Indian Journal of History of Science, 47.4 (2012) 699-717

VEṆVĀROHA FROM A MODERN PERSPECTIVE

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(Received 21 December 2010; revised 23 October, 2012)

Candravākyas of Vararuci (4th Cent. AD) give a method of computing the position of the Moon at sunrise on any day at any place. For finding the position at a time other than the sunrise, linear interpolation is inadequate. So Mādhava of Saṅgamagrāma (14th Cent. AD) devised a method for finding the position corresponding to 9 parts of the day. The mathematical idea of this method is studied in this paper.

Key words: Anomalistic period, *Candravākyas*, *Dhruva*, *Mandocca*, Periodic function, *Sphuṭa*,

1. INTRODUCTION

The well-known *Candravākyas* of Vararuci (4th Cent. AD) provide a method of finding the longitude of Moon for any *Kali* day, at sunrise at Laṅkā. Corrections like *cara*, *deśāntara* etc. can be effected to get the longitude of the Moon for any place at sunrise on any day. For a time other than sunrise one can use linear interpolation. But the results obtained thus are not accurate.

To rectify this inaccuracy, Mādhava of Saṅgamagrāma (14th Cent. AD) devises an ingenious method. In this, the day is divided into nine parts and the longitude corresponding to the beginning of each part is obtained. From a modern point of view the method can be interpreted in terms of periodic functions with certain properties. Mādhava also gives revised *Vākyas* in the place of *Candravākyas* of Vararuci.

2. COMPUTATION OF PLANETARY POSITIONS

For the computation of planetary positions, two methods were generally followed. The former is arithmetical and the latter geometrical. In

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the former the position of the concerned planet is recorded for a long period till it comes back to original position nearly after a certain number of days. The same table can be used to calculate the position for the successive periods and a correction is made to remove the accumulated error.

***Candravākyas* of Vararuci**

We shall illustrate this with *Candravākyas* of Vararuci. This table which gives the positions of the Moon for 248 days at Sunrise at Laṅkā. (the zero position on earth), starting at the instant when *Mandocca* of Moon, the Moon and *meṣādi* coincide. Thus at the starting point the longitude of the Moon at *Mandocca* is zero. The Vararuci's *vākyas* run thus:

ग्रीर्नः श्रेयः (12^0 03')
Gīrnaḥ śreyaḥ

धेनवः श्रीः (24^0 09')
Dhenavh śrīaḥ

रुद्रस्तु नम्यः (1^r 06^0 22')
Rudrastu namyaḥ

The table gives the longitude of the Moon at sunrise for 248 days at Laṅkā. *Mandocca* is the position at which it is farthest from the earth. Anomalistic period of the Moon – the period for moving from *Mandocca* to *Mandocca* is 27.554501 days. In about 248 days it makes 9 revolutions round the earth with respect to *Mandocca* and at that time the longitude would be 27^0 44′ as given by the last *Vākya*, भवेत् सुखम् (*bhavet sukham*).

We can use the *Candravākyas* for finding the longitude of the Moon for any day at sunrise at any place. First, we find the longitude at sunrise at Laṅkā. For this we find the number of days elapsed since the commencement of *Kaliyuga* or what is called *Ahargaṇa*. The year used is sidereal year the duration of which is 365 days 6 hours 12 minutes 38.56 seconds and this differs from the modern figure by less than three minutes. To get the number of *kali* days over, we proceed as follows.

Multiply the figure representing the year of *Kali* elapsed by 2103890, subtract 12372 and divide by 5760. This gives the number of days elapsed at the commencement of the current *Kali* year represented by the solar ingress into the *rāśi Meṣa*. Then we add the *Vākyas* giving the number of days in each month starting from *Meṣa*. This is given by:

कुलीन (*kulīna*) (31) रूक्षज्ञ (*rūkṣajña*) (62) विधान (*vidhāna*) (94)

मात्रया (*mātrayā*) (125) क्षणस्य (*kṣaṇasya*) (156) सिंहस्य (*simhasya*) (187) सुपुत्र (*suputra*) (217) चत्वरः (*catvarah*) (246)

तथाद्रि (*tathādri*) (276) मीनाङ्ग (*mīnāṇga*) (305) मृगाङ्ग (*mṛgāṇga*) (335) मातुलः (*mātulaḥ*) (365)

These *Vākyas* suggest that by the end of month *Meṣa* 31 days by *Vṛṣabha* 62 days by *Mithuna* 94 days are over and so on. If the date is 10th of *Karkaṭaka*, the number of days that has passed over since the commencement of the year is: 94+9=103. This has to be added to the figure representing the number of days that have elapsed in the *Kali* year. Thus we get the *Ahargaṇa*. But it may differ from the actual figure by a day or two. To obviate this difficulty, use the fact that *kali Yuga* started on a Friday. By dividing the *Ahargaṇa* by 7 and noting the remainder and the actual weekday, the error can be rectified. Now we have to find the *Vākya* to be used for the day. For this, subtract from the number of *kali* days elapsed, 1741650 and divide by 12372, 3031 and 248. Note the quotients q_1, q_2 and q_3 respectively and note the final remainder. This suggests the *Vākya* to be used. If the remainder is 195, then the 195th *Vākya* indicating $1^r17°14'$ is to be used. Now we have to find *Dhruva* to remove the accumulated error. For this multiply q_1 by $9^r27°48'\ 9''\ 44''$ multiply q_2 by $11^r7°31'\ 10''\ 16'''$ and q_3 by $0^r27°43'\ 28''\ 39'''$ and add. To the sum add $1^r6°31'\ 41''\ 31'''$. Note the sum. When added to the *Vākya* the longitude of the Moon at Sun rise at Lanka is obtained. For a particular place *Deśāntara Samskāra* and *Cara Samskāra* are required. *Bhujāntara Samskāra* can also be effected. Thus the longitude of the Moon at any place at sunrise can be obtained. To get the longitude of the Moon at some time of the day or night, (20 *Nāḍikās* 43 *Vināḍikās* after sunrise for example) linear interpolation can be made. But this method is not accurate. Mādhava of *Saṅgamagrāma* has suggested a method in *Veṇvāroha* to get this at any 9th part of the day. The author has studied this from the modern perspective and the method of Mādhava is equivalent to the use of properties of periodic functions.

The procedure of finding the *Vākya* and *dhruva* is detailed in *Pañcabodha* (5.12-14) thus:

अमितयवोत्सुकहीनं
द्युगणं रसगैरिकैः कुलीनाङ्गैः ।
देवेन्द्रैरपि हृत्वा
तच्छिष्टं भवति वाक्यसंख्येन्दोः ॥
विविधं निजवसुरोधं
तापेनोह्यं कुलासनैपुण्यम् ।
धिगहरलघुसत्रोनं
चैतान् हारास्तैः फलैः क्रमशः ॥
हत्वा तेषां योगः
कौलटभूपालतनयसंयुक्तः ।
देशान्तर विघटीकृत
रत्नाप्रायेन्वितो ध्रुवो ज्ञेयः ॥

amitaya votsuka hīnam
dyugaṇam rasagairikaiḥ kulīnāñgaiḥ |
devendrairapi hṛtvā
tacchiṣṭam bhavati vākyasaṃkhyendoḥ ‖ 12 ‖
vividhaṃ nijavasurodhaṃ
tāpenohyaṃ kulāsanaipuṇyam |
dhigaharalaghusatronaṃ
caitān hārhāhṣtaiḥ phalaiḥ kramaśaḥ ‖ 13 ‖
hatvā teṣām yogaḥ
kaulaṭa bhūpālatanayasaṃyuktaḥ
deśāntaravighaṭīkṛta |
ratnāprāyenvito dhruvoḥ jñeyaḥ ‖ 14 ‖

Rationale: *Vākyas* themselves can be derived. In the beginning of *Kali* the three points *Meṣādi*, the Moon and the *Mandocca* of the Moon coincided.

The mean longitude of the Moon after one day = 790' 35"

The longitude of *Mandocca* after one day = 6' 41"

Manda Kendra = 783' 54"

$$Mandaphala = \frac{7}{80} \times R \sin (783'54'')$$

= 67′54″ (negative being *Meṣādi*)

Therefore the longitude of the Moon = 790' 35" – 67' 54"

= 722' 41"

= 12° 2' 41"

This is approximated to 12° 03' (*gīrnah śreyah*).

In this way other *Vākyas* can be found out.

It is to be observed that *Candravākyas* can be used for from any instant of coincidence of the Moon and its *Mandocca*. Starting from the beginning of *Kaliyuga* any multiple of the anomalistic period of the Moon corresponds to the coincidence of the Moon and its *Mandocca*. The numbers 1741650, 123723031 and 248 correspond to these. The longitude of the Moon when 1741650 days have elapsed is found thus:

Longitude of the Moon	=	$1^r 6°27' 54'' 49''' 52''''$
Longitude of *Mandocca*	=	$1^r 7°11' 5'' 31''' 41''''$
Manda Kendra	=	$12^r – (43' 10'' 41''' 49'''')$
Manda Phala	=	$3' 46'' 43''' 10''''$
		(Positive being *Tulādī*)
Longitude of the Moon	=	$1^r 6°31' 41'' 31'''$
		retaining up to *tatpara*

This is indicated by *kaulaṭabhūpāla tanaya* (verse 14).

Also *mandocca* and the Moon are very near indicating the end of anomalistic cycle.

In a similar way we can show that the longitude of the Moon when 12372 days have elapsed is $3^r 27°48' 9'' 44'''$ (*vividham nijavasurodham*, verse 13). These numbers themselves are obtained from the continued fraction corresponding the anomalistic period

$$\frac{188611}{6845}$$

Novel Method in *Veṇvāroha*

The method described above is useful in finding the longitude of the Moon at any place on earth, at sunrise. But, for finding the longitude of the Moon at any time on a day accurately, this method does not help. One can use the method of linear interpolation but the results are not very accurate. Mādhava of *Saṅgamagrāma* devices is an ingenious method to circumvent this difficulty. The method is described here under. For details see [2 and 3]

The anomalistic period of the Moon or the time taken by the Moon to move from *Mandocca* to *Mandocca* once is 27 days 33 *Nāḍikās* $16\frac{587}{1389}$ *Vināḍikās*. Mādhava has given it as $\frac{188611}{6845}$ days. *Candrvākyas* start from a position when the *Mandocca*, the Moon and *Meṣādi* coincided at sunrise at Laṅkā. Let *a* full days and *g* part-day be elapsed at sunrise since the coincidence of the Moon and *Mandocca*. This means that we can use the *Candravākyas* from the moment which was exactly *a+g* days before the current day. Consider a moment which is *g* of a day before the current day. The moment falls on the previous day, its *nāḍi-vināḍi* etc. being the same as the end of the cycle *a* days ago. Thus if we get the *dhruva* of the Moon at the end of the cycle, add to the *vākya a* of the Moon, we get the longitude of the Moon for the moment on the previous day. In this way can get the longitudes at the moments at the end of (*a* + *h*) days + 1 cycle, (*a* + *h*) days + 2 cycles etc. before sunrise on the current day. Only thing is, for every moment for which the moment is pushed backwards, the *dhruva* $3°4'7\frac{22''}{33}$

has to be subtracted. The procedure for computing them is given in *Veṇvaroha* (Sarma 1956) and *Sphuṭacandrāpti* (Sarma 1973) which is detailed below. We thus get a set of *Vākyas* corresponding to the nine moments the first *Vākya* being *a* and the successive *vākyas* differing by 27 or 28. Using a suitable *Vākya* one can get the longitude of the Moon at $g + f$ on that day and so on. Thus one can get the nine instants and the corresponding *Vākyas*. Adding it to the *dhruva* one gets the longitude of the Moon at the instant. The method given in *Veṇvaroha* (Sarma, 1955) and *Sphuṭacandrāpti*(Sarma, 1973) is detailed below.

Let *N* be the number of *kali* days elapsed at some instant of the day. Subtract 1502008 from *N*. *N* – 1502008 is called *Khaṇḍaśeṣa*. Let it be equal to *k*. Find

$$k \times \frac{\text{शिवदूत}}{\text{पयप्तिहृदय}} = k \times \frac{6845}{188611}$$

In fact

$$k \times \frac{6845}{188611} = \frac{k}{\frac{188611}{6845}}$$

where $\frac{188611}{6845}$ days = anomalistic period of the Moon. Let q_1 be the quotient and r_1 be the remainder. Then q_1 is the number of anomalistic cycle over. Find $r_1/6845$. Let q_2 be the quotient and r_2 be remainder. Then q_2 is the number of the first *Vākya* to be used. To find the next *Vākya*, find $\frac{r_2 + 188611}{6845}$.

Then quotient q_3 is the next *Vākya*. Find $\frac{r_3 + 188611}{6845}$. The quotient q_4 gives the next *Vākya*. The instants are given by

$$\frac{6845 - r_1}{6845}, \frac{6845 - r_2}{6845}, etc$$

Thus we get the nine instants and the corresponding *Vākyas*.

To the *Vākyas* obtained, *Dhruva* has to be added. In *Veṇvaroha*, the *Khaṇda* or the date from which calculations are made is 15002008. The *dhruva* on this date can be calculated. To this is to be added *dhruva* as prescribed below: For 5105 anomalistic periods the *dhruva* is $5^r 27°47'$. For each 69 anomalistic periods the *dhruva* is $7^r 1°47'$ and for one anomalistic period it is $3°4'5'' \frac{22}{25}$. Adding the sum of the *Dhruva* so obtained to the *Vākya*, the longitude of the Moon is obtained. This longitude is for *Laṅkā*. By making *Deśāntara* and *Cara* corrections one gets it for the desired place.

The *dhruva* $5^r 27°47'$ includes that for the *Khaṇḍa*. Therefore, while calculating *dhruva*, 5105 should be subtracted only once. For the remaining use the *dhruva* for 69 cycles and cycles for the remaining. We calculate *dhruva* for *agrimaphala* the first quotient and for the remaining subtract at the rate of $3°4'65 \frac{22}{23}$.

3. Periodic Functions and their Properties

We discuss in this section real valued functions of real variables. In what follows R stands for the set of real numbers. A function $f: R \rightarrow R$ is called a periodic function if there exists a real number λ such that $f(x + \lambda) = f(x)$ for every $x \varepsilon R$. The number λ is called a period of f. The smallest positive number λ is called the period of f.

Consider a periodic function f for which $f(1), f(2),..., f(n)$ are known for some positive integer n. Let the period of the function be $m + l$, where m is positive integer and $0 < l < 1$. We develop a method for finding $f(x)$ when x takes certain non-integral values.

Lemma 1: Let $f: R \rightarrow R$ be a periodic function with period $m + l$. Then for any positive integer r.

$f(x + (r - rl) - [r - rl]) = f(x - (m + 1)r - [r - rl])$

[ ] denoting the greatest integer function.

In particular, $f(x + (1 - l) - [1 - l]) = f(x + (m + 1) - [1 - l])$ etc.

Proof: Let r be a positive integer. Then,

$$f\,(x + (r - rl) - [r - rl]) = f\,(x + (r - rl) - [r - rl] + r\,(m + l))$$
$$= f\,(x + r\,(m + 1) - [r - rl])$$

Lemma 2: Let $l = \frac{p}{q}$ in lowest terms where $0 < l < 1$.

Let $k\ \varepsilon\ \{0, 1, 2, ..., q\text{-}1\}$.

Then $k - kl = a + {}^{i}\!/_{q}$ where k takes the values 0, 1,

and j takes the same values in some order and a is a positive integer or 0.

Proof: Since $\frac{p}{q} = l$ is in lowest terms. $kl = \frac{kp}{q} = a + \frac{j}{q}$ where takes these values 0, 1, 2, ..., q - 1 in some order or other.

When k = 0, a = 0 and i = 0 when k = 1, $1 - l = \frac{q-p}{q}$ when $k > 1$,

$$k - kl = k - \left(a + \frac{j}{q}\right)$$

$$= (k - a - 1) + 1 - \frac{j}{q}$$

$$= (k - a - 1) + \frac{q - j}{q}$$

As k takes the values 0, 1, ..., q-1, q-j also takes the same values in some order or other. The result now follows.

Lemma 3: Let $l = \frac{p}{q} + \varepsilon$ where $\frac{p}{q}$ is in its lowest terms and

$$|\varepsilon| < \frac{1}{q(q-1)}$$

then for any r, s, ε {0, 1, 2, ..., q-1}

$rl - [rl] = sl - [sl]$ if and only if $r = s$.

Proof: The 'if' part is trivial. To prove the 'only if' part let

$$rl = r\frac{p}{q} + r\varepsilon = a + b + r\varepsilon$$

where a is a positive integer or zero and $b = \frac{j}{q}$ for some

$j \varepsilon \{0,1,2,...,q-1\}$.

Clearly

$$b + r\varepsilon < \frac{q-1}{q} + \frac{q-1}{q(q-1)}, \text{ since } |\varepsilon| < \frac{1}{q(q-1)}$$

$$= \frac{q}{q} = 1$$

Thus we can write,

$rl - [rl] = b + r\varepsilon$

Similarly,

$sl - [sl] = d + s\varepsilon$ for some d.

Let now, $rl - [rl] = sl - [sl]$. Then

$b + r\varepsilon = d + s\varepsilon$

Therefore,

$\frac{i}{q} + r\varepsilon = \frac{j}{q} + s\varepsilon$ for some

$i, j \varepsilon \{0,1,2,...,q-1\}$.

Thus,

$$\frac{j-i}{q}+(r-s)\varepsilon=0$$

Since we can take

$$|\varepsilon|=\frac{f}{q(q-1)} \text{ where } 0 < f < 1.$$

Thus we get

$$\frac{j-i}{q}+\frac{(r-s)f}{q(q-1)}=0.$$

This implies

$$(j-i)+\frac{(r-s)f}{q-1}=0$$

Since $\frac{(r-s)f}{q-1}<1$ and j - i is and integer or zero, it follows that $i = j$ and $r = s$.

The result now follows.

The above lemmas lead to

Theorem 1: Let $f = R \to R$ be a periodic function with period equal to $m + l$ where m is a positive integer and $0 < l < 1$. If $l=\frac{p}{q}$ is in its lowest terms, then the points

$1 - l$, $(2 - 2l) - [2 - 2l]$, $(3 - 3l) - [3 - 3l]$, ..., $(q - 1) - (q - 1) l - [(q - 1) - (q - 1) l]$, divide the interval [0,1] into q equal parts.

If $l=\frac{p}{q}+\varepsilon$ where $|\varepsilon|<\frac{1}{q(q-1)}$, then these points divide [0,1] into q parts, not necessarily equal in length. Moreover, if x is any integer and p is any point of division, then $f\,(x+p)$ can be expressed as the value of f at some integral value.

Proof : The major part of the theorem follows immediately.

When $l = \frac{p}{q} + \varepsilon$

and

$|\varepsilon| < \frac{1}{q(q-1)}$

then successive points of division are of the form $\frac{i}{q} + n_1\varepsilon$ and $\frac{i+1}{q} + n_2\varepsilon$ and the difference is of the form $\frac{1}{q} + (n_2 - n_1)\,\varepsilon$ which varies with the values of n_1 and n_2. Thus the points of division partition [0,1] into sub intervals which are not necessarily equal in length. The last part follows from Lemma 1.

Lemma 4: Let $x\ \varepsilon$ [0,1] and [0,1] be partitioned by $k\left(\frac{p}{q} + \varepsilon\right)$, k = 0, 1, ..., q - 1 as described above. Then the points of division $x + \frac{i}{q} + k\,\varepsilon$ are distinct.

Proof: First of all we shall consider the case when e = 0. Then $x + \frac{i}{q} + f$ where 0 < f < 1. We note that $\frac{i}{q} + f < 1$ and we can take $f < \frac{1}{q}$

$\frac{i}{q} + f + \frac{j}{q} = \frac{i+j}{q} + f$. If $\frac{i+j}{q} + f < 1$, the points of division are as given earlier. When $\frac{i+j}{q} + f > 1$ the fractional part is given by

$\frac{i+j}{q}+f-1=\frac{i+j-q}{q}+f=\frac{h}{q}+f$ where h = 0, 1, 2, ..., q - 1, since

$$\frac{l+j}{q}+f=\frac{i}{q}+f+\frac{j}{q}<1+\frac{i}{q}<2$$

We shall show that h > 0. We note that

$\frac{i+j}{q}+f>1$ and therefore $\frac{i+j}{q}+f=1+g$ where 0 < g < 1

Consequently,

$$\frac{i+j}{q}=1+g-f>i+g+\frac{1}{q}$$

Thus

$$\frac{i+j}{q}-1>g+\frac{i}{q}>0$$

Therefore h > 0.

It can be shown that

$$\frac{i+j}{q}+f-1=\frac{i+k}{q}+f-1\Rightarrow j=k$$

Also

$$\frac{i+j}{q}+f=\frac{i+k}{q}+f\Rightarrow j=k$$

If

$\frac{i+j}{q}+f-1=\frac{i+k}{q}+f$, then $\frac{i+j}{q}-\frac{i+k}{q}=1$

Thus

$$\frac{j-k}{q}=1$$

which is impossible.

Thus q takes the values 0, 1, 2, ..., q - 1

Thus the points of division are

$f, f+\frac{1}{q}, \ldots, f+\frac{q-1}{q}$ which are distinct. As in lemma 3, the case when $\varepsilon \neq 0$ is dealt with.

If

$$f+\frac{i}{q}+s\varepsilon=f+\frac{j}{q}+t\varepsilon$$

then

$$\frac{i-j}{q}=(t-s)\varepsilon$$

Therefore for some $0 < f < 1$

$$i-j=(t-s)q.\varepsilon=\frac{(t-s).q.f}{q(q-1)}$$

$$=\frac{(t-s)f}{q-1}<1$$

This can hold good only if $i = j$ and $t = s$

Theorem 2: Let $f\ \ R \rightarrow R$ be a periodic function with period $m + l$, where m is a positive integer and $0 < l < 1$. Let $x = k(m + l) + a + g$ where a is a positive integer and $0 < g < 1$. Then $f(x - 1 + 1 - g) = f(a)$

Also

$f(x - 1 + (1 - g + r - rl) - [1 - g + r - rl])$

$= f((k - r)(m + l) + a + r(m + 1) - [1- g + r - rl])$

Proof: We have

$x = k(m + l) + a + g$

Thus

$x - 1 + 1 - g = k(m + l) + a$

Therefore

$f(x - 1 + 1 - g) = f[k(m+l) + a] = f(a)$

Also

$x - 1 + (1 - g + r - rl) - [1 - g + r - rl])$

$= (k - r)(m + l) + a + r(m + l) + r - rl - [1 - g + r - rl]$

$= (k - r)(m + l) + a + r(m + 1) - [1 - g + r - rl]$

Therefore

$f(x - 1 + (1 - g + r - rl) - [1 - g + r - rl]$

$= f((k - r)(m + l) + a + r(m + 1) - [1 - g + r - rl])$

In fact, the function treated in *Veṇvāroha* is not strictly periodic. But the *Vākyas* can be reduced to a periodic function. Because of its intrinsic interest, it is treated like that.

A function $f : R \to R$ is called number λ with period λ is called δ periodic if $f(x + \lambda) = f(x) + \delta$. Accordingly we get the analogues of the results in Theorem 2 thus:

$f(x - 1 + 1 - g) = f(a) + k\,\delta$

$f(x - 1 + (1 - g + r - rl) - [1 - g + r - rl]$

$= f[(k - r)(m + 1) + a + r(m + 1) - (1 - g + m - rl)] + (k - r)\,\delta$

This simply indicates the fact that *dhruva* has to be added to the *vākyas*. In the cycles in 248 days the *dhruva* is added after the period is over in effect.

In fact, the *vākyas* of Vararuci can be reformulated in which the first set of *vākyas* for one cycle can be repeated without adding the *dhruva*. In the case the periodic functions apply. Otherwise one can directly apply δ periodic functions.

4. Mādhava's Method from a Modern Perspective

We shall examine the method of Mādhava in the light of the preceding section. First of all, the anomalistic period of the Moon is

$$= \frac{188611}{6845} = 27 \text{ days } 30 \textit{ nādikas } 16 \frac{551}{1339} \textit{ vinādikas} = 27.5545 \text{ days.}$$

Noting that $\frac{5}{9} = 0.5555$ and $\frac{5}{9} = \frac{p}{q}$ we can write $0.5545 = 0.5555 - 0.0010 = \frac{p}{q} - \varepsilon$

where ε = .0010

Clearly, $.0010 = \frac{1}{1000} < \frac{1}{8.9} = \frac{1}{72}$ as required.

Thus the conditions required for the interpolation described in the preceding section are satisfied.

Let N be an integer representing the number of *Kali* elapsed over. Subtract *Khaṇḍa*. Let the remainder be *r*. Then

$$\frac{x}{\frac{188611}{6845}} = k + \frac{f}{188611}, \text{ where f} < 188611$$

$$= k + \frac{\frac{f}{6845}}{\frac{188611}{6845}}$$

$$= k + \frac{a + \frac{d}{6845}}{\frac{188611}{6845}}, \text{ for some } a$$

Therefore, $x = k\left(\frac{188611}{6845}\right) + \left(a + \frac{d}{6845}\right)$

Comparing with the expression in Theorem 2, $g = \dfrac{d}{6845}$.

The number of the *Vākya* to be used first is *a* and the time concerned is $1 - \dfrac{d}{6845} = \dfrac{6845 - d}{6845}$ of the day after Sunrise.

Again $\dfrac{188611}{6845} = 27 + \dfrac{3796}{6845} = (m + l)$

We shall now prove the equivalence of the method we developed with Mādhava's.

We denote the current day by *x* and start the cycles from the one to which belongs.

First of all,

$x = 0\ (m + l) + a + g$

and

$x - 1 + 1 - g = a + g - 1 + 1 - g = a$, the number of the first *Vākya*.

Also

$f(x - 1 + 1 - g + r - rl) - [1 - g + r - rl]$

$= f\ (r\ (m + 1) + a - [1 - g + r - rl])$ ....A

From Mādhava's procedure we get that the increase in the number of *Vākyas* in the $r + 1^{th}$ cycle from the start

$= 27\ r + [g + rl]$

But $[g + rl] = r - [1 - g + r - rl]$, if $[g + rl]$ is not an integer.

The increase in the number of *Vākyas* according to (A) is

$27r + r - [1 - g + r - rl] = 27\ r + [g + rl]$

Thus the two methods are equivalent.

Note: When $[g + rl]$ is an integer the time concerned is 0 or 60 and two methods give different answers. The method (A) gives the time as 0 and Mādhava's gives as 60 *ghaṭikās*.

Example

Consider

The number of *Kali* days elapsed = 1862452

Kali days – *Khaṇḍa* =1862452 – 1502008 = 360444

Find $\frac{360444 \times 6845}{188611}$

The quotient is 13081 (*agrimaphala*)

The remainder = 18689

Dividing 18689 by 6845 the quotient is 2 and remainder is 4999. The first *Vākya* to be used is 2nd and the *dhruvakāla* = $= \frac{(6845 - 4999) \times 60}{6845}$

$$= \frac{1846 \times 60}{6845} = 16 \; gh \; 10 \; vigh$$

Proceeding thus we can determine all the *dhruvakālas* and *vākyas*. The *dhruat* can be calculated for 360444 according to the rules given and we get the following table.

Time Statis- ***vighaṭis***	**No. of the** ***Vākya***	***Vākyas***				***dhruva***				***sphuṭa***			
		r	**deg°**	**min′**	**sec″**								
3 – 5	112	1	3	46	10	5	7	43	35	6	11	29	45
9 – 37	57	0	28	57	13	5	13	51	49	6	12	49	02
16 – 10	2	0	24	8	39	5	20			6	14	8	39
23 – 16	195	1	17	3	34	4	28	31	14	6	15	34	48
29 – 49	140	1	12	15	1	5	4	39	28	6	16	54	29
36 – 22	85	1	7	26	34	5	10	47	42	6	18	14	16
42 – 55	30	1	2	38	12	5	16	55	53	6	19	34	05
49 – 59	223	1	25	33	31	4	25	27	7	6	21	0	38
56 – 32	168	1	20	45	21	5	1	35	21	6	22	20	42

These give the longitude of the Moon on the day preceding the current day. Mādhava's revised *Candravākyas* are used above.

Historical Note

Mādhava of Saṅgamagrāma (c. 1340-1425) who had the title '*golavid*' (master of spherics) is the author of *Veṇvāroha*. Saṅgamagrāma is generally indentified with Irinjalakkuda near Kochi. His works include *Lagnaprakaraṇa* and *Agaṇita*. His *Mahājyānayanaprakāra* and *Madhyamānayanaprakāra* for which short commentaries are available give novel theorems. It is likely that he wrote an exhaustive astronomical work, though not available now. He discovered the infinite series for the circumference of circle (and effectively for π) and infinite series for *R sine* and *R cosine* much before they were known in the west. His *Veṇvaroha* had been commented upon by Acyuta Piṣaraṭi (c. 1500-1621). By any standards, Mādhava was acclaimed as very great astronomer and mathematician of the world (Sarma 1972).

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