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Syamadas Mukhopadhyay (1866-1937): A Reputed Geometrician of India

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Abstract

Syamadas Mukhopadhyay of the Pure Mathematics Department of Calcutta University made important research work in various branches of Geometry. His famous theorem, eponymously called "Mukhopadhyay's Four Vertex Theorem" is a notable piece of research and acted as a catalyst for further investigations in that area. Mukhopadhyay's research works have created wide interest amongst researchers in Topology and Algebraic Geometry the world over. This article is both a tribute to the great geometer of India, as well as an effort to bring to the fore the price-less research contributions that he made, working in India under colonial rule.

Key words: Cyclic points, Differential geometry, Hyperbolic geometry, Osculating conics, Sextactic points.

1. Childhood and Early Education

Syamadas Mukhopadhyay (SD Mukhopadhyay) was born at Haripal in the Hooghly district of West Bengal on the 22nd June, 1866. His father Babu Ganga Kanta Mukhopadhyay was in the state judicial service and was transferred to different places in connection with his official duties. As a consequence Syamadas studied in different institutions at different times. He completed his graduation from Hooghly College and obtained his M. A. degree in Mathematics from the Presidency College, Calcutta in 1890. In 1909 he won the Griffith's Prize of Calcutta University for his mathematical dissertation entitled "On the infinitesimal analysis of an arc". In 1910 he was awarded the Ph. D. degree by the University of Calcutta for his original work in Differential Geometry. His thesis was entitled "Parametric Coefficients in Differential Geometry of Curves". It may be noted that S D Mukhopadhyay was the first Indian to obtain a doctorate degree in Mathematics in India. It is needless to mention that he was the first recipient of the Ph. D. degree in Mathematics from the Calcutta University.

Syamadas Mukhopadhyay (1866-1937)

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2. Teaching Career

Soon after completing his M.A. in Mathematics, Syamadas Mukhopadhyay joined a private college in Calcutta as a faculty member and served there for several years. Thereafter he got an appointment in Government Service as a Professor of Mathematics in the Bethune College, Calcutta. In Bethune College, which happened to be the first women's College in Asia, S D Mukhopadhyay had to undertake very heavy teaching load. There apart from Mathematics, he had to regularly take classes on English and Philosophy. In 1904, he was transferred to the Presidency College. He served there for eight years till 1912. In 1912, the then Vice-Chancellor of Calcutta University, Sir Asutosh Mookerjee, invited Syamadas Mukhopadhyay to join the newly set up Department of Pure Mathematics in the Calcutta University. S D Mukhopadhyay accepted the offer and joined the department. He served the department to the satisfaction of Sir Asutosh and worked there till 1932. During the last six years of his tenure, Syamadas held the post of a university professor of the Calcutta University.

S D Mukhopadhyay was a talented teacher and his fame as a teacher of eminence spread far and wide. R C Bose after completing his M.Sc. in Applied Mathematics from the Delhi University came and joined the Pure Mathematics Department of Calcutta University as a student, just in order to be taught by Syamadas Mukhopadhyay. He later became a researcher under S D Mukhopadhyay and earned eminence as a great mathematician. Amongst many students who were taught by Syamadas Mukhopadhyay, three deserve special mention. They were G Bhar, who was a Professor of Mathematics in the Presidency College, Calcutta and a teacher of great repute, R N Sen, who became the Hardinge Professor of Pure Mathematics in Calcutta University and also a famed researcher in the field of Differential Geometry, and Professor M C Chaki who held the Chair named "Sir Asutosh Birth Centenary Professor of Higher Mathematics" in Calcutta University and also did good research in many areas of geometry.

3. Academic Career and Research Contributions

Syamadas Mukhopadhyay was possibly inspired during his undergraduate student days in Hooghly College by his teacher William Booth who was quite well known for his researches in geometry. He was the man who had influenced young Asutosh Mookerjee during the latter's student days in the Presidency College. M C Chaki, a famed mathematician of Calcutta University in his article on Syamadas Mukhopadhyay, wrote that "In a very short time after taking his B. A. degree he (S D Mukhopadhyay) solved an intricate geometrical problem, a fact recorded in M'celland's 'Geometry of the Circle'." He has further written that S D Mukhopadhyay's research contributions "were outstanding for their originality and novelty of treatment".

Syamadas Mukhopadhyay's research work may be broadly classified into Use of Synthetic Geometry to solve properties of plane curves; Non-Euclidean Geometry; Differential Geometry and Stereoscopic representations of 4-dimensional space.

In the first part, he has dealt with properties of plane curves, especially in their infinitesimal regions using synthetic methods. Here he has developed new methods. These methods led to a number of interesting theorems on the existence of minimum number of cyclic and sextactic points between two points of a given curve on a convex oval etc. In this context two of his theorems deserve special attention. But before that it would be necessary to understand what are cyclic and sextactic points. A cyclic point is a singular point, on a plane curve, where the circle of curvature passes through 4 consecutive points instead of 3. On the other hand a sextactic point is a singular point, where the osculating conic passes through

6 consecutive points instead of 5. At a cyclic point, the circle of curvature may touch the given circle internally or externally. In the former case, the point will be called in-cyclic and in the latter case, ex-cyclic. Similarly at a sextactic point, the osculating conic may touch the given curve internally or externally. In the former case, the point is called in-sextactic and in the latter case it is called ex-sextactic. With these definitions, S D Mukhopadhyay first demonstrated a number of interesting propositions. Later they led to his famous two theorems, which have already been mentioned above. Now they are being stated below.

Theorem I states that "the minimum number of cyclic points on a convex oval is 4" and Theorem II states that "the minimum number of sextactic points on a convex oval is 6". These two theorems were first published in 1909 in the *Bulletin of the Calcutta Mathematical Society (BCMS).* But initially, at that time not much attention was paid to this piece of research. Only the eminent French mathematician J S Hadamard (1865-1963) referred to this work in the memoirs of College de France. However, much later, these theorems were re-discovered in Europe. Since then many noted mathematicians have taken them up as subject of investigations. W Blaschke, a noted German geometer gave credit to Syamadas Mukhopadhyay for giving the original first proof of the Theorem I stated above. In modern literature of Geometry, this theorem is now eponymously quoted as "Mukhopadhyay's Four Vertex Theorem". This celebrated theorem is stated in various important mathematical treatises such as Differential Geometry by H Guggenheimer, McGraw Hill, New York, 1930; Differential Geometry by J J Stoker, Wiley Interscience, 1969 and Elements of Differential Geometry by R S Millman and G D Parker, Prentice-Hall, Englewoods Cliffs, 1977.

As already mentioned, W Blaschke gave the credit of proving the "Four Vertex Theorem" to Syamadas Mukhopadhyay. He has mentioned this in his book "Vorlesungen uber Differential Geometry", Springer, Berlin, 1924. Further references connected to this theorem are also given there. Later on S D Mukhopadhyay generalized these two theorems. The generalized Theorem I states that "If a circle C intersects an oval V in 2n points (n 2) then there exists at least 2n cyclic points in order on V, of alternately contrary signs, provided the oval has continuity of order 3". The generalized Theorem II states that "If a conic C intersects an oval V in 2n points (n> or = 2), then there exist at least 2n sextactic points in order on V, which are alternatively positive and negative, provided V has continuity of order 5". Thus S D Mukhopadhyay placed his earlier investigations on more rigorous basis.

Apart from these investigations, Syamadas Mukhopadhyay published a number of research papers on the general theory of osculating conics. In this context, the comments made by his student G Bhar of Presidency College may be noted. Bhar wrote "It is highly probable that Mukhopadhyay was led to the study of the theory of osculating conics by Sir Asutosh's work on the differential equations of all parabolas and his beautiful geometrical interpretation of the Mongean equation".

In the second part of his research work, Syamadas Mukhopadhyay contributed to a type on Non-Euclidean geometry namely Hyperbolic Geometry. Actually hyperbolic geometry is the geometry obtained by assuming all the postulates of Euclid except the fifth one. The fifth postulate of Euclid is replaced by its negation. In the context of S D Mukhopadhyay's research in Non-Euclidean geometry, the comments made by his student G Bhar, is very relevant. Bhar writes:

> "Professor Mukhopadhyay found himself in his elements when he was called upon to teach the principles of Non-Euclidean Geometry to the Post-Graduate students of the University".

Serious research work in Non-Euclidean geometry in India was first undertaken by R

Vaithyanathaswamy (1894-1960) of Madras University. His first two research papers[1] in this field were published in the *Journal of the Indian Mathematical Society* (*JIMS*) in 1914. S D Mukhopadhyay in collaboration with his student G Bhar published his first research paper on hyperbolic geometry in the *Bulletin of the Calcutta Mathematical Society* in 1920. The paper was titled "Generalization of certain theorems in the hyperbolic geometry of the triangle". This publication was followed by a number of research papers on this kind of geometry. The investigations carried out by him resulted in the important discovery of the "Rectangular Pentagon" and his beautiful geometrical interpretation of the Engel-Napier rules. In collaboration with his two famous students R C Bose and G Bhar he made interesting generalizations of the ideas of concurrence and colinearity of lines and points in Non-Euclidean Geometry. To be more specific, S D Mukhopadhyay, extended the well known concurrency theorems of the angle bisectors and the right bisectors of the sides of an ordinary triangle, to all types of hyperbolic triads of lines and points. It may be noted that by a hyperbolic triad, it is meant a group of three elements (points or lines) lying upon a hyperbolic plane. In this context, his three research papers titled "Geometrical investigations on the correspondence between a right-angled triangle, a three-right-angled quadrilateral and a rectangular pentagon in hyperbolic geometry" [*BCMS*, 13(4), (1922-1923):211-216]; "On general theorems of co-intimacy of symmetries and hyperbolic triad" [*BCMS*, 17(1), (1926): 39-55] and (with R C Bose) "Triadic equations in hyperbolic geometry" [*BCMS*, 18, (1927), 99-110] deserve special attention.

In the first paper mentioned above, S D Mukhopadhyay concluded "We have thus the closed series of 5 associated right-angled triangles and the Engle-Napier Rules are shown to possess a real geometrical basis in the rectangular pentagon". It was an exquisite piece of mathematical research.

In the third part of his research and investigations, Professor Syamadas Mukhopadhyay while dealing with Differential Geometry of curves, introduced for n-dimensional space curves certain differential forms. He named them parametric coefficients and using them he expressed many invariant properties of the curves. In this matter, another eminent geometer M C Chaki has commented "The distinct merit of the method of parametric coefficients of Mukhopadhyay lies in the fact that it achieves by elementary method, results which have been obtained by advanced analysis".

The fourth and final part of his research comprises a suggestion of a steoroscopic device for visualizing figures in four-dimensional space and his discussion with Bryan in this matter.

In 1912-1913, S D Mukhopadhyay had published his note on the steoroscopic representation of four-dimensional space [*BCMS*, 4, (1912-1913):15]. Around the same time, Bryan had also published a paper in the same journal suggesting another kind of device. He had also claimed that it was superior to the one suggested by S D Mukhopadhyay. A reply to this criticism was also given by Syamadas Mukhopadhyay and it too was published in the *Bulletin of the Calcutta* Mathematical Society [*BCMS*, 6, (1914):55-56]. In that reply, Syamadas Mukhopadhyay wrote "I do not however see any good in further prolonging the controversy between us. Both of us have fairly stated our methods. It would lie with other mathematicians interested in this problem of four dimensions to accept or reject either".

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1. The first two research papers of Professor Vaithyanathaswamy were titled "Parallel straight lines" [*JIMS*, 6(1), (1914):58-61] and "Length of a circular arc"[*JIMS*, 6(1), (1914):220-221]. These were in all likelihood the first research papers on Non-Euclidean Geometry published by an Indian.

Toward the end of his research career, S D Mukhopadhyay published two important papers entitled “Lower Segments of M-curves” and “Cyclic curves of an ellipsoid” in the *Journal of the Indian Mathematical Society*. Mukhopadhyay explained that the name M-Curve (Monotropic Curve) was due to the German geometrician Stackel H Mohrmann (1907-1934), who in adopting the name, gave the following precise definition. He said “A singularity-free (that which does not intersect itself) real branch of an analytic curve, which divides the Euclidean plane into two and only two regions, and for which the curvature at every finite point is limited and different from zero, will be called a limited monotropic curve or simply an “M-curve”. [*Mathematische Annalen*].

In his investigations, S D Mukhopadhyay considered M-curves which are not necessarily analytic. He has characterized M-curves in a different way and investigated the associated problem.

4. Impact of Research (Reviews and Accolades)

W Blaschke, with regard to the paper “Cyclic curves of an ellipsoid” suggested in his lecture on “Selected Problems of Differential Geometry” in 1932 and said that the object of Dr. Mukhopadhyay’s paper is to study certain properties of cyclic curves on an ellipsoid, which is known to possess six vertices.

J S Hadamard of the Institute of France had a fair amount of interest in the researches carried out by S D Mukhopadhyay. In a letter dated February 23, 1923, he opined on Professor Syamadas Mukhopadhyay’s contributions in Synthetic, Non-Euclidean and Differential Geometry and wrote “The interest of your researches on osculating conics, on even non-Euclidean geometry and parametric formulae in differential geometry of curves have been increased by comparison with memoirs published in a slightly different line by a Dane, Juel. Indeed the conjunction of both kinds of works (Juel dealing with straight lines, you with circles and conics), is likely at my seminar, to prove of great power and bearing for further improvement of geometry”.

Syamadas Mukhopadhyay published 30 original papers in different national and international journals. Barring a few most of them were published by the University of Calcutta as “Collected Geometrical Papers by Professor Syamadas Mukhopadhyay” in two parts. The volumes were simultaneously reviewed in high-end journals like Nature and the *Bulletin of the American Mathematical Society* and received favourable comments. The original texts from the journals are quoted below.

The review of Part I of the collection, as published in *Nature* of 1931 stated “The papers in this collection number ten on plane curves and seven on non-Euclidean, mainly hyperbolic geometry. The papers of the first group include six dealing with such topics as the geometrical theory of a plane non-cyclic arc, cyclic and sextactic points and a generalized form of Böhmer’s theorem, in which methods of pure geometry are employed, in several cases new methods of considerable interest. In this group, there are also four papers on the general theory of osculating conics, in which the methods of differential geometry are applied in rather a novel manner. The papers of the second group also offer some new features, and amongst a number of interesting results may be noted an extension of the well-known correspondence between a right-angled triangle and a three-right-angled quadrilateral in hyperbolic geometry, so as to include a regular pentagon. The book can be recommended to all who are interested in geometry, whether Euclidean or not, and wish to learn something of the progress of geometrical studies in Indian Universities.”[*Nature*, no. 3205, vol. 127, April 4, (1931):516]

The same collection of papers, were reviewed by Professor Virgil Snyder, in the *Bulletin of the American Mathematical Society* in 1931 and the exact text of the review is quoted below. Professor Snyder wrote

> "The present collection contains 17 papers, previously published in Asiatic periodicals in the interval 1908-1928. The topics considered fall into three general heads, those concerning topological questions, including cyclic points, sextactic points, etc. of plane curves, those concerning triangles and quadrilaterals in hyperbolic geometry, and those on methods of visualizing representations of four way space. The methods are mostly those of elementary geometry, although differential expressions are used freely in the treatment of osculation. The proofs are strikingly direct and simple, and many of the theorems were first published previously to those obtained by others. For workers in topology, the Papers will be of real service." [*Bulletin of the American Mathematical Society*, 36(No. 9), (1931):614]

The second volume of the collected papers was again reviewed in *Nature* in 1932. The review is quoted exactly as the published version was. It said

> "This volume is a continuation of the volume of papers by the same author published in 1929 and reviewed in *Nature* of April 4, 1931, p. 516. There are two papers on plane convex ovals, but the chief part of the book consists of seven papers on the differential geometry of curves in an N-space. The latter are of special interest both on account of the original methods employed and the results obtained. They deal with parametric coefficients and their properties, the extension of the Serret-Frenet formulae to curves in the N-space, the expression of the co-ordinates in terms of the arc, curvatures at a singular point and osculating spherics. Unfortunately, the investigation is restricted to Euclidean space, but the author claims that, by the use of a certain distance formula, it can be adapted to any kind of non-Euclidean space without insurmountable difficulties. It would be of some interest if such a programme were actually carried out, if only for the four- and five-dimensional spaces used in relativity theory. The abstract nature of the topics dealt with makes the papers difficult to read, but students of algebraic geometry should find much to interest them. The book is clearly printed and unusually free from misprints, and is a credit to the Calcutta University Press."[*Nature*, No. 3323, Volume 132, July 7(1933):48]

The review of the same collection of papers was done by Professor Virgil Snyder in the *Bulletin of the American Mathematical Society* in 1932. He wrote "Part I of the Collected Papers was published in 1930, and reviewed in this Bulletin, Vol. 36(1931), p. 614. In Part II the pagination continues, and the make up is the same as that of Part I. It contains two recent essays on plane topology which appeared in the *Mathematische Zeitscrhrift* in 1931 and the *Tohoku Mathematical Journal* in 1931 respectively, and seven on parametric representation of curves in n-space, all but one of which (the Griffiths Memorial Prize Essay of 1910) were published in the *Bulletin of the Calcutta Mathematical Society* from 1909 to 1915. The argument and point of view of the papers on topology are similar to those in Part I. Only elementary methods are employed, but with striking originality and richness in new results. Most of these concern cyclic and sextactic points on continuous ovals.

The other essays are on differential geometry of analytic curves in a Euclidean n-space. Properties are expressed in terms of determinants of derivatives of various orders of the coordinates as to the parameter.

The first intrinsic parameter is the arc length. The second is the projection of the area of the triangle formed by three points which approach coincidence on the curve, summed over the interval of integration, etc. A curve in S has n such intrinsic parameters. They are independent of the coordinates chosen and of the parameter. Any $n-1$ independent equations connecting these parameters will determine a curve in S, intrinsically. The generalized idea of curvature, spheric of osculation, quadric of osculation etc. can now be expressed. The results in the case of plane curves are compared with those obtained by projective differential geometry. The same ideas are then extended to curves in *S*. At times the amount of machinery necessary seems a bit bewildering, but one is soon consoled by an unexpected general theorem evolving from the maze of formulas. The various kinds of singular points and the associated parametric representation in series are treated in great detail. The papers contain a powerful weapon with which to attack metric problems on analytic curves of hyperspace". [*Bulletin of the American Mathematical Society*, 38(7), (1932), p. 480]

The reviews from internationally reputed journals quoted above, as well as the comments made by the noted mathematicians of the time, clearly reflect the importance of the research conducted by Syamadas Mukhopadhaya and the interest that it generated in the mathematical arena of the thirties of the twentieth century. After his retirement from the Department of Pure Mathematics in 1932, S D Mukhopadhyay availed of the Ghosh Traveling Fellowship of the University of Calcutta and proceeded to Europe to study the methods of education followed there.

This also gave him a chance to personally interact with the eminent mathematicians of Europe. He utilized this opportunity and expounded his invented principles to the mathematical circle of the West. He also carefully studied the methods of education prevalent in the countries of Europe that he visited. On his return to India he wrote extensive memoirs based on his observations. After the death of Ganesh Prasad in 1935, Syamadas Mukhopadhyay was elected the President of the Calcutta Mathematical Society. He took a keen interest in all matters connected with the Society, till his death in 1937.

5. Concluding Remarks

Apart from the mathematician Syamadas Mukhopadhyay, a closer view of the man himself is very nicely portrayed by his eminent student G Bhar. He wrote

> "Mukhopadhyay was a man of broad sympathies and wide culture. He had an artistic bend of mind and was at one time a keen amateur photographer of no mean order. His love for the beautiful and the sublime found expression in his ardent passion for rose culture. His collection of roses in his country house at Mihijam is unique in India, and he enriched it every year by directly importing rose trees from England, France, Holland and other European countries".

He was man of simple habits and a very private person who preferred to live away from the glamour of public life. He remained absorbed in his own work. Very correctly his another famous student M C Chaki observed "Mukhopadhyay left an example of plain living and high thinking". On the 8th May, 1937, this great mathematician of India breathed his last.

This article is written in an effort to highlight and bring to the knowledge of the present generation the achievements of a great geometrician of India and also to preserve for posterity the memory of a man who contributed so handsomely to the world of mathematics.

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