's introduction of the Hindu number system in day-to-day business calculations became so fruitful and appreciated that it attracted attentions of the Kings and Queens and generated greater and greater momentum to establish direct interactions with India via unknown and uncharted sea routes not just for spices but also for advanced mathematics etc. The fact that European knowledge about the size of the earth and navigation through its high seas in late fifteenth century was very primitive remains immortalized in Christopher Columbus's (1451-1506) 'Discovery' of the 'New World' (the Americas) in 1492 while on a voyage to India through an uncharted and unknown western route.

It is intuitively obvious that the knowledge of advanced mathematical calculations that makes more accurate navigation through high seas possible will be of immense interest to the European navigators and their sponsors. By the time direct link between Europe and India was established in the early sixteenth century through high seas bypassing the Middle East, mathematical knowledge originating in India in Sanskrit language, was getting translated into Arabic and then into Latin and appearing in Europe for many centuries, albeit slowly. It is indeed Mādhava's works that came into the attention of James Gregory (1638-1675) during 1664-1668 when he was sent (upon the recommendation of Robert Moray (1608-1673), one of the founders of the Royal Society, Great Britain and its first

President, to study advanced mathematics from Stefano degli Angeli in the University of Padua (Italy). Brook Taylor (1685-1731) was not yet born when Gregory published his power series expansion work in 1668, soon after returning to Scotland. By this time, Mughal Emperor Shāh Jahān in India had already built (1632-1653) the Taj Mahal. Immediately after the famous Lucasian Chair professorship of mathematics was created at the Cambridge University of Great Britain by Henry Lucas, the member of British parliament in December 1663, Isaac Barrow (1630-1677) was selected as the first occupant of the Chair. Barrow went (1656) to study mathematics from Vincenzo Viviani in Florence (Italy). Barrow transferred the acquired knowledge to Isaac Newton whose development of infinitesimal calculus was kept secret and unpublished for a long time with Newton not knowing that the knowledge of advanced mathematics that was imported into various points of what is now Italy was also accessible to Leibniz through some contemporary parallel routes including direct interactions with Vincenzo Viviani himself in 1689.

The great grandfather of recognition of mathematics originating from the Hindu civilization in India and conquering Europe during the first half of the second millennium is the brilliant work of Fibonacci (Leonardo of Pisa). Progress of the Hindu civilization in India was severely disrupted by the arrival of Islamic invaders of various kinds from the northwest. The marching baton of progress in mathematics and astronomy got abruptly transferred to the Europeans. If one needs further discriminating and conclusive DNA evidence of paternity of the Gregory series and Taylor series expansions of mathematical functions, one should check what mathematical knowledge Stefano degli Angeli of the University of Padua and Vincenzo Viviani of the University of Florence possessed and from where, when, and how! Because it is precisely through these two Italian mathematicians that the pioneering works of Mādhava and Jyeṣṭhadeva (ca.1500 - ca. 1575) of the Hindu civilization entered Great Britain in the second half of the seventeenth century. Possibility of practical usefulness of newly imported knowledge (for example, in navigation) is a strong valid reason for keeping the origin secret.

Arrival of increasingly powerful electronic digital computers from the middle of the twentieth century onwards has brought numerical and computational mathematics on the center stage of importance very rapidly. Finite difference methodology employed by Hindu mathematicians from Āryabhaṭa through Brahmagupta onwards to Mādhava and beyond can now be fully appreciated for their practical importance and advanced nature for the time period during which it was invented and applied.

In the Hall of fame for science and technology, Sir Isaac Newton (1642-1727), the second Lucasian Chair Professor of mathematics at the Cambridge University, Great Britain, appears very tall partly because he, according to his very own admissions, stands on the shoulders of Giants. Mādhava (1350-1425) of Saṅgamagrāma, India, who invented the power series expansion of various elementary functions is indeed one such Giant of mathematics. Some three hundred years after Mādhava, mathematician Brook Taylor (1685-1731) of Great Britain built (1715) a floor above Mādhava's work by adding what is now widely known as the Taylor Series expansion of a well behaved function that require existence of derivatives of the function. Beyond Mādhava and Jyeṣṭhadeva of India, the concept of derivatives of a well behaved function was further developed and advanced by Newton, Leibniz and many others in Europe. Therefore, Mādhava by virtue of his advanced work, as explained in Chapter 3 of this book, can and must rightfully be considered and accepted as the grandfather of calculus.

Present existence of the majestic pyramids in Egypt (the Great Pyramid of Giza was built around 2560 BC) conclusively proves that geometry was not invented by Greek mathematicians and scientists like Euclid, Pythagoras (ca. 570 BC – ca. 495 BC), Archimedes (ca. 287 BC – ca. 212 BC) or Claudius Ptolemy. Euclid was definitely not a Christian and since his original work does not exist, one can definitely doubt whether all the '*Elements*' really go back all the way to the time period of his stated existence. Professor Raju's apprehensions, permeating throughout this book, that true history of mathematics has been severely distorted by Christian religious zealots of medieval times are somewhat exaggerated. It is incumbent upon the able descendants of the Hindu civilization and not the Europeans, to study and narrate the true history of its glorious contributions in mathematics for educating the world. Professor Raju has assumed that responsibility and done his duty to the best of his ability. This book will serve as a starting source for further research in the history of mathematics. Inclusion of this book in the collections of all libraries around the world is strongly recommended.