GEORGE PEACOCK'S *ARITHMETIC* IN THE CHANGING LANDSCAPE OF THE HISTORY OF MATHEMATICS IN INDIA

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The changing landscape of the history of numbers and arithmetic in India at the beginning of the XXth century is examined, after a detour in mid-XIXth century France, through the debate that opposed G. R. Kaye to a group of Indian historians of mathematics and astronomy on the origins of the decimal place value position. This study highlights how Peacock's historical analysis of algebra and arithmetic's genesis seems to have been singular and isolated in mainstream histories of science. It also chronicles the birth of the scholarly field of the history of mathematics in India.

[Cet article examine, après un bref détour dans la France du milieu du XIXème siècle, le paysage de l'histoire des nombres et de l'arithmétique dans l'Inde du début du XXème siècle en décrivant le débat qui y opposa G. R. Kaye à un groupe d'historiens des mathématiques indiens. Cette étude met en lumière comment le point de vu historique de Peacock sur la genèse de l'algèbre et de l'arithmétique semble être resté isolé et singulier dans les milieux plus institutionnalisés de l'histoire des sciences. Cet article raconte aussi la naissance du champs savant de l'histoire des mathématiques en Inde].

Key words: Chasles, Das, Datta, Ganguly, History of Arithmetics, History of the decimal place-value notation, History of Mathematics in India, Kaye, Libri, Orientalism and history of mathematics, Peacock, Singh

INTRODUCTION

This paper was originally an inquiry into the Indian reception of Peacock's *Arithmetic*. It quickly became an attempt to understand why Peacock's historical

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enterprise never met a public: why was it not taken into account, during the XIXth century and at the beginning of the XXth century, in the debates that took place first in Europe, then in the subcontinent, on the origins of the decimal place value notation? Peacock's (history of) *Arithmetic* was an English mathematician's vigorous plea in favor of a sub-continental origin of the decimal place value notation, grounding arithmetic on algebra in a fashion reminiscent of the Sanskrit author Bhāskara II. His historical writings could have been a sure reference for Indian scholars attempting to valorize their past tradition in mathematics. To understand this absence of Peacock's point of view in the debates, one had to chronicle the changing landscapes of the history of mathematics in the Indian subcontinent. The topics and questions raised at the end of the XIXth century will form the basis of Indian scholarship during the XXth century. The vast landscape uncovered and the debates encountered show how, retrospectively, Peacock's attempt was singular and isolated.

The *Encyclopaedia Metropolitana*, Peacock's *Arithmetic* and India

A haze surrounds the publication and (lack of) reception of the *Encyclopaedia Metropolitana*. As for all encyclopaedias, its aimed readership was universal[1]. The *Metropolitana*, however seems to have been specifically devised to cater the needs of those who were forging the British Empire far from the British Isles:

> 'An Encyclopaedia is indispensable to every library (…) As a concentration of human knowledge; (…) while to the Voyager, the Naval and Military Officer, the Colonist, and that numerous class of enterprising Britons whose want of a settled residence may isolate them from the world of letters, it is the only possible substitute for all other books'[2].

Once published, the *Metropolitana* was probably shipped through out the British Empire. I have not found any figures regarding its diffusion in India, or elsewhere. To what sort of institutions was it shipped? Did private colleges or missionary funded schools pursue copies? Fieldwork at the Indian Office gave no information. The libraries I visited in India, which were created under the British Rule, and belonged to British institutions[3], all had copies of the *Metropolitana*[4]. Lest do we know if this encyclopedia was read, who read it, or what part of it was read. The *Metropolitana* was overwhelmingly a commercial failure[5], this may be why so little information on its posterity is available.

Peacock's point of view on Indian Arithmetics

As noted in M-J. Durand-Richard and Dhrub Raina's articles, George Peacock (1791-1858) in his (history of) *Arithmetic* believed that the decimal place value notation because of its close relation to the body (base ten) and its operative structure was an important stage in the development of mathematics. The use of separate symbols for numbers was also a landmark but less important. Little of this point of view can be found in Peacock's arguments in favor of an Indian origin of the decimal place value notation. A precise rigorous historian he was intent on the details of each case study. He did not sweep from specific cases to general statements. As mentioned previously by D. Raina, for Peacock the early use of names for very big numbers in base ten was the main reason to believe in an Indian origin[6]:

> This luxury of names for numbers, much greater than what are required for the ordinary uses of life, or even for the most extended astronomical calculations, is entirely without example in any other language, whether ancient or modern; and implies a familiarity with the classification of numbers according to the decimal scale and the power of indefinite extension which it possesses, which could only arise from some very perfect system of numeration, such as that "with device of place" …we should be inclined to assign to the Sanskrit terms for such numbers, and consequently to the system of numeration, upon which they are founded an antiquity at least as great as their most ancient literary monuments; as the arbitrary imposition of so many new names, for the most part independent of each other, and in number also so much greater than could be required for any ordinary application of them…

He also argues that since Sanskrit authors consider it a divine invention (together with zero and algebra), it must be very old[7]. This thus gave an important place to "Hindoo" mathematics in History[8]. Consequently, Peacock's description of Sanskrit arithmetical procedures (elementary operations including root extractions and computations with fractions, Rule of Three, Rule of alligation) insist on their ingenuity and their proximity with contemporary practices[9], noting at times however that they were limited by an absence of adequate symbolism for elementary operations[10].

Peacock's historical arguments concerning the origin of the decimal place value notation then are somewhat separate from his theoretical point of view on the genesis from natural structures of arithmetic and then algebra. Thus to trace his influence, we will look for direct reference to his work, and indirectly at the

way historians situated the decimal place value notation in a link to further arithmetical and algebraical developments.

Multiple landscapes in XIXth century historiography of science

During the time Peacock wrote his text (1826), had it published (1845) and saw it shipped within the *Metropolitana* to India, the intellectual landscape had underwent many changes in Great Britain and in India. The algebraist's influence had wavered in Cambridge[11], and the radical "anglicists" who did not believe in the "orientalist" engrafting of civilization on Indian culture had administratively won the battle over policies of colonial education in India[12]. Of course, such failures did not prevent the existence of niches, where European and Indian scholars could continue to interact and exchange[13], or algebraist to network and debate. In this landscape, who could have been interested in the history of mathematics in the Indian subcontinent and in Peacock's *Arithmetic?* Indeed, in the following, the presence/absence of Peacock's ideas in histories of arithmetic written in the Indian sub-continent will be but a guide to fray a path in the more general topic of how the history of arithmetic was first narrated by Indians: to what assertions were they replying? What were their historiographical positions? In the end, how Peacock's *Arithmetic* was concerned with a set of different problems will become obvious, explaining partially how little reception it indeed had[14].

Debating on the origins of numbers and notations

The question of the origins of "Indian numerals"[15] and of the decimal place value notation was a recurrent theme of the history of arithmetics in Europe and in the Indian subcontinent during most of the XIXth century. It gave rise to fierce debates. The origin of present day numerals and numerical notation was but one in the midst of others controversies, such as the origins of algebra, of Diophantine indeterminate equations, or of the signs of the zodiac. The discussions on the origins of the decimal place value notation culminated with a wave of Indian criticism of the "western origin" point of view in the year 1927. One of the key polemical texts of this debate in the Indian sub-continent, G. R. Kaye's "*Notes on Indian Mathematics- Arithmetical Notations.*" published in 1907, refers to Peacock in its introduction[16]. G. R. Kaye, an amateur indologist and historian of science will become the favored Punch of the emerging community of Indian historians of mathematics. The publication, in 1935, in Lahore, of Datta and

Singh's *History of Hindu Mathematics[17],* will slowly put a term to these debates[18]. Revealingly, this book devotes approximately 120 pages to Indian numerals, another 120 to arithmetic and 400 pages to algebra.

A first part of this article evokes the debates in Europe on the origin of numbers, providing the backdrop on which Indian scholars will take a stand in the second half of the XIXth century. A second part looks closely at the positions taken by G. R. Kaye, while a third observes how Ganguly, Das, Datta and Singh replied on the origin of the decimal place value notation. The marginal position of Peacock's genesis and history of *Arithmetic* in this context will be underlined.

I. DEBATING IN EUROPE

François Charette and Dhruv Raina have shown that in Europe at the beginning of the XIXth century a debate opposed (English) philologists and (French) astronomers/mathematicians over who had the authority to write the history of astronomy and mathematics of non-western science. By the 1840's however, with the progress of orientalism and philology this debate had disappeared and seemed to be rather fueled by nationalistic points of view[19]. Concerning the history of arithmetic in India, the original link of linguistics and history of mathematics described in Dhruv Raina's article remained pre-eminent. If by the 1920's mathematical arguments will find their way in historical arguments, most exchanges will be on the philology and history of texts, not on their technical contents. Debates will fist be concerned with the authenticity of sources ascertaining the early use of the decimal place value notation. Indeed many fakes seem to have been forged during this period, highlighting the material and symbolic value of the subcontinent antique culture textual proofs. Patriotism and political antagonisms were the discussions underlying fuel. As the XIXth century will come to an end, several different technical arguments will emerge. None use theoretical or practical arguments concerning the generation of arithmetic from natural structures, or the generation of algebra from arithmetical structures, that would indicate Peacock's imprint on the debate.

I.A The debate on the origin of "Arabic Numerals"

By the time the *Metropolitana* was published, the question of the Indian origin of the « Arabic numerals », from their script to the use of a positional decimal notation had been challenged, as we will see. A concert of conflicting opinions voiced by authors who do not always seem to know that their publications

contradict one another can be heard in succeeding articles from the late 1830's to 1907, the date of Kaye's key polemical article on the origin of Indian numbers. Thus, Peacock's arguments to demonstrate their Indian origin, might have been considered dated by the second half of the XIXth century. Whether in Paris, London, Rome or Calcutta scholars speculate on the Arabic, Greco-roman or Indian origins of the numeral system that we use today. Obviously several groups and separate networks were at play in these discussions. The way they interacted, the values they defended, still needs very much to be mapped out.

I have listed so far a total of 99 titles dealing with the question of the origin of numbers and arithmetic published between 1827 and 1907 in Europe and in India. I have not looked at all of them and do not think this list is exhaustive. The conflict was mainly on the linguistic and epigraphic evidence that would prove the existence of systems of numeration.

I.B Chasles and Libri

A famous, often briefly chronicled[20], virulent argument on the origin of numbers and of algebra opposed in the late 1830's and the beginning of the 1840's Guglielmo Libri (1803-1869) and Michel Chasles (1793-1880) at the academy of science in France[21]. The feud lasted until Libri's death.

In 1836, Chasles read an apocryphal part of Boece's « Geometry » as a proof that by the Vth century AD, people in Europe, especially in France new of a decimal place value system, "*le système de l'abaque*", used in computations but not for noting numbers. This part of Boece's "Geometry" was already well known to those interested in the history of numbers. It was understood as representing a multiplication table ("*mensa pythagorica*") using "Arabic" numbers. Peacock actually mentions this passage in a note. He discards it as not belonging to Boece's originial text, since it uses "Arabic" numbers that are not used elsewhere in the text or in older manuscripts[22]. Chasles provided a new interpretation of the passage, understanding it as describing a tabular (*abaque*) system for computing with numbers. According to Chasles then, later Italian authors such as Fibonnaci would have been influenced both by Arabic authors and by this "occidental" system. Libri was publishing the first volumes of his "*Histoire des sciences en Italie*" [23]. An important part was devoted to the question of the Indian origins of arithmetic, algebra and astronomy used by Arab and subsequently Latin authors. Libri maintained an "oriental" origin of the positional notation in his argument with

Chasles, on the grounds that a single dubious text could not prove such an origin. Chasles and Libri also disagreed on the origin of algebra, Chasles believed Viete (and France) was a pioneer in algebra downplaying the influence of what in Libri's point of view was an Indian influenced Arabic algebra, transmitted to Europe first by the work of Fibonacci.[24] They quibbled on other themes as well, they quibbled among themselves and with others[25].

A strong emotional ring strikes the modern reader. Chasles went as far as ridiculing himself publicly on the question of the "*système de l'abaque*". He asked in 1839 the authoritative opinion of an English scholar, J. O. Halliwell (1820-1869)[26]. Halliwell replied, declaring his interest in the theme explored by Chasles, but highlighting the dubious nature of the part of Boece's "Geometry" that grounded Chasles' system. In a case of delusion that we will meet later on, Chasles seemed to have brushed aside the part of Halliwell's observations that bothered him. Presenting publicly Halliwell's "approval", he thus had to face Libri's wry ironic clarification in front of other members of the academy[27].

If debate between Libri and Chasles there was, it was no dialogue. Rather a public, vocal, opposition of what might have rather been true political, social and personal antagonisms on which not very explicit epistemological differences were harnessed. Chasles was the son of a revolutionary bourgeois, a vibrant catholic French patriot. Libri a liberal aristocrat, who took an active part in the 1830 French restoration of monarchy[28], was also a keen Italian patriot in history of science. The Chasles-Libri feud finally played itself out in institutional battles[29]. Chasles, but a "correspondant" of the *Académie*, wished to be a full member. This became possible when Libri, who had a position in the geometrical section since 1833, was expelled sometime in between his flight from Paris in 1848 and his judgement in 1850, for stealing manuscripts in French public Libraries[30]. From a distance, old and new conflicts continued to be fought[31].

Historiographically, Chasles subscribed to the primer of synthetic geometry over algebra[32]. Libri, on the other hand, was interested in linking mathematical developments to mainstream history. In the beginning of his *Histoire des sciences en Italie*, he sets forth a program striking for the importance it gives to the relation of mathematics with people[33.] Libri further argues that symbolism was one of the main strengths of Indian arithmetic and algebra[34]. Theoretically then Libri appears closer than Chasles to the "algebraical network". Indeed, De Morgan's letters show that he took an early interest in Libri's historical endeavors, following from

afar Libri's and Chasle's exchanges from the mid-1850's onwards.[35] Morgan's loose connection to Libri does not however seem to extend to any influence of Peacock's historical and genealogical theory. No influence of Peacock can be found on Libri and in the arguments exchanged with Chasles. Rather, Libri's benevolence toward an Indian past in the history of mathematics has probably equally to do with his "enlightenment" culture and values, and with the fact that this Indian presence could put Italian scholars into the limelight.

In the end, this debate will be especially famous for opposing a thief to a forgerer's victim with acrimony on both sides. Stories of fakes and thefts, political affiliations and institutional reconnaissance will be a constant feature in the history of positional numeration in India, and for this reason important to underline here.

I.C European Indologists also take up the arguments

By 1841, Libri, Chasles, and Peacock all belonged to the newly founded and shortly lived *Historical Society of Science*. Its creator was none but O. Halliwell. Augustus De Morgan was in the original council[36]. Thus, however conflicting, however different in intellectual backgrounds, there was a loose European network of scholars interested in the history of arithmetic and algebra.

Libri's public engagement with history of mathematics had further consequences, because he had an extended network of colleagues and friends. For instance, he inspired Joseph Toussaint Reinaud (1795-1867), a friend of his, professor of Arabic at the *Langues Orientales* and member of the *Academie des Inscriptions et Belles Lettres*, to include all that he could find relating to numbers while compiling Arabic sources on India[37]. Reinaud thus published in the mid 1840's information on Al-Bîrūnî (973-1048), a Persian astronomer whose testimonies on the numeration system used in India were often discussed subsequently to defend an Indian origin of the decimal place value notation[38]. Al-Bîrūnî's text is still today a landmark testimony on the history of mathematics and astronomy in India during the XIth century[39].

English Indologists seem to have ignored the *Arithmetic* as well. Thus, a debate will oppose James Prinsep (1799-1840), once head of the *Asiatic Society of Bengal* and Edward Thomas (1813-1886), an East Indian Company employee specialist of numismatics on the interpretation of numerical data on early copper plate inscriptions. James Prinsep published an article in 1834 reading some

inscriptions as containing proto-symbols for numbers, which he supposes might have been derived from letter symbols. Edward Thomas replies in 1848 and 1856, showing that these symbols are independent from any alphabet and are not positional. In 1858, Edward Thomas will compile posthumously James Prinsep's articles, including new extensive replies and corrections.

This technical discussion, which incidentally provides quotations of Reinaud's work[40], will open an epigraphic strand in the debate on the origins of Indian numerals. Libri and Chasles did not discuss numerals appearing on copper plates, coins and stone inscriptions. Epigraphic notations testify of lay uses of numbers; Sanskrit scholarly texts preserved in palm-leaf manuscripts testify of scholarly practices of numbers. At a given time, lay practices and scholarly ones could be different. This distinction was often blurred by polemical historians, increasing the confusion of the debate.

The social networks drawn by these examples, from the *Historical Society of Science*, l'*Académie des Sciences*, *Académie des Inscriptions et Belles Lettres*, to the *Asiatic Society* and the "Algebraical network" questions how separate these worlds were, how they interacted and what ideas circulated. These arguments show how little those who engaged with them seemed preoccupied with the set of theoretical questions raised by Peacock. Patriotic historiography of science mingled with the unreliability of sources. Emotion then might be the most important feature of these historical discussions, fueling the energy to write and rewrite answers and arguments even when those concerned are in exile or dead.

I.D Technical aspects of the discussions

By the end of the XIXth century, arguments to establish the origin of the decimal place value had taken on three threads. They will be described, explaining how they are understood today, before looking back at how G.R. Kaye and his opponents dealt with them.

The first thread wondered whether different traces of alphabetical notations in India had Greco-latin origins or not. Alphabetical notations are those that use a letter (in Sanskrit a syllable) to represent a number. An alphabetical notation will thus concatenate letters as in a word (and sometimes allow puns) to represent a number. For instance in the *kaṭapayādi* system of noting numbers, the expression *bhavati* which means «he/she is/becomes» notes 644[41]. The first European scholars

who encountered alphabetical systems of noting numbers in the Indian sub-continent, such as James Prinsep, were tempted to analyze these notations as traces of an early Greek influence, since Greeks used to note numbers with letters[42]. Peacock did not believe in their existence, since it seemed to him much more inconvenient than the simple decimal place value notation. He described Anquetil du Perron's testimony of such notations as "one of his numerous other dreams which have been found to have no foundation in fact"[43]. Alphabetical notations were often used in scholarly contexts: they enabled to list numbers in verse form. Lay inscriptions, more often used regular number names or notations. Chasles and Libri do not seem to have taken such instances in account in their debates. After Prinsep's hunches, most exchanges were concerned with epigraphic rather than with Sanskrit scholarly treatises. A specific way of expressing big numbers, on copper plate inscriptions as well as in astronomical texts, was often studied and described: numbers were noted as we do, with the smallest digit on the right and then from right to left advancing in power ($\leftarrow$). For instance: «664». A list of digits nominally came with these notations in order to prevent mistakes in their transmission; it usually listed them in reverse order, from left to right ($\rightarrow$). For instance, the *kaṭapayādi* notation given previously, *bhavati,* actually reads: 'four *(bha)*, six *(va)*, six *(ti)*'. This way of stating larger numbers, even when no noted numbers comes behind it, is a proof of the use of a place value notation. G. R. Kaye will focus on this double way of stating numbers, insisting on the mysterious difference in order of the digits.

The second thread wondered whether the « invention » of zero and of the decimal place value notation came from India. Indeed, «zero» as «an empty space» in a positional notation, was thus a sign of the use or not of the decimal place value notation. Zero as a number on which elementary operations can be computed was less discussed. For Peacock, although he felt the need to argue in favor of this, there was no doubt possible: the zero was of Indian origin.

Numerous ways of noting numbers were found in old inscriptions. Endless bickering on the date and reliability of copies of copper plates fuelled a large number of exchanges. Thus even Peacock notes (op. cit):

> "If the royal grant of land engraved on a copper plate found in the ruins of Mongueer, and translated by Dr. Wilkins, be not a forgery, it would furnish evidence of the existence of this notation at a much earlier period than any which we have mentioned…"[44].

Until today the date of the earliest epigraphic testimonies for the decimal place value or zero are still unclear[45]. Increasing scholarship will evolve from a search for very old inscriptions hoping to uncover a numeral scripts with a decimal place value notation and a zero, to intricate analysis of Brāhmī and Kharoṣṭi inscriptions, Gurjara copper plates and the way they represent digits. Medieval Sanskrit scholarly texts will also progressively be edited and translated during the second half of the XIXth century. Consequently, part of the debate will fade away since these texts included definitions of the decimal place value notation. The oldest scholarly mathematical text to define the decimal place value notation in India is the *Āryabhaṭîya*[46], composed at the end of the Vth century. Following scholarly texts will contain such definitions or include algorithms which imply its use, such as the procedure to extract square roots. The *Bhakshālî Manuscript* (ca. VIIth-IXth century), Brahmagupta's work (VIIth century) or Bhāskara II's (XIIth century) will often be quoted by European and Indian scholars alike.

Finally, the third thread debated the existence of an early abacus in India. Its existence was seen as the principal vector of transmission of the decimal place value notation, in one way or an other. For an obscure reason the company officer and indologist Edward Clive Bayley (1821-1884) stated in an article of the Royal Asiatic Society that abacuses were of use in every small bazaar in India[47]. European historians of sciences, such as Léon Rodet (1850-1895), or Indologists, such as A. C. Burnell (1840-1882), used this text to argue in favor of an Indian origin of the decimal place value notation. Nowhere is the abacus considered, Peacock style, as characteristic of lay people's computation or that counting in base ten can be seen as a "natural" thing to do. The kind of device such a word refers to is not defined[48].

In these threads, the history of numbers is not thought of as belonging to a theory of knowledge another of Peacock's characteristic ideas.

II. G. R. Kaye's booming opinions

G. R. Kaye was a member of the Department of Education of the Government of India, (« Bureau of Education », Simla in North India) with a passion for history of mathematics, astronomy and astrology. In July 1907 he published an article in the Journal of the Asiatic Society of Bengal entitled *Notes on Indian Mathematics.- Arithmetical Notation*[49]. The article starts as follows:

'We are told that our modern arithmetical notation is of Indian Origin. Peacock, Chasles, Woepcke, Cantor, Bayley, Bühler, Macdonnell and others

state this more or less emphatically, and the encyclopaedias and dictionaries follow suit.'

Note here that mathematicians (Chasles, Peacock) and philologers (Bühler, Macdonnell) of all origins (German, American, French and Scottish) are grouped together. In a typical G. R. Kaye mode, the list includes a *faux sens*: Chasles was in fact against the idea of an Indian origin for numbers. The reference to Peacock probably echoes in his allusion to encyclopaedias. Similarly, in the first part of his article, Kaye starts by dismissing commentaries as proofs of the antiquity of the rules they comment. He remarks that they are of much later origin than the texts and that they have "fanciful" statements. Althought Kaye quotes here Colebrooke and Rhys Davids he may also be thinking of Peacock, who uses these kinds of arguments.

We know very little about George Rusby Kaye (1866 -1929), but that an English man, he worked at the Department of Education of the Government in India from 1899 to 1923, before retiring back to Great Britain where he would have died in 1929. His file at the Indian Office in London testifies that as a bibliophile he was subsequently employed to compile manuscript catalogues for the British Library. After the 1907 article, G. R. Kaye will continuously publish in the early teens of the XXth century, before achieving in 1915 a synthetic publication on "Indian Mathematics"[50], and another one on "Hindu Astronomy" in 1924[51]. He will then turn to the study of a birch wood manuscript, the *Bhakshālî Manuscript* that he will edit and publish in 1927[52]. This is roughly the time when his opinions will prompt a vehement reaction from S. Ganguly, S. Das, A. N. Singh and B. Datta.

G. R. Kaye wanted to establish that the numerals and the decimal place value position did not come from India. He will argue in this direction by trying to invalidate the arguments put forth in all three threads.

The first thread, the question of alphabetical notations, he will tackle in the above mentioned 1907 article on Indian numerical notations. This will be followed by arguments developed in his 1908 article on Āryabhaṭa's mathematics. He will come back to this point, back-referring to this article in his 1915 article called 'Indian Mathematics'. His main argument rests on a misinterpretation of Āryabhaṭa's alphabetical notation. H. Kern (1833- 1917) had edited Āryabhaṭa's *Āryabhaṭîya* in 1874[53]. The first translation in an European language, was made by Leon Rodet (1850-1895), in French, in 1879[54]. The first English translation

appeared much later, due to P. C. Sengupta (1876-1962) in 1927[55]. This text contains two rules to note numbers, one "alphabetical" stated in its first chapter, the *gîtikāpāda*, a second, which defines the decimal place value notation in its second chapter, the *gaṇitapāda*. Kaye will concentrate on the "alphabetical notation", first omitting the second definition, later referring to it seemingly not realizing how essential it was to his demonstration that it invalidated.

Āryabhaṭa's alphabetical notation has been extensively described, discussed and interpreted, with or more less clarity in all secondary literature on the subject[56]. It uses base ten, and is semi-positional[57]. Kaye seems to have not understood how the notation worked. He repeatedly declares that Āryabhaṭa's notation is not positional. He also discusses non-positional alphabetical notations existing in India. Confusion attains its peak when he looks for epigraphic of these scholarly devices. Because he does not find any, he concludes that in any case these alphabetical notations cannot be of Indian origin.

Concerning the second thread, Kaye aims at establishing that Arabic sources pointing towards an Indian origin for their numerals and arithmetic have been misinterpreted. He thus denounces spurious copper plates, absurd statements in commentaries, and uses philological arguments of all sorts. For instance, in his 1907 article, Kaye discusses Woepcke's interpretation of the adjective « hindasi » used in Arabic to qualify computations. Does « hindasi » mean « Hindu », or is derived from the word « measure » (*andazah*)? Kaye devotes four pages of discussions quoting numerous Arab philologers to defend the second point. His main argument then is to note the discrepancy in between the way the numbers are enumerated and the way they are noted, implicitly considering that the notation was imported from a place where people wrote from right to left[58]. Kaye includes in his discussion a number of texts of the scholarly Sanskrit tradition. He discusses Colebrooke's interpretation of Brahmagupta (VIIth century), Bayley's interpretation of Āryabhaṭa (Vth century) and Hoernle's dating of the *Bhakshālî Manuscript* (VIIth-IXth century) denying in all these texts the presence of a decimal place value notation. G. R. Kaye, all along, alludes to an Arabic origin for these notations. The 1907 article ends by stating vocally:

> « We can go further and state with perfect truth that, in the whole range of Hindu mathematics, there is not the slightest indication of the use of any idea of place-value before the tenth century A. D. »[59]

We are here in one of these strange but familiar moments that history of science encounters, usually in stories of science: the denial of facts. How can we

understand G. R. Kaye's attitude? He certainly had access to texts that discredit such a claim. His mastering of Sanskrit may have been insufficient. Furthermore, he seems to have systematically overlooked any evidence that went against his convictions. This attitude is especially clear if we watch how he treats Āryabhaṭa's definition of the place value system. While this definition is not alluded to in the 1907 article, in 1908 he devotes an article to this astronomer, and gives a translation of the definition (p. 117[60]). He first concentrates on the names given to the different powers of ten, and then questions the existence of specific symbols to note numbers (p. 118), omitting any reference to positions that are however clearly stated in the translation he gives of the verse. It is true that he doubts that the author of the second chapter is indeed Āryabhaṭa (pp. 115-117). By 1915, never stating explicitly that the known text of Āryabhaṭa gives a definition of the place value notation, he attempts to show that "the work of Indian mathematicians from Āryabhaṭa to Bhāskara are essentially based on western knowledge"[61]. "Western knowledge" meaning in this case a Greco-Latin origin transmitted through Arabic intermediaries. The existence of a definition of the place value in early Sanskrit mathematical texts certainly nagged at him. He used all possible resources to disqualify such a definition, without ever clearly referring to it.

Finally, concerning the third thread, Kaye published a separate article on the abacus in 1908[62]. This article collects evidences of all sorts of different modes of computations, from written notations to computing instruments, all grouped under the name 'abacus'. He shows that there is no testimony of their use in ancient India.

G. R. Kaye, then, does not need to discuss and refute Peacock. Just one abrupt statement seems to be addressed to the readers of the *Arithmetic,* in the 1907 article. Kaye reasons that the decimal place-value notation is a sign of culture, not of nature.

> 'The popular idea that the order of our (European) arithmetical notation
> is the more natural and convenient order is not correct.'[63] .

Note the paradox: the idea is qualified as popular, but in relation to the decimal place-value notation in India, we have encountered it in two texts only, this count includes G. R. Kaye's article. Such an idea, then may have been current in circles that were not writing on the history of mathematics in India.

G. R. Kaye's articles were read. His works were quoted by D. E. Smith, Karpinsky[64] and Cajori[65]. Sarton will publish him in *Isis*[66]. G. R. Kaye's popularity

will prompt a new generation of Indian historians of mathematics to appear loudly on the scene.

III. Indian Historians take the stage

III.A Forging an Indian History of Science

How history of science in the XIXth century was written by Indian scholars has been given some attention since the 1990's[67]. As shown by Dhruv Raina[68] the two first histories of Science in India, published at the end of the XIXth century and beginning of the XXth century by B. N. Seal and P. C. Ray (1861-1944) adopted, with some nuance, the values of positivist history of science to oppose a growing literature that tended to describe India as having no scientific tradition of its own. P. C. Ray was a scientist while B. N. Seal was professor of philosophy at Calcutta College[69]. When turning to mathematics, in the same way, during the XIXth century the first Indians to engage in debates were both Sanskritists and trained mathematicians, although they seemed to have interacted more with indologists then with mathematicians. By the beginning of the XXth century however, Kaye's positions will prompt Indian mathematicians turned historians of mathematics to answer him, claim by claim. Their main integration network will less be indologist circles than a burgeoning Indian historical scholarship with a nationalist tinge on the one hand and Indian and later American mathematical circles on the other. Debates previously resting on uncertain epigraphic evidences will now integrate reliable and meaningful scholarly texts produced by XIXth century scholarship.

The use of Indian informants, had been in the shaded background of those who wrote the history of mathematics in India[70]. For instance, James Prinsep tells us that he uses a Pandit, called Kamalākānta. One can hardly measure how much he participated in Prinsep's work however. Before Kaye's 1907 article an increasing number of scholars of Indian origin published in the *Journal of the Asiatic Society of Bengal* on subjects related to the history of Indian mathematics and astronomy. Their works consisted almost exclusively in the edition of Sanskrit texts. They also wrote translations in other Indian languages and some synthetic essays. Some joint publications were made. Bāpu Deva Śāstrî (1821-1900) translated with Wilkinson in 1861 two astronomical texts the *Sūrya Siddhānta* and the *Siddhānta Śiromāṇi*[71]. Sudhākara Dvivedî (1855-1910/1911) published with G. Thibaut in 1888 the *Pañcasiddhāntikā*[72]. More often these Indian scholars published alone. Thus Bhau Dāji published in 1865 an article on the dates of

Āryabhaṭa, Varāhamihira and Bhāskara. In 1877 Bhāgvantlal Indraji published a Hindi text on the history of numbers in India, in 1893, H.C. Bannerji published a new translation and edition of the *Lῑlavātî*, in 1896, S.B. Dikshit published a *History of Indian Astronomy*, in 1902, Dvivedi edited the *Brāhmsphuṭasiddhānta*, in 1904, Sita Ram published a Hindi version of *Lîlavātî* and in 1907 a Hindi version of *Bîjagaṇita[73]*. These Hindi translations were probably not the first, but they where referenced by scholars of the *Asiatic Society*, sign that they were considered as worthy academic scholarship. They testify of a north Indian scholarship in the history of mathematics at the time[74].

The texts published by these Indian authors were so far very courteous and quite unlike the polemical debates of their European counterparts, at least when they published in English[75]. Slowly Indian scholars where entering the scene of the history of mathematics and astronomy in India, without engaging in direct front headed debates with their European colleagues.

With the publication of Kaye's edition of the *Bhakshālî Manuscript* things will change. A new set of mathematicians will challenge his claims, vehemently.

III.B 1927: The stage is set

1927 is the year in which a wave of criticism hit Kaye's works. Arguments had been forged and developed before, they continued to be voiced after that. But the year 1927 seems to be a turning point. Several Indian scholars took the stage, with the explicit aim to counter Kaye's point of view. They argued with him on a diverse range of themes including the origin of the decimal place value notation. As noted previously, in 1927 the first English translation of the *Āryabhaṭîya*, the oldest Sanskrit astronomical treatise defining the decimal place value, was published in a local Bengali journal by P. C. Sengupta (1876-1962). The publications of this historian of Indian astronomy[76] trace by its crossing bibliographical references a network of Indian mathematicians turning to their history, in which we will encounter A. N. Singh, S. K. Ganguly and B. Datta.

Saradakanta Ganguly (b. 1881)[77] was a mathematics teacher at the Ravenshaw College in Cuttack. He published several articles in the *Bulletin of the Calcutta Mathematical Society (*created in 1909) in the mid 1920's[78], and in the United States (in the *American Mathematical Monthly*[79] and in *Isis*[80]). In 1932[81], he published a decisive article in the *American Mathematical Monthly* entitled « the Indian origin of the modern place value arithmetical notation ».

Ganguly's arguments are almost solely based on Sanskrit scholarly texts, some of which are the astronomical and mathematical texts that had been edited and published in the 50 years preceding him. He thus inaugurates a movement that will slowly push epigraphic data in the background. The focus of history of mathematics will then be scholarly mathematics rather than the administrative uses of mathematics that inscriptions testify of[82].

S. Ganguly accumulates evidence of the way numbers are named. His reasonings quotes examples from the *Āryabhaṭîya*, the *Pāṭîgaṇita* and the *Brāhmasphuṭasiddhānta* but also from non astronomical and non mathematical texts such as Patañjali's *Yogasūtra* or Śankara's commentary on the *Brahmasūtra*. All these texts use the decimal place value notation before the IXth century AD. Ganguly also explains extensively Brahmagupta's (VIIth century) rules for computations on zero. These provide him with a proof of the existence of zero as a number in India prior to Al-Bîrūnî's visit. He furthermore uses « mathematician's » arguments. He reasons that quoting numbers from the smallest decimal value to the highest enables one to represent progressively the value of the number. Lists of names of numbers, then, do not need to be linked to writing sides (from left to right or vice versa). He also remarks that the non-existence of an abacus does not prove that the decimal place value notation is not of Indian origin.

Another network is drawn by S. R. Das's publications. We have little to know information on Sukumar Ranja Das (fl. 1930).[83] His publications in the second half of the 1920's will be first restricted to the *Indian Historical Quarterly*, a Calcutta based review both academic and nationalist, before a publication in the early 1930's in *Isis*[84]. After an article on the origins of Indian numerals, published in two parts in 1927[85], S. R. Das will concentrate on astronomical lore[86]. S. R. Das has read Peacock's *Arithmetic* and uses his rhetoric as he argues for an Indian origin of the decimal place value notation. His article opens on a synthesis of his point of view on arithmetic and what numbers are for[87]:

> "The chief use of numerals is for reckoning. The use of visible signs to represent numbers and aid reckoning in not only older than writing, but also older than the development of numerical language on the denary system. We count by tens because our ancestors counted on their fingers and named the numbers accordingly. So used, the fingers were with our ancestors really numerals, that is visible numerical signs; and in remote antiquity the practice of counting by these natural signs were in vogue in all classes of society."

The paragraph is followed by a quotation of Peacock, the very passage reproduced earlier in the Introduction of our article. S. R. Das's article aims to tackle all threads of the debate on the origins of the decimal place value notation, invalidating one by one Kaye's arguments. As in a theater play, B. Datta is in the footnotes and thanks of S. R. Das's article[88]: The next article published in the *Indian Historical Quarterly* on numbers will have B. Datta for author.

B. Datta (1888-1958), a mathematician trained in Calcutta with a mystical bent[89], had written numerous articles on the history of Indian mathematics before his joint book with A. N. Singh, the *History of Hindu Mathematics,* published in 1935, came out as the crown of his career as a historian of mathematics. He had published in a great diversity of journals from the *Bulletin of the Calcutta Mathematical Society,* the *Indian Historical Quartely* to the *Journal of the Asiatic Society of Bengal,* including an article in the American *Isis*[90]. He finally associated himself with A. N Singh (1901-1954) a former student like himself of G. Prasad (1876-1935), a mathematics professor with a keen interest in the history of mathematics[91]. Singh had also published in the year 1927 an article critical of Kaye, on the question of square root extractions[92]. Although not directly on the decimal place value notation, the procedure rests on such a notation.

The structure of the section of Datta and Singh's book devoted to the numerical systems in India, can be seen as a long and systematic effort to synthesize all the debates and answer every objection. Until today Datta and Singh's book is a reference manual because it is all-encompassing and rigorous in treating mathematical questions.

Datta, Ganguly and Singh were trained Indian mathematicians. Their colleagues seemed to have been mathematicians rather than indologists. After them, almost all Indian historians of mathematics will be trained mathematicians[93]. To challenge Kaye, they seemed to have operated from outside the European realm, as Biot might have done almost a hundred years before[94]. Thus after publishing in the *Bulletin of the Calcutta Mathematical Society,* they will appear in the *American Mathematics Monthly* before providing articles to *Isis*.

No trace of Peacock's *Arithmetic* can be found in Datta and Singh's manual. Actually, none of the authors of the "mathematician's network" seem to have read it. Datta and Singh's textbook inaugurates a new moment in the history of mathematics in India, giving rise to a tradition of technical history of mathematics,

which argues priorities in a nationalist mode, that remained quite lively until the end of the XXth century.

CONCLUSION

Peacock's Arithmetic was not received in total silence: it echoed faintly in the Indian subcontinent. G.R Kaye and S. R. Das, fleetingly, evoke both his theoretical thesis of a natural structure of arithmetic, and his conviction that the decimal place-value notation came from India. However the main themes of the debate on the origins of the decimal place value notation remained elswhere. Discussions focussed on the reliability of epigraphic sources, the direction in which numbers were written, the understanding and dating of Sanskrit mathematical rules.

Who then had read and taken into account Peacock's *Arithmetic* in India? Indian indologists and scientists who had relations with English scholars would have had access to the text. But then they would have had access to more specialized writings as well, and may not have turned to an Encyclopaedia to reflect on the history of mathematics. The hostile anglicist atmosphere could partially explain the fact that the Peacock enchantment with Indian arithmetic will not echo in the scholarly *Asiatic Societies* of the 1860's. In other words, the colonisor, the intended readership of the *Metropolitana*, might have not read or discussed the *Metropolitana*. Dhruv Raina and S. Irfan Habib have shown that Peacock's colleague and friend Augustus De Morgan was in contact with Ramachandra, a mathematician at Delhi University[95]. Ramachandra used algebra as a way of pedagogically approaching calculus, in a fashion that would have probably appealed to Peacock. Maybe then Peacock's works were familiar to the teachers and administrators who reflected on questions of science education. Working for the «Bureau of Education», G. R. Kaye might have been part of a network, less prestigious than that of the orientalists and mathematicians of the *Asiatic Societies*. Other threads then to explore Peacock's influence include the authors of histories of mathematics in Indian languages, and more generally the teachers of Colleges and Universities of small Presidency towns. Knowledge of S. R. Das's background would probably help in having an idea of the circles of Peacock's readership. In other words, the different networks of historians of science, orientalists and scientists whether in Europe, the United States or India, still needs to be mapped out.

For Peacock, a civilized country's politics and economy should reinforce the natural organization of thought. Thus, mathematics, its teaching and history, could be understood in relation to an economical and political organization of society. Reforms of the first could promote changes in the second and reciprocally. In the emergence of new Indian historiographies of arithmetics that have been chronicled here, politics is also on the table. However, the aim of the actors encountered here was not to reorganize the world and knowledge around algebra. For Ganguly, Das, Datta and Singh the politics of history of science has to do with narration: what is important is to rid the field of prejudices, to rigourously, rationally, distinguish right from wrong, the aim being the recognition of the intellectual feat of past Indian mathematical thinkers. So that if political concerns unite Peacock with these Indian historiographers, there aims and ideas do not coincide.

In a bout of history of science fiction, we could fantasize on what could have happened if Peacock had had a more direct access to Bhāskara II (XIIth century). This author seems to have had a conception of the links of algebra to arithmetic which echoes Peacock's relation of arithmetical algebra to general algebra. Indeed, elementary arithmetical operations are often adequated by this Sanskrit author to elementary algebraical ones, and reciprocally. Bhāskara II opens his text devoted to algebra with the following statement, as translated by Colebrooke:

> .. the arithmetic of apparent [or known] quantity, (...), is founded on that of unapparent [or unknown] quantites.[96]

For Colebrooke however the use or not of symbols was more important than the common operative structure of arithmetic and algebra. Consequently, this potential common ground remained unnoticed by Peacock. More than a century later, Datta and Singh, quoting Eugene Smith, pay attention to the use of symbols in Indian mathematics: they thus note existing symbols for numbers, operations and unknowns in equations. They also remark, without extending its theoretical significance, Bhāskara II's conception of the links of arithmetic to algebra, leaving thus unanswered the faint echo of Peacock in the Indian subcontinent.

Notes and References

1. S.T. Coleridge, "'Prospectus' to the *Encyclopaedia Metropolitana*" in *The Collected Works of S. T. Coleridge 11; Shorter Works and Fragments,* London & Princeton, 1995, 586, characterizes his readers as "Friends and Patrons of Art, Science, History and General Literature". He also evokes parts of the Encyclopaedia "interesting to every member of the purchaser's family". See also, Yeo, Richard, *Encyclopaedic Visions, Scientific Dictionaries and Enlightenment Culture*, Cambridge, 2001, p. 57.

2. Coleridge, *op. cit*., p. 579.

3. The Asiatic Society Mumbai, the University of Mumbai Library, the University of Madras Library, the Asiatic Society of Bengal, the Benares Sanskrit University Library.

4. I omitted to check however what was their date of publication, and when these copies had arrived.

5. Yeo, op.cit. p. 282 notes its ideological anachronisms in respect to ordered encyclopaedism, by the time it came out.

6. Peacock. "History of Arithmetic." In *Encyclopedia Metropolitana*, edited by Coleridge, 1847 : 374-b, 375-a.

7. Peacock, op.cit. pp. 412-413.

8. Peacock, op. cit. p. 407, thus declares the superiority of "Hindoo (…) numeral symbols" over all others "in Europe and in Asia".

9. Peacock, op. cit, for squaring and extracting square roots, p. 435, for the Rule of three, pp. 450-51, for the rule of alligation, pp. 464-465.

10. Peacock, op. cit, p. 438.

11. Pycior, Helen "Augustus De Morgan's algebraic work : The three stages". *Isis* 74 (1983) 211-226.

12. Kopf, David British *Orientalism and the Bengali Rennaissance, The Dynamics of Indian Modernization, 1773 - 1835*: Regents of the University of California,1969.

13. Bayly, C. A. *Empire and information : intelligence gathering and social communication in India, 1780-1870*. Cambridge University Press, 1996 . Raj, Kapil. « Colonial Encounters and the Forging of New Knowledge and National Identities: Great Britain and India, 1760-1850 ». *Osiris* 15 (Nature and Empire: Science and the Colonial Entreprise): (2000) 119-134 . Raj, Kapil *Relocating Modern Science: Circulation and the Construction of Scientific Knowledge in South Asia and Europe, 17th and 19th Centurie*s. Delhi: Permanent Black, 2006. M. S. Dodson. *Orientalism, Empire, and National Culture; India, 1770-1880*. Palgrave Macmillan, 2007.

14. Of course other reasons should be taken in account as well for instance Peacock could have only be read in literate-in-English circles.

15. The authors examine here use several expressions: «hindoo numerals», «indian numerals», «arabic numerals». A study of who used what expression, of how these name evolved still needs to be carried out.

16. Kaye, G. R. "Notes on Indian Mathematics- Arithmetical Notations." *Journal of the Asiatic Society of Bengal, New Series* III(7) (1907) 475-508.

17. B. Datta and A. N. Singh, *History of Hindu Mathematics*, Lahore, 1935.

18. Although the book was at first critically received. See notably, Gandz, Salomon, 'Review of History of Hindu Mathematics', *ISIS*, Vol. 25, No. 2 (1936) 478-488.

19. Raina, Dhruv. "Jean-Baptiste Biot on the History of Indian Astronomy (1830-1860): The Nation in the Post Enlightenment Historiography of Science." *IJHS* 35, no. 4 (2000): 336-337. Charette, François, *Orientalisme et histoire des sciences, L'historiographie européenne des sciences islamiques et hindoues*, Université de Montréal, 1995.

20. See Charette, op. cit. (7). J. Peiffer, France, in *Writing the History of Mathematics, Its Historical Development*, J. W. Dauben and C. J. Scriba (eds.), Basel, Boston, Berlin, 2002, pp. 20-22. K. Chemla, "Chasles", in *Dauben and Scriba (eds.) op. cit*., pp. 396-398.

21. A codex at the Bibliothèque nationale in France has compiled most of the debate: BNF Res-Z-1249/ Livres Rares.

22. Peacock, op. cit., p. 415.

23. Libri, Guillaume. *Histoire des science en Italie : depuis la renaissance des lettres jusqu'à la fin du dix-septième siècle*, vol. 1: Jules Renouard, Paris (1838) 117-135. He explains in the foreword that the book should have been published in 1835 but it was delayed by a fire. He actually claims that all ideas were already established in 1831 when he had to flee Italy.

24. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences* 12 (1841) 741-756; 13 (1841) 560-563, 601-626, 627-628.

25. The *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences* of 1841 thus sees them fighting over interpretations of star catalogs and medieval description of celestial phenomena, Arago then secretary of the Academy siding for Chasles, while Libri quarrels with Biot on the use of symbolism to solve finite difference equations.

26. Halliwell was nineteen at the time, while Chasles was in his mid-fourties! See also Rice, Adrian, "Augustus de Morgan: Historian of Science", *History of Science* (xxxiv) (1996) 223-224, and Dickinson, H. W, "J. O. Halliwell and the Historical Society of Science (London 1841)", *Isis* 18.1 (1932) 127-132.

27. Chasles, Michel and Libri, G. « Réponse de M. Chasles, au sujet de M. Libri sur la brochure de M. Halliwell. Réponse de Libri. Réponse de Chasles à la réponse de Libri. » *Comptes rendus hebdomadaires des Séances de l'Académie des Science. Séance du 14 octobre 1839*.

28. Biographical notice, in Dauben, Joseph W., and J. Christoph Scriba (eds.), *Writing the History of Mathematics : Its Historical Development*. Bâle, Birkhäuser, 2002 : p. 466.

29. Rice, Adrian. "Brought to Book: The Curious Story of Guglielmo Libri (1830-69)." *EMS*, June 2003 (2003): 12-14.

30. A whole codex is devoted to the "affaire Libri" at the Bibliothèque Nationale de France, under the registration number, 8-LN27- 83210.

31. Libri publicly doubted the veracity of manuscripts presented by Chasles at the *Académie des sciences* in 1867: *Lettre de G. Libri à M. Chasles, membre de l'Institut*, 1867. BN 8 -Q Piece- 3195 support. These were indeed fakes sold to Chasles by the forger Vrain Lucas who wrote for him false autographed manuscripts of letters from Pascal to Boyle, Pascal to Galilée, Socrates to Euclides, among others. See Prevost, Marie Laure « Vrain Lucas, le Balzac du Faux. » *Revue de la Bibliotheque Nationale de France* 13: 59-69, 2003 . Chasles published his last article on the "système de l'abaque" more or less at the same time; Chasles, Michel. « Recherches des traces anciennes du système de l'Abacus. Calcul de Victorius et Commentaire d'Abbon. » *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences* 64 (1867) 1059-1067.

32. Raina, Dhruv. "Nationalism, Institutional Science and the Politics of Knowledge; Ancient Indian Astronomy and Mathematics in the Landscape of French Enlightenment Historiography." Göteborgs University, 1999, part 6.2 pp. 240-257.

33. Libri, Guillaume. *Histoire des science en Italie : depuis la renaissance des lettres jusqu'à la fin du dix-septième siècle, vol. 1*: Jules Renouard, Paris : xii, 1838,:

    « Rebuté par l'aridité de ces écrits [d'histoire des sciences] où l'on voyage sans cesse d'une étoile à une autre, du triangle au cercle, sans qu'on lui fasse jamais apercevoir les hommes qui sont derrière la science, j'ai senti d'abord la nécessité de montrer que l'état intellectuel des peuples est toujours lié à leur état moral et politique ; et j'ai dû m'appliquer à faire marcher de front l'histoire des idées et celle des hommes, pour les éclairer l'une par l'autre ».

34. Libri, op.-cit., p. 121 note 1. Libri however does not seem to have in general favored the use of symbolical notations to solve geometrical questions, as might show his exchanges with Biot in *Comptes Rendus Hebdomadaires de l'Académie des Sciences* (1841) p. 519-523.

35. M.–J. Durand-Richard has kindly gave me access to her notes on De Morgan's correspondence. In a letter to Henry Brougham (1778-1868) in 1855 De Morgan notes that the only person who could understand Libri at the institute would have been Chasles (Notes taken by M. J. Durand-Richard, Brougham Papers, University College Library, London):

    "I never had any idea that the members of the Institute would have any feeling of justice in Libri's case**.** He was a scholar among the technicalists of the Institute, men brought up in one thing, études spéciales, they had an ineffable dislike of the man

to whom literature and bibliography were as familiar as civil law & philosophy to Leibnitz, and to whom their staple sciences were hors d'oeuvre, as mathematics was to Leibnitz also. His <u>history</u> is the true offence. The only scholar among them, Chasles, the only man who had any chance with Libri (not much) was kept out of the Insitute for many years".

Furthermore, De Morgan wrote a piece in support of Libri over Chasles in the late 1860's. His letter claimed that an exchange of letters between Pascal and Newton published by Chasles were probably forgeries and that Libri had nothing to do with it. One can feel the tinge of irony in De Morgan's letter as translated in French in this public letter for francophone readers (Manuscript of the Bibliothèque Nationale de France : *Lettre de G. Libri à M. Chasles, membre de l'Institut*, 1867. BN 8 -Q Piece-3195 support):

Lettre de M. de Morgan a M. Libri

91 Adelaide road, N. W.

6 septembre 1867

Mon cher Monsieur,

Vers le 10 aout vous m'avez montré l'Indépendance du 3, contenant deux lettres attribuées à Pascal. Vous avez fortement exprimé votre opinion que c'étaient des falsifications (...) La remarque que 1652 était une époque trop ancienne pour qu'on pu parler de cette façon du café, fut faite par vous (...) L'assertion que cette falsification vous est due est une autre bêtise digne de figurer à côté des autres bêtises relatives à Pascal.

Votre sincèrement,

A. de Morgan

36.See Rice, *op-cit.* Dickinson, o*p.-cit.*

37.Charette, op. cit. (7).

38.Reinaud, Joseph Toussaint *Fragments arabes et persans inédits relatifs à l'Inde, antérieurement au XIe siècle de l'ère chrétienne.* Paris, 1845: Imprimerie royale. Reinaud, Joseph Toussaint « Analyse d'un mémoire géographique, Historique et Scientifique sur l'Inde. » *Moniteur Universel*, 1846. Reinaud, Joseph Toussaint. « Mémoire géographique, historique et scientifique sur l'Inde, antérieurement au milieu du XIe siècle de l'ère chrétienne, d'après les écrivan arabes, persans et chinois ». *Mémoires Ac. Inscr. et Belles Lettres* 18 (1849).

39.See for instance, Plofker, Kim (2007)« Mathematics in India ». In: Katz (ed.), *Mathematics in Egypt, Mesopotamia, China, India and Islam* : p. 435.

40.Prinsep, J. "On the ancient Sanskrit numerals." *Journal of the Asiatic Society of Bengal.* 1834. Prinsep, J., E. Thomas, et al. *Essays on Indian antiquities, historic,*

numismatic, and palaeographic, of the late James Prinsep, to which are added his useful tables, illustrative of Indian history, chronology, modern coinages, weights, measures, etc. London, 1858.

41. See Datta and Singh, op.cit. Volume I, p. 69 sqq.

42. See Prinsep 1834, *op.cit.*

43. Peacock, op.-cit. p. 412.

44. Op. cit. Wilkins (1749-1836) was one of the first British Indologists who read and published Sanskrit Inscriptions.

45. Salomon, Richard, *Indian Epigraphy, A guide to the study of inscription in Sanskrit, prakrit, and the other indo-aryan languages*. Delhi: Munishram Manorhal (1998) 57-63.

46. See for instance, Shukla, K. S. & Sarma, K. V, *The Āryabhaṭîya of Āryabhaṭa*, INSA, New-Delhi (1976) p. 33

    (the translation is my own adaptation of Shukla and Sarma's translation: closer to the Sanskrit, but keeping their choices of translations of Sanskrit words and syntax):

    Ab.2.2 *ekaṃ ca daśa ca śataṃ ca sahasraṃ tv ayutaniyute tathā prayutam/*

    koṭ yarbudaṃ ca vṛndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt//

    Units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, thousands of millions are, respectively, from place to place, each ten times the preceding.

47. E. C. Bayley, "On the genealogy of modern numbers", *Journal of the Royal Asiatic Society*, xiv (1882) 20. Observers of the time agree with him, although today, to my knowledge such instruments are of no common use. These abacus were probably the abacus's common to merchants in the Arabic peninsula.

48. No counting instrument seems to have been used in the past in India, but tabular dispositions, maybe written in the dust, or noted with seeds and shells are sometimes evoked.

49. Kaye 1907, op. cit.

50. G. R. Kaye, "Notes on Indian Mathematics. no. 2. Āryabhaṭa." *Journal of the Asiatic Society of Bengal* IV(8) (1908) 111-141. G. R. Kaye, "The Use of the Abacus in Ancient India." *Journal of the Asiatic Society of Bengal, New Series*. IV(6) (1908) 293-297. G.R. Kaye, "A brief bibliography of Hindu Mathematics." *Journal of the Asiatic Society of Bengal, New Series* VII(10): 1911. G. R. Kaye, *Indian Mathematics*, Calcutta & Simla, 1915.

51. G. R. Kaye, *Hindu astronomy*. Calcutta, Gov. of India Central Pub. Branch. Archaeological Survey of India: Memoirs; v.18: 1924.

52. G. R. Kaye, *The Bhakhshāli Manuscript*. Calcutta, 1927.

53. Kern, H. *The Āryabhaṭîya, with the commentary Bhaṭadîpika of Parameśvara*, Leiden, 1874.

54. Rodet, Léon, *Leçons de calcul d'Âryabhata*, Imprimerie nationale, Paris : 1879.

55. Sengupta, P. C."The *Aryabhatiyam*, translation." *Journal of the Departments of Letters of the University of Calcutta.* **XVI**.1927

56. See Datta & Singh, op.cit. (1), 64-69 and Shukla, K. S. and Sharma, K. V. *Āryabhaṭîya of Āryabhaṭa, critically edited with translation* New-Delhi (1976) 3-5, for a synthetic presentation.

57. Indeed, power of tens are denoted in pairs by vowels (thus *a* denotes both 1 and 10, *i* both a 100 and a 1000), while consonants enables one to associate to these powers a value and a parity (thus *ga* is 3, and *ya* 30). In practice, Āryabhaṭa's verses do not mingle different pairs of powers ten in a haphazard order, but gives them all in decreasing order, as when noted with the decimal place value notation (thus 33 is *yaga* and not *gaya*). This is a sign among others that the device was closely related to the decimal place value notation.

58. Note that this remains open to discussion until today, see Salomon, *op. cit.* who quotes Kaye favorably.

59. Kaye 1907, op. cit. p. 493.

60. Compare his translation with note 46:

    Kaye, G. R, "Notes on Indian Mathematics. no. 2. Āryabhaṭa", *Journal of the Asiatic Society of Bengal* (IV), March (1908) p. 117:

    Units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, thousands of millions. In these such succeeding place is ten times the preceeding.

    G. R. Kaye comments on this verse from the point of view of the name of higher numbers, but does not gloss on the word *sthāna*, «place», which makes the verse a definition of a place-value notation.

61. Kaye 1915, op. cit. p. 44.

62. Kaye, G. R."The Use of the Abacus in Ancient India." *Journal of the Asiatic Society of Bengal, New Series.* IV(6) (1908) 293-297.

63. He refers then to Perry's *Practical Mathematics*, to make his point.

64. Smith, D.E. & Karpinski, L.C, *The hindu-arabic numerals*, Boston: 1911.

65. Cajori, Florian, *A History of mathematical notations*, Chicago : 1928.

66. Kaye, G. R, Indian Mathematics, *Isis*, 2 (2) (1919) 326-356.

67. Charette, op.cit. (7). D. Raina. *Images and contexts: the historiography of science and modernity in India*, OUP, New-Delhi, 2003. Raina and Habib, *Domesticating Modern Science, A social history of science and culture in Colonial India*, OUP, 2004. One can also note R.C. Gupta, 'India', in *Writing the History of Mathematics, Its Historical Development*, J. W. Dauben and C. J. Scriba (eds.), Basel, Boston, Berlin (2002) 307-315. S. N. Sen briefly evokes the different authors evoked here in 'Survey of Studies in European languages', in the *History of Astronomy in India*, INSA, S. N. Sen & K. S. Shukla (eds.), (1985) 92-93.

68. Raina 2003, op. cit. 'Prafulla Chandra Ray and Marcelin Berthelot:Chemist-Historians', p.20.

69. It is noteworthy that as William Jones a century earlier, B. N. Seal in his « Positive Science of the Hindus » does not include mathematics or astronomy as «science».

70. For an account of the use of pandit scholarship in burgeoning indology, see M. S. Dodson. *op.cit.* and also the case study of Madhav Deshpande on Marathi pandits: M. M. Deshpande, "Pandit and Professor: Transformations in the 19th century Maharashtra", in *The Pandit, traditional scholarship in India, Axel Michaels (ed.)*, Delhi, 2001.

71. B. D. Śāstrî and L. Wilkinson, The Sūrya siddhānta or, An Ancient system of Hindu astronomy, followed by the Siddhānta Śiromaṇi, Calcutta, *Bibliotheca Indica* (32), 1861. On this collaboration see also Bayly, *op. cit*, p. 235.

72. G. F. Thibaut and S. Dvivedi, *Pañcasiddhāntika*. 1888. Reprint Varanasi, 1968.

73. S, Dvivedi, "The Brhmasphuṭasiddhānta of Brahmagupta." *The Pandit*, 1902.

    One can also note: S, Dvivedi, *History of Mathematics (in Hindi)*. Benares. 1910. S, Dvivedi . "The Mahāsiddhānta of Āryabhaṭa." *Benares Sanskrit Studies* (1910) 148-150.

74. South Indian texts may have been published as well, and need to be documented.

75. For harsh debates in Sanskrit on questions of cosmology see Minkowski, C, "The Pandit as public intellectual: the controversy over *virodha* or inconsistency in the astronomical sciences.", in *The Pandit. Traditional scholarship in India. Edited by Michaels, Axel,* (2001) 79-96 Delhi.

76. Gupta, Radha Charan, "Prabodh Chandra Sengupta (1876-1962), Historian of Indian Astronomy and Mathematics", *Gaṇita Bhāratî*, 1 (1979) 31-35.

77. According to Shukla, op. cit.

78. Ganguly, S. K, "Alphabetical system of Āryabhaṭa", *Bull. Calcutta Math. Soc.* (4), XVII, (1926) 66-67. Ganguly, Sarada Kanta, "Bhāskarcārya's references to previous teachers", *Bulletin of the Calcutta Mathematical Society* (xviii), 2, 1927.

79. S. Gnguli, "On the modern place-value notation in the Āryabhaṭiyam." *American Mathematical Monthly* XXVII. 1927.

80. Ganguly, S, "Notes on Indian Mathematics. A criticism of George Rusby Kaye's Interpretation", *Isis* (12), 1 (1929) 132-145.

81. S. Gānguli "The Indian Origin of the modern Place-value arithmetical notation." *American Mathematical Monthly* XXXIX (1932) 251-256/389-393.

82. Kaye had also interacted with American Scholars, since his 'Indian Mathematics' was initially to be published in *Isis*. The publication was temporarily stopped by the war, he thus decided to publish it in India, before having it re-published in Isis in 1919, as specified by an Editor's note in Ganguly's polemical review of Kaye's text in *Isis*: Ganguly, S, "Notes on Indian Mathematics. A criticism of George Rusby Kaye's Interpretation", *Isis* (12), 1 (1929) 132-145.

83. It is possible that this Sukumar Ranjan Das can be indentified with the cousin of the freedom fighter Chittaram Das (Aurobindo Ghosh's barrister) also called Sukumar Ranjan Das, who wrote a criticism of his cousin's opinion in 1922.

84. Das, Sukumar Ranjan, "Some Notes on Indian Astronomy", *Isis*, 14 (2) October1930 : 388-402

85. Das, S. R. (1927) "The Origin and Development of Numerals, part I and II" *Indian Historical Quaterly,* III (2): 356-375

86. Das, S. R. (1928) "Astronomical Instruments of the Hindus", *Indian Historical Quaterly* IV (2) : 256-269.

    Das, S. R. (1928) "Seasons and the Year-beginning of the Hindus", *Indian Historical Quaterly,* IV (4): 653-666.

    Das, S. R. (1928) "Hindu Calendar", *Indian Historical Quaterly,* IV (3): 483-510.

87. S. R. Das, "The Origin and Development…*op. cit,* p. 98:

88. S. R. Das, *op. cit*, footnote, p. 371 "For the concluding portion of this chapter, I am indebted to an article by Dr. Bhibuti Bhusan Datta of the University College of Science, Calcutta, which was published in the American Matheamtical Monthly, Nov. 1926 ».

89. He cast off a career in mathematics to become a renunciate (*sanyasi).* Gupta, Radha Charan, "Bibhutibhusan Datta (1888-1958), Historian of Indian Mathematics", *Historia Mathematica*, 7 (1980) 126-133.

90. Among Datta's rich bibliography, one can note: Datta, Bibhutibhusan, "Early History of the Arithmetic of zero and infinity in India", *American Mathematical Monthly* (XVIII), 4 (1927) 165-176. Datta, Bibhutibhusan, "The present mode of expressing numbers", *Indian Historical Quartely* (3), 3 (1927) 530-540. Datta, Bibhutibhusan, "Āryabhaṭa, the Author of the "Ganita", *Bulletin of the Calcutta Mathematical Society*, (March 1927) 5-18. Datta, B, "The Bakhshālī manuscript", *Bull. Calcutta*

Math. Soc. (21) (1929) 1-60. Datta, Bibhutibhusan, "The scope and development of Hindu Ganita", *IHQ* V (1929) 470-512. Datta, Bibhutibhusan, "Early literary evidence of the use of the zero in India", American Mathematical Monthly, (1931) 565-572.

91. Chapter on India, written by R. C. Gupta in Dauben, Joseph W. & Scriba, Christoph J, *Writing the History of Mathematics : Its Historical Development*, Birkhäuser, Historical Studies. Science Networks, Bâle : 2002.

92. Singh, Avadhesh Narayan, "On the Indian Method of Root Extraction", *Bulletin of the Calcutta Mathematical Society* (XVII), 3 (1927) 123-140.

93. For the mathematical bent of subsequent history of mathematics see D. Raina 2003, op. cit. .

94. For a description of a network of indologists and scientists in and outside of India interacting with one another, see : "Prafulla Chandra Ray and Marcelin Berthelot: Chemist-Historians" and "The Sarton and Coomaraswamy Dialogue" in Raina, op. cit (2003). See also Raina, 2000, op. cit, 340-341.

95. Raina and Habib 2004, op. cit. Especially the reprint of D. Raina and I. S. Habib, 'Ramachandra's Treatise "The Haze of the Golden Sunset": An Aborted Pedagogy', *Social Studies of Science*, 2.3 (Aug. 1990) 464-467.

96. Translation of *Bîjagaṇita*1.2, in Colebrooke, H. T. (trans.), *Algebra, with Arithmetic and mensuration* , J. Murray, London : 1817: 151.

   B. Datta & A. N. Singh *op. cit* translate more clearly maybe:

   "The science of calculation with unknowns is the source of the science of calculation with knowns"

   They explain: "Both (arithmetic and algebra) deal with symbols. But in arithmetic the values of the symbols are *vyakta*, that is, known and definitely determinate, while in algebra they are *avyakta*, that is unknown, indefinite" (pp.2-3).