

**PROFESSOR GANESH PRASAD: AN EPITOME OF TEACHING AND  
RESEARCH IN MODERN MATHEMATICS IN INDIA**

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Ganesh Prasad was the most distinguished mathematician in India in the beginning of the twentieth century. He was born on November 15, 1876 in the town of Ballia in Uttar Pradesh state. Prasad obtained his M.A. degree in Mathematics both from Allahabad and Calcutta universities. Subsequently, he took the D.Sc. degree from Allahabad University in 1898. After serving as a Professor of Mathematics for a brief period in the Kayastha Pathshala, Allahabad, he went to the Christ Church College, Cambridge University in 1899 for further studies as a Government of India scholar. At Cambridge, Prasad studied mathematics with men like Hobson, Forysth and Whittaker, and took B.A. degree of the Cambridge University. Later he left Cambridge to continue his research at Gottingen in Germany. There he was a pupil of Felix Klein and came in contact with mathematicians like Hilbert and Sommerfield. Ganesh Prasad returned to India in 1904. After a brief stint of service at the Muir Central College, Allahabad as an Additional Professor of Mathematics, he was appointed Professor of Mathematics at the Queen's College, Banaras in 1904. He occupied this position till 1914, when he was invited by the Calcutta University for the newly created Ras Behari Ghosh Professor in Applied Mathematics.

Ganesh Prasad's main interest was in the theory of functions of real variable, and Fourier series. He worked much on the convergence or summability of Fourier series. On the suggestion of David Hilbert, he worked on curvature of surfaces. He also worked on potential theory, where he made many significant contributions. He looked at the expansion of functions in a series of spherical harmonics. Ganesh Prasad was basically a pure

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mathematician, but he did work on many problems of applied mathematics in detail. It has been approximately a century since the researches of Dr. Ganesh Prasad on the above subjects started appearing in international and national journals. It is appropriate to revisit some of the significant contributions made by Dr. Ganesh Prasad in mathematical research.

E.W. Hobson of the Cambridge University has referred to the work of Ganesh Prasad in his book *The Theory of Spherical and Ellipsoidal Harmonics*, first published by the Cambridge University in 1931, and later reprinted in 1955. Hobson referred to the work of Dougall, published in 1913, on the expression for polynomial function of degree  $n$ . To be honest to himself, Hobson stated that a similar work has already been published by Ganesh Prasad a year earlier in 1912. He writes:

“A somewhat different, but equivalent, form for  $f_n$  (polynomial function of degree  $n$ ) had already been given by G. Prasad. By this formula the value of  $f_n$  over a sphere  $r = a$  is expressed as a sum of surface harmonics.”<sup>1,2</sup>

In 1851, the famous mathematician Reimann presented his Ph.D. dissertation entitled “Foundations of a general theory of functions of a complex variable” to the University of Berlin. This dissertation was much appreciated by Gauss. Reimann constructed an example of a continuous function, which was differentiable at no point. This was a great advance over the notions of Cauchy and Dirichlet, both of whom were great mathematicians. Ganesh Prasad went a step ahead of Reimann’s work. He constructed a continuous function which was mean differentiable at no point. In this connection, it is important to remember that if a function is differentiable at some points, then it has a mean differential coefficient at that point also. Ganesh Prasad’s paper entitled “On the existence of the mean differential coefficient of a continuous function”, published in 1912, contains the details of this work (Prasad, 1912).

Ganesh Prasad published a paper entitled “On the present state of the theory and application of Fourier series” in the *Bulletin of the Calcutta Mathematical Society* in 1910 (Prasad, 1910-11). In the concluding remarks section of the paper, he presented three problems mentioned as (A), (B) and (C) before the mathematical community, as follows:

- (A) To find the condition which is both necessary and sufficient for the validity of the equation

$$f(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\alpha) d\alpha + \frac{1}{n} \sum_1^{\infty} \int_{-\pi}^{\pi} f(\alpha) \cos n(\alpha - x) d\alpha, \quad -\pi < \alpha < \pi.$$

- (B) To find a continuous function  $f(x)$  for which the series

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(\alpha) d\alpha + \frac{1}{n} \sum_1^{\infty} \int_{-\pi}^{\pi} f(\alpha) \cos n(\alpha - x) d\alpha,$$

diverges for every value of  $x$  in the interval  $(-\pi, \pi)$

- (C) To find the condition which is both necessary and sufficient in order that, for a given value of  $x$  and for  $t > 0$ ,

$$\left| \sum_1^{\infty} n\alpha_n \sin nxe^{-n^2t} \right|$$

may have a finite upper limit in the neighbourhood of  $t = 0$ ,  $-\pi < x < \pi$ .

The above problems, proposed by Ganesh Prasad, are fundamental to the theory of Fourier series. A Russian mathematician named Lusin conjectured about the problem (B). His conjecture was that the Fourier series of a square integrable function including continuous functions in particular, converge almost everywhere. This hypothesis was proved by Lennart Carleson to be true in 1966. Carleson was awarded the Field Medal for this work and later Abel Prize recently in 2006.

It is important to note in this context that Ganesh Prasad was very much aware of the fundamental problems of his subject, and drew attention of the mathematical community through his papers. The above example shows that the solution of the problem posed by him in 1910 was obtained ultimately in 1966 by Carleson.

Ganesh Prasad was an epitome of teaching and research in modern mathematics in those days in India. He was a source of inspiration to young researchers. It was against his background that sir Ashutosh Mukherjee invited him to Calcutta University for the post of Ras Behari Ghosh Professor in

Applied Mathematics in 1914. He held the post with high distinction till 1918. After serving the Banaras Hindu University for a few years, he was once more called to Calcutta University in 1923. This time he was invited to the Chair of “Hardinge Professor of Higher Mathematics”. In this context, it should be noted that the Calcutta University started the Chair of Hardinge Professor of Higher Mathematics in 1912. It was named after the then Governor-General and Chancellor, Lord Hardinge, out of the grant made by the Government of India to the Calcutta University. The first occupant of this Chair was Prof. W.H. Young, FRS, who was appointed in 1913. After the retirement of Prof. Young, Prof. Cullis was appointed on this Chair and he continued to hold the post till his retirement in 1923. Thus, Dr. Ganesh Prasad was the first Indian Professor to occupy the Hardinge Professor of Higher Mathematics at the Calcutta University.

Ganesh Prasad was always of the view that the findings of the researches carried out throughout the world must be made available to the young generation to encourage them for carrying out research in Mathematics in India. His special lectures delivered from time to time at different places were full of such materials. I would like to discuss a few such lectures delivered by him.

After joining the Calcutta University in 1914 as the Ras Behari Ghosh Professor of Applied Mathematics, Ganesh Prasad delivered the inaugural address entitled “From Fourier to Poincaré: A Century of Progress in Applied Mathematics” (Prasad, 1912, pp. 1-4). He divided the address in two parts. In part I, he talked about some researches relating to the logical foundation of applied mathematics from the point of view of the continuous theory of matter as well as that of molecular theory. He was not very happy at the way it was being done in the literature. In part II, he talked about the solution of boundary value problems. This presentation of Dr. Ganesh Prasad was a reflection of his depth of understanding of the subject.

Ganesh Prasad delivered six lectures on Fourier series at the Calcutta University, which was later published by the same university in 1928 (Prasad, 1928). The above monograph was an authentic review of the subject of Fourier series at that time. He himself stated in the opening remark that “the six lectures which I propose to deliver on the present state of the convergence

problem in the theory of Fourier series may be considered to be a review of nearly a thousand publications on Fourier series which have appeared in the last twenty-seven years in various countries and in various language<sup>3</sup>. This shows his dedication to his profession, and the depth of scholarship in Mathematics. The above monograph was an up-to-date review of the Fourier series prevalent in those times. It also included the contributions made by the author in the subject.

On one hand Ganesh Prasad was very much appreciative of the new research findings of western scholars, but on the other hand he did not hesitate to criticize the anomalies in their researches as and when noticed by him. For example, Prasad critically examined the findings of Lebesgue on criteria for functions with discontinuities of the second kind, and put forward his own findings on the subject (Prasad, 1928, pp. 9-11).

Ganesh Prasad succeeded in attracting a large number of talented Indians from different parts of the country to carry out research in modern mathematics, in particular the theory of functions of a real variable, under his guidance. Amongst his research students, mention must be made of those who distinguished themselves by their valuable contributions. They are: Prof. B.N. Prasad of Allahabad University, Prof. A.N. Singh of Lucknow University, Prof. Braj Mohan of Banaras Hindu University, Prof. Hariprasanna Banerjee of Calcutta University.

Ganesh Prasad published more than 50 research papers in international and national journals. He also authored 10 books on the subject of Mathematics. He was elected as the Foundation Fellow of the Indian National Science Academy in 1935 for his significant contribution in Mathematics. Ganesh Prasad breathed his last on 9<sup>th</sup> March 1935 due to cerebral hemorrhage at Agra, while he was attending a University meeting.

The present Indian mathematical community owes a great deal of gratitude to Professor Ganesh Prasad for the initiation of the modern mathematical research in India way back in the early part of the twentieth century. The foundation made by him played an important role in building the present structure of mathematical teaching and research in India.

### Notes and References

1. Hobson, p. 148.
2. Prasad, *Math. Annalen*, 1912, p. 136.
3. Prasad, Ganesh, *ibid*, p. 1.

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