Precise Determination of the Ascendant in the *Lagnaprakaraṇa* - I

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Abstract

The determination of the ascendant (*udayalagna*) or the rising point of the ecliptic is an important problem in Indian astronomy, both for its astronomical as well as socio-religious applications. Thus, astronomical works such as the *Sūryasiddhānta*, the *Brāhmasphuṭasiddhānta*, the *Śiṣyadhīvṛddhidatantra*, etc., describe a standard procedure for determining this quantity, which involves a certain approximation. However, Mādhava (c. 14th century) in his *Lagnaprakaraṇa* employs innovative analytic-geometric approaches to outline several procedures to precisely determine the ascendant. This paper discusses the first method described by Mādhava in the *Lagnaprakaraṇa*.

Key words: Ascendant, *Dṛkkṣepajyā*, *Dṛkkṣepalagna*, *Lagna*, *Lagnaprakaraṇa*, *Madhyakāla*, *Madhyalagna*, Mādhava, *Paraśaṅku*, *Rāśikūṭalagna*, *Udayalagna*.

1 Introduction

As the name implies, the *Lagnaprakaraṇa* (Treatise for the Computation of the Ascendant) is a text exclusively written to outline procedures for the determination of the ascendant (*udayalagna*) or the rising point of the ecliptic. To our knowledge, it is the first text to give multiple precise relations for finding this quantity. We have defined the *udayalagna* and discussed its significance in an earlier paper.[1] In the same paper, we have briefly noted the state of *udayalagna* computations in Indian astronomy prior to Mādhava, and also remarked upon the approximations involved therein.[2]

In the first chapter of the *Lagnaprakaraṇa*, Mādhava discusses several procedures (many of them novel) to determine astronomical quantities such as the *prāṇakalāntara* (difference between the longitude and right ascension of a body), *cara* (ascensional difference of a body), and *kālalagna* (the time interval between the rise of the vernal equinox and a desired later instant). The physical significance of these quantities, the crucial role they play in the computation of the ascendant, as well as the import of Mādhava's procedures in their determination have been discussed in earlier papers.[3] It may be briefly noted here that the *kālalagna* is an ingenious and novel concept, apparently first introduced by Mādhava in the *Lagnaprakaraṇa*, which greatly facilitates precise determination of a number of astronomical quantities, including the *udayalagna*.

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[1] See the introduction to [7].

[2] The standard procedure adopted by Indian astronomers prior to Mādhava to determine the *udayalagna* is perhaps best described in the *Tripraśnādhikāra* of *Śiṣyadhīvṛddhidatantra*. For a detailed discussion of this technique, see [10, pp. 61–69].

[3] See [8], [9], and [7] respectively.

From the second chapter onwards, the *Lagnaprakaraṇa* describes several techniques of precisely determining the *udayalagna*. These techniques are fairly involved, spread over many verses, and require the calculation of numerous intermediary quantities. The current paper focuses only on the first technique of determining the *udayalagna* described in the second chapter of the *Lagnaprakaraṇa*.

Besides this introduction, this paper consists of two more sections. In Section 2, which consists of several subsections, we provide the relevant verses of the *Lagnaprakaraṇa* which describe the first method of the computation of the *udayalagna*, along with their translation and detailed mathematical notes. In the third and last section, we make a few concluding remarks.

2 Precise determination of the ascendant

The second chapter of the *Lagnaprakaraṇa* commences with the definition of two quantities known as the *rāśikūṭalagna* and *madhyalagna* (meridian ecliptic point). Through eight verses (31 to 38), Mādhava successively and systematically defines several quantities such as the *madhyakāla*, *madhyajyā*, *dṛkkṣepajyā*, *paraśaṅku*, *dṛkkṣepalagna*, and finally the *udayalagna*. As we go through the verses, one cannot but help conclude that Mādhava's approach is quite meticulous and methodical.

As a prelude to our discussion, it may be mentioned that in the following discussion we employ the symbols λ, α, δ, and z to respectively refer to the longitude, right ascension, declination, and zenith distance of a celestial body. The *kālalagna*, the latitude of the observer, and the obliquity of the ecliptic are denoted by the symbols α_e, ϕ, and ϵ respectively. It may also be mentioned that all the figures in this section depict the celestial sphere for an observer having a northerly latitude ϕ.

2.1 Obtaining the *rāśikūṭalagna* and the *madhyalagna*

निजप्राणकलाभेदं
कुर्यात् कालविलग्नके ।
राशिकूटविलग्नं तत्
त्रिभोनं मध्यलग्नकम् ॥३१॥

nijaprāṇakalābhedaṃ
kuryāt kālavilagnake |
rāśikūṭavilagnaṃ tat
tribhonaṃ madhyalagnakam ||31||

One should apply the *nija-prāṇakalābheda* (*nija-prāṇakalāntara*) to the *kālavilagna* (*kālalagna*). That is the *rāśikūṭavilagna*. That decreased by three signs is the meridian ecliptic point (*madhyalagnaka*).[4]

This verse (in the *anuṣṭubh* metre) shows the method to determine the *madhyalagna*, or the meridian ecliptic point, in degrees. Towards this end, the verse first gives the following relation to determine the *rāśikūṭalagna*, which is a point on the ecliptic ninety degrees from the meridian ecliptic point:

$$\begin{aligned} rāśikūṭalagna &= kālalagna \pm nija\text{-}prāṇakalāntara \\ \text{or,} \quad \lambda_r &= \alpha_e \pm |\lambda_r - \alpha_e|, \end{aligned} \tag{1}$$

where α_e and λ_r represent the *kālalagna* and the longitude of the *rāśikūṭalagna* respectively. Then, the verse notes that the *madhyalagna* can be simply determined as follows:

$$\begin{aligned} madhyalagna &= rāśikūṭalagna - tribha \\ \text{or,} \quad \lambda_m &= \lambda_r - 90. \end{aligned} \tag{2}$$

Note on *nija-prāṇakalāntara* and *koṭi-prāṇakalāntara*

The verse states that the *rāśikūṭalagna* can be determined by applying the *nija-prāṇakalāntara* (lit. own *prāṇakalāntara*) to the *kālalagna*. The *nija-prāṇakalāntara* refers to the magnitude of the difference of the longitude and right ascension of any body,[5] not necessarily lying on the ecliptic. The term *nija-prāṇakalāntara* is used in contrast to the term *koṭi-prāṇakalāntara*, which appears in later verses, to differentiate between the two possible *prāṇakalāntaras* for a point B on the equator shown in Figure 1. Here, we have

$$nija\text{-}prāṇakalāntara = |\lambda_f - \alpha_b|,$$

[4] The term *madhyalagnaka* employed in the verse refers to the *madhyalagna* only. In other words, the suffix *ka* is not meant to modify the meaning of the noun here (स्वार्थे).

[5] Though the definition is generic, and is quite inclusive to be applicable to celestial bodies that are off the ecliptic, it may be noted that the term *prāṇakalāntara* discussed in the first chapter of the *Lagnaprakaraṇa* assumed the body (typically the Sun) to lie on the ecliptic. See [8].

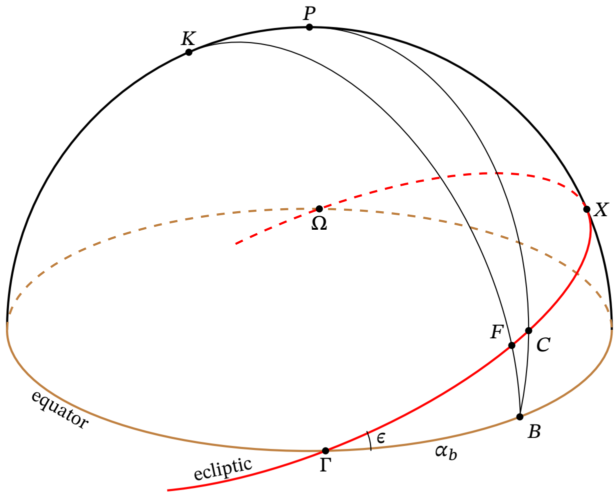


Figure 1 The significance of *nija-prāṇakalāntara* and *koṭi-prāṇakalāntara*.

which gives the difference in terms of point B's 'own' longitude and right ascension, while the

$$koṭi\text{-}prāṇakalāntara = |\lambda_c - \alpha_b|,$$

gives the difference in the longitude and right ascension of point C on the ecliptic whose right ascension (α_b) corresponds to that of point B.[6] Thus, only one kind of *prāṇakalāntara* is applicable for a point on the ecliptic, while the two kinds discussed above are possible for a point on the equator.

Perhaps assuming the procedure to be straightforward (though it doesn't seem to be so), the text does not describe how to determine the *nija-prāṇakalāntara*. Hence, for the convenience of the readers, here we outline how to obtain this quantity using modern spherical trigonometrical results. Applying the cosine rule of spherical trigonometry in the spherical triangle $F\Gamma B$, where

$$B\hat{F}\Gamma = 90, \quad B\hat{\Gamma}F = \epsilon, \quad \Gamma B = \alpha_b, \quad \Gamma F = \lambda_f,$$

yields

$$\cos\alpha_b = \cos\lambda_f \cos BF.$$

Applying the sine rule in the same triangle, we get

$$\sin BF = \sin\alpha_b \sin\epsilon.$$

[6]The term *koṭi-prāṇakalāntara* may have been employed as it refers to the *prāṇakalāntara* corresponding to the *koṭi* or the upright CB of the triangle $C\Gamma B$, which is right angled at B.

From the above two relations, we have

$$\lambda_f = \cos^{-1}\left(\frac{\cos\alpha_b}{\cos[\sin^{-1}(\sin\alpha_b\sin\epsilon)]}\right)$$

$$\text{or,}\quad 90 - \lambda_f = \sin^{-1}\left(\frac{\cos\alpha_b}{\cos[\sin^{-1}(\sin\alpha_b\sin\epsilon)]}\right).$$

The above relations can be used to determine the *nija-prāṇakalāntara* in the form of

$$\lambda_f - \alpha_b,$$

where α_b is already known. The same relations can also be arrived at using planar geometry, and would surely have been known to the author of the text.

Deriving the expressions for *rāśikūṭalagna* and *madhyalagna*

The verse states that the *nija-prāṇakalāntara* is to be applied to the *kālalagna* to obtain the *rāśikūṭalagna*. This can be understood from Figure 2, where the great circle arc KRE is the secondary[7] from the pole of the ecliptic (K) to the ecliptic, and also passes through the east cardinal point (E). Let the right ascension of E be α_e. This secondary meets the ecliptic at the *rāśikūṭalagna* (R). Let the longitude of R be λ_r. The points E and R are analogous to points B and F in Figure 1. Therefore, applying the *nija-prāṇakalāntara* to α_e gives λ_r, or

$$\lambda_r = \alpha_e \pm nija\text{-}prāṇakalāntara = \alpha_e \pm |\lambda_r - \alpha_e|,$$

[7]The secondary would of course be perpendicular to the ecliptic.

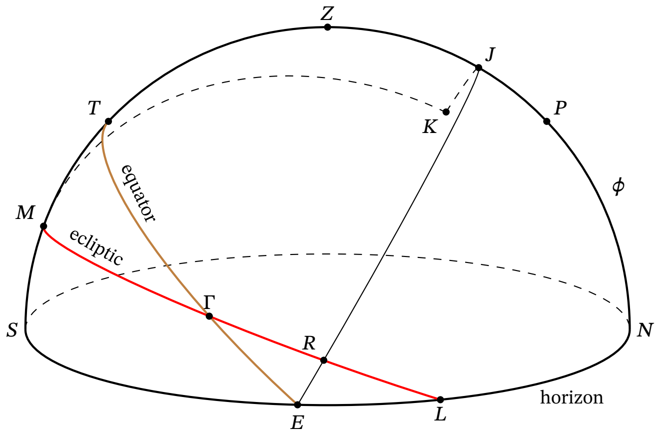


Figure 2 Determining the *madhyalagna* from the *kālalagna* and the *rāśikūṭalagna*.

which is the relation given by (1).

Now, the *madhyalagna* is the longitude of the point M at the intersection of the ecliptic and prime meridian in Figure 2. As the arcs $ME = MK = 90$,[8] one can conclude that M is the pole of the great circle arc KRE.[9] Therefore, we have $MR = 90$, which is the relation stated in (2).

It may be noted that since the *kālalagna* is the same for all observers on a given longitude,[10] the *rāśikūṭalagna* and the *madhyalagna* are the same too for these observers.

2.2 Computation of the *madhyakāla*

मध्यलग्ने पुनः कुर्यात्
निजप्राणकलान्तरम् ।
मध्यकालो भवेत्सोऽयं
त्रिभाढ्यं काललग्नकम् ॥३२॥
काललग्नं त्रिराश्यूनं
मध्यकालः प्रकीर्तितः ।

madhyalagne punaḥ kuryāt
nijaprāṇakalāntaram |
madhyakālo bhavet so'yaṃ
tribhāḍhyaṃ kālalagnakam ||32||
kālalagnaṃ trirāśyūnaṃ
madhyakālaḥ prakīrtitaḥ |

One should again apply the *nija-prāṇa-kalāntara* to the meridian ecliptic point (*madhyalagna*). That would be the *madhyakāla*. That increased by three signs would be the *kālalagna*. The *kālalagna* diminished by three signs is stated to be the *madhyakāla*.

The above verses (in the *anuṣṭubh* metre) essentially introduce the concept of *madhyakāla*, which is the right ascension of the point at the intersection of the equator and the prime meridian, represented by point T in Figure 3. They also present expressions detailing the relationship between the *madhyakāla* and the *kālalagna*.

In Figure 3, let the point M represent the *madhyalagna*, which has been discussed in the previous verse. Taking α_t as the right ascension of T, and λ_m as the longitude of M, the relation given in the verse for the *madhyakāla* can be expressed as:

$$\textit{madhyakāla} = \textit{madhyalagna} \pm \textit{nija-prāṇakalāntara}$$
$$\text{or,} \quad \alpha_t = \lambda_m \pm |\lambda_m - \alpha_t|. \tag{3}$$

Having defined the *madhyakāla* thus, starting with the latter half of verse 32, the author describes the connection between the *kālalagna* and the *madhyakāla* through the following relations:

$$\textit{kālalagna} = \textit{madhyakāla} + \textit{tribha}$$
$$\text{or,} \quad \alpha_e = \alpha_t + 90, \tag{4}$$

and

$$\textit{madhyakāla} = \textit{kālalagna} - \textit{trirāśi}$$
$$\text{or,} \quad \alpha_t = \alpha_e - 90. \tag{5}$$

[8] $ME = 90$ as E is the pole for any point on the prime meridian. $MK = 90$ as K is the pole of any point on the ecliptic.

[9] Except when the points are separated by 180 degrees, two points are sufficient to define a unique great circle on a sphere.

[10] See verse 30 in [7].

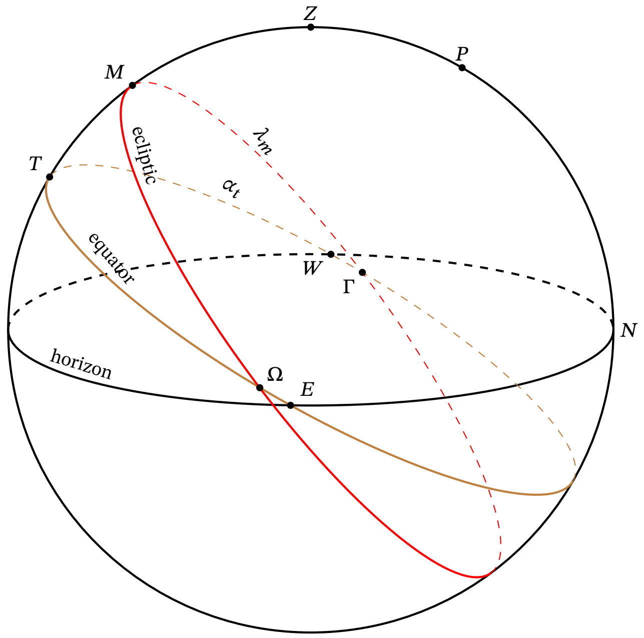


Figure 3 Determining the *madhyalagna* and the *madhyakāla*.

The relation between the *madhyakāla* and the *madhyalagna* given in (3) can be easily understood with the help of Figure 3. Here, both the points T and M lie on the prime meridian, which immediately indicates that α_t is the right ascension of a body at M. Thus, if the *madhyalagna* (λ_m) is already known, then the *madhyakāla* (α_t) can be readily found by applying the *prāṇakalāntara* to it.[11] That is,

$$\alpha_t = \lambda_m \pm |\lambda_m - \alpha_t|,$$

which is the first relation given in the verse.

As the east cardinal point (E) is the pole for any point on the prime meridian, it is also evident from the figure that the *kālalagna* (α_e) is given by

$$\alpha_e = \alpha_t + 90,$$

which explains the relations (4) and (5) stated in the verse.

2.3 An alternate way of obtaining the *madhyalagna*

मध्यकाले पुनस्तस्मिन्
कोटिप्राणकलान्तरम् ॥३३॥
व्यस्तं कुर्यात् तदा वात्र
मध्यलग्नमवाप्यते ।

madhyakāle punastasmin
koṭiprāṇakalāntaram ||33||
vyastaṃ kuryāt tadā vātra
madhyalagnamavāpyate |

Again, one should apply the *koṭi-prāṇa-kalāntara* to that *madhyakāla* reversely. Then also the meridian ecliptic point (*madhyalagna*) is obtained here.

The latter half of verse 33 and the first half of verse 34 (both in the *anuṣṭubh* metre) together present the following expression to determine the *madhyalagna* or the meridian ecliptic point from the *madhyakāla*:

$$\textit{madhyalagna} = \textit{madhyakāla} \mp \textit{koṭi-prāṇakalāntara}$$
$$\text{or,} \quad \lambda_m = \alpha_t \mp |\lambda_m - \alpha_t|. \qquad (6)$$

It is easily seen that the above relation is a corollary of (3). The *koṭi-prāṇakalāntara* is applied here as we want to convert the meridian ecliptic point's right ascension into its longitude.[12] Furthermore, it must be noted that the *prāṇakalāntara* is applied reversely here for the same

[11] Note that for a point on the ecliptic, there is only one kind of *prāṇakalāntara*.

[12] Refer to our discussion on *nija-prāṇakalāntara* and *koṭi-prāṇakalāntara* in verse 31. The points T and M here are analogous to points B and C in Figure 1.

reason, as we want to convert right ascension into longitude.[13]

2.4 Determining the *madhyajyā*

> मध्यलग्नात् पुनस्तस्मात्
> दोःकोट्योः क्रान्तिमानयेत् ॥३४॥
> दोःक्रान्तिकोट्याक्षममुष्य
> कोट्या दोःक्रान्तिजीवां च निहत्य[14] भूयः ।
> तद्योगभेदात् समभिन्नदिक्त्वे
> त्रिज्याहृतं मध्यगुणं वदन्ति ॥३५॥
>
> *madhyalagnāt punastasmāt*
> *doḥkoṭyoḥ krāntimānayet* ||34||
> *doḥkrāntikoṭyākṣamamuṣya*
> *koṭyā doḥkrāntijīvāṃ ca nihatya bhūyaḥ* |
> *tadyogabhedāt samabhinnadiktve*
> *trijyāhṛtaṃ madhyaguṇaṃ vadanti* ||35||
>
> Again, from that meridian ecliptic point (*madhyalagna*), one should derive the Rsine and Rcosine of its declination. Having multiplied (i) the [Rsine of the] latitude (*akṣa*) with the Rcosine of the declination corresponding to the longitude (*doḥkrāntikoṭi*) [of the *madhyalagna*], and also (ii) the Rsine of the declination corresponding to the longitude (*doḥkrāntijīvā*) [of the *madhyalagna*] with the Rcosine of this [latitude], their sum or difference—depending upon [whether the equator and the zenith are in the] same or opposite direction [with respect to the ecliptic]—divided by the radius (*trijyā*), is stated to be the *madhyaguṇa*.

In one and a half verses (half in the *anuṣṭubh* and full in the *indravajrā* metres) Mādhava gives the relation for calculating the *madhyajyā* (i.e. *madhyaguṇa*) or the Rsine of the zenith distance (z_m) of the meridian ecliptic point (*madhyalagna*). To this end, the verses instruct that first the Rsine and Rcosine of the declination (δ_m) corresponding to the longitude of the *madhyalagna* should be calculated.[15] The relation for the *madhyajyā* in terms of these two quantities, as well as the Rsine and Rcosine of the latitude (ϕ), is stated as follows:

$$\textit{madhyaguṇa} = (\textit{akṣajyā} \times \textit{doḥkrāntikoṭi} \pm \textit{akṣakoṭijyā} \times \textit{doḥkrāntijīvā}) \div \textit{trijyā}$$

or,

$$R\sin z_m = \frac{(R\sin\phi \times R\cos\delta_m \pm R\cos\phi \times R\sin\delta_m)}{R}. \tag{7}$$

The verses further state that the sign in the above relation has to be taken as positive or negative depending upon whether the latitude and declination are in the 'same' or 'opposite' directions. This remark, as well as the validity of the above expression, can be understood from Figure 4. The figure depicts the *madhyalagna* (M) when it has northern as well as southern declination. The figure also depicts its declination (δ_m), as well as zenith distance (z_m), in these two cases. As can be seen from the figure, the zenith distance of M is equal to $\phi + \delta_m$ when the ecliptic is in the same direction with respect to both the zenith and the equator, and $\phi - \delta_m$ when the equator and the zenith are on either side of the ecliptic.[16] We therefore have

$$\sin z_m = \sin(\phi \pm \delta_m) = \sin\phi\cos\delta_m \pm \cos\phi\sin\delta_m,$$

or,

$$R\sin z_m = \frac{(R\sin\phi \times R\cos\delta_m \pm R\cos\phi \times R\sin\delta_m)}{R}$$

which is the same as (7).

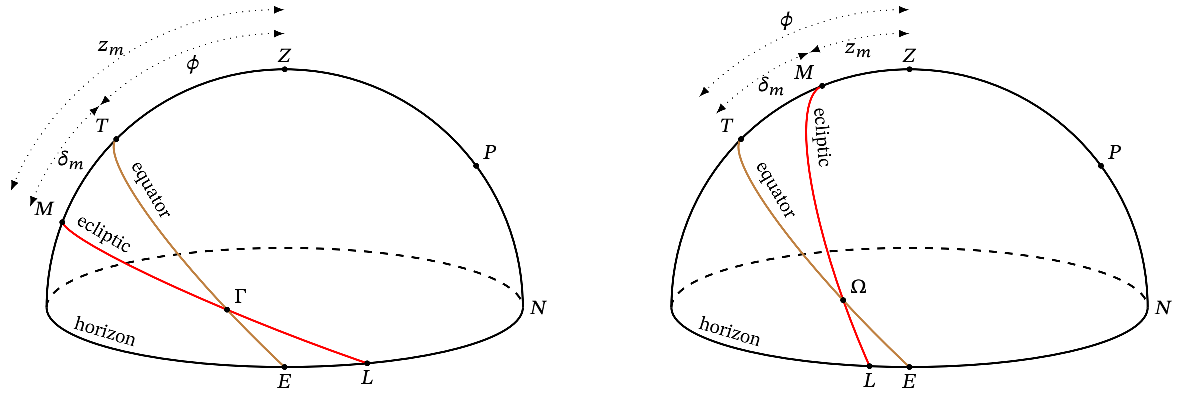
Now, a brief note on the advantage of the choice of form of the rule prescribed by (7). Naively, it may appear that expanding $\sin(\phi \pm \delta_m)$ and determining the sum or difference of products of the sine and cosine functions could be more cumbersome than directly determining the sine of the total quantity. However, this need not be the case if the constituent terms of the expansion were already known to the practitioners, who could then easily determine the result by the simple summation of two products. For instance, the sine of the declination of the *madhyalagna* ($\sin\delta_m$) can be derived directly using the relation

[13] The *prāṇakalāntara* as defined in the first chapter of the *Lagnaprakaraṇa* is for converting the longitude of an ecliptic point to the corresponding right ascension. Thus, it is prescribed to be applied 'reversely' here.

[14] The available manuscripts give the reading as निहृत्य. This however appears to be a transcribing error as the correct relation requires multiplication and not division.

[15] Knowing the longitude λ_m of the *madhyalagna*, the sine of its declination can be determined easily using the well known relation $\sin\delta = \sin\lambda\sin\epsilon$. This would be the *doḥkrāntijīvā*. Its cosine would be the *doḥkrāntikoṭi*.

[16] Here, δ_m represents only the magnitude of the declination of the *madhyalagna*.



(a) Same direction. **(b)** Opposite directions.

Figure 4 The direction of the equator and the zenith with respect to the ecliptic for determining the *madhyaguṇa*.

$\sin\delta = \sin\lambda \sin\epsilon$. Its cosine too can be easily determined using basic arithmetic, while the sine and cosine of the latitude would be readily available for a given location. On the other hand, to directly determine $\sin(\phi \pm \delta_m)$, one would have to first calculate the inverse sine of the quantity $\sin\delta_m$ to obtain δ_m, add or subtract this to the latitude, and then determine the sine of the composite quantity. The complex and time consuming task of determining the inverse sine can be avoided by following the prescribed procedure.

The quantity *madhyajyā* thus determined is now used to determine the *dṛkkṣepajyā* in the next verse.

2.5 Determining the dṛkkṣepajyā

कोटिक्रान्तेर्मध्यजीवाहताया
लब्धं बाहुक्रान्तिकोट्या तु बाहुः ।
मध्यज्याया वर्गतो बाहुवर्गं
त्यक्त्वा शिष्टं स्याच्च दृक्क्षेपवर्गः ॥३६॥

koṭikrāntermadhyajīvāhatāyā
labdhaṃ bāhukrāntikoṭyā tu bāhuḥ |
madhyajyāyā vargato bāhuvargaṃ
tyaktvā śiṣṭaṃ syācca dṛkkṣepavargaḥ ||36||

The result obtained from the *koṭikrānti*, which is multiplied by the *madhyajīvā* (*madhyajyā*), and divided by the *bāhukrāntikoṭi* is *bāhu*. The residue obtained after the subtraction of the square of the *bāhu* from the square of the Rsine of the *madhyajyā*, would be the square of [the Rsine of] the *dṛkkṣepa*.

The *dṛkkṣepajyā*, or simply the *dṛkkṣepa*, is the Rsine of the zenith distance of the *dṛkkṣepalagna* or the nonagesimal.[17] This verse (in the *śālinī* metre) gives an expression for determining the *dṛkkṣepa* in terms of the *madhyajyā* and another intermediary quantity called the *bāhu*, which is defined as follows:[18]

$$\begin{aligned} bāhu &= \frac{madhyajyā \times koṭikrānti}{bāhukrāntikoṭi} \\ &= \frac{R\sin z_m \times R\cos\lambda_m \sin\epsilon}{R\cos\delta_m}. \end{aligned} \tag{8}$$

Now, the *dṛkkṣepajyā* is defined as:

$$\begin{aligned} (dṛkkṣepajyā)^2 &= (madhyajyā)^2 - (bāhu)^2 \\ \text{or,} \quad (R\sin z_d)^2 &= (R\sin z_m)^2 - (bāhu)^2, \end{aligned} \tag{9}$$

where z_d corresponds to the zenith distance of the *dṛkkṣepalagna*.

Rationale behind the expression for dṛkkṣepajyā

The expression for the *dṛkkṣepajyā* given in (9) can be arrived at as follows. In Figure 5, D represents the *drkkṣepalagna* or the nonagesimal, which corresponds to the highest point of the ecliptic lying above the horizon. In other words, its zenith distance ZD (which is the

[17] The nonagesimal is a point on the ecliptic above the horizon which is ninety degrees from the rising ecliptic point, and is also the highest point of the ecliptic.

[18] The standard relation for *krānti* or declination is $\sin\lambda \sin\epsilon$. The term *koṭikrānti* is to be instead understood to be $\cos\lambda \sin\epsilon$. The term *bāhukrāntikoṭi* here is to be understood to mean the cosine of the declination of the *madhyalagna*, i.e., $\cos\delta_m$.

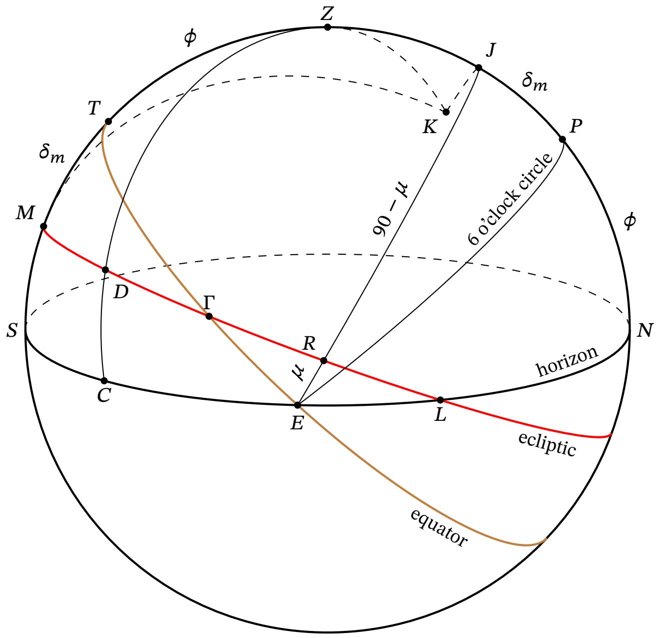


Figure 5 Visualising the *dṛkkṣepa*.

dṛkkṣepa), is the least compared to any other point on the ecliptic. This is only possible when ZD is perpendicular to the tangent to the ecliptic at D.[19] This in turn implies that the secondary KD from the pole of the ecliptic (K), which is perpendicular to the ecliptic at D, passes through the zenith (Z).[20]

Also, in the figure, the ecliptic points M and R correspond to the *madhyalagna* and the *rāśikūṭalagna* respectively, while the equatorial point T corresponds to the *madhyakāla*. As E is the pole for any point on the prime meridian, we have $EJ = 90$. Now, let $ER = \mu$. Then, obviously $RJ = 90 - \mu$. As M is the pole of the great circle arc $KJRE$,[21] the arc RJ also corresponds to the angle between the prime meridian and the ecliptic.

From the form of (9), it is evident that the author visualised a right-angled triangle, with the *madhyajyā* as the hypotenuse, and the *dṛkkṣepajyā* and *bāhu* as sides. To help visualise this triangle, the celestial sphere is depicted from the point of view of the ecliptic plane in Figure 6a. Here, ZD' and ZM' correspond to the Rsines of the *dṛkkṣepa* and the zenith distance of the *madhyalagna* respectively. Taking $ZD = z_d$ and $ZM = z_m$, we have

$$ZD' = R \sin z_d, \quad \text{and} \quad ZM' = R \sin z_m.$$

We also have

$$OD' = R \cos z_d, \quad \text{and} \quad OM' = R \cos z_m.$$

Now, as KZD is perpendicular to the ecliptic, the planar triangle $ZM'D'$ is right-angled at D', and lies in a plane perpendicular to the ecliptic. Therefore, we have

$$(R \sin z_d)^2 = (R \sin z_m)^2 - (M'D')^2,$$

which is the relation given in (9), where the side $M'D'$ has been called *bāhu*.

Rationale behind the expression for *bāhu*

The relation for the *bāhu* given by (8) can be understood from the same right-angled triangle $ZM'D'$. In this triangle, the angle

$$Z\hat{M}'D' = 90 - \mu$$

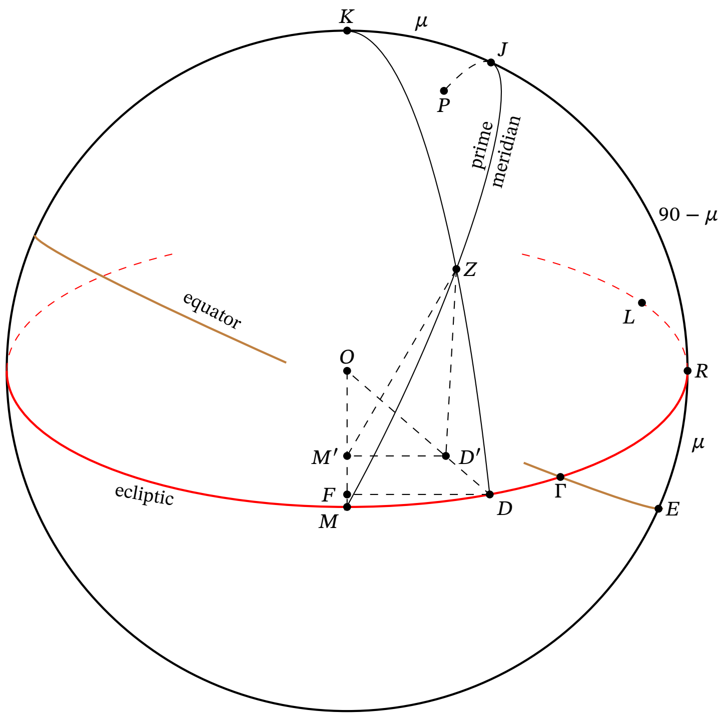
corresponds to the angle between the prime meridian and the ecliptic. Using simple trigonometry, we have

$$M'D' = R \sin ZM \cos(90 - \mu),$$

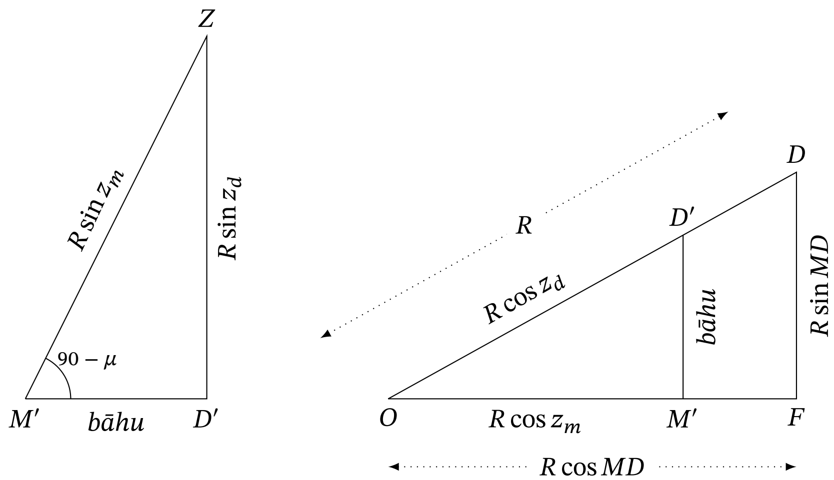
[19] The arcs corresponding to the zenith distances of other points on the ecliptic will not be perpendicular to it, implying that they will be longer than ZD.

[20] The great circle containing the arc KZD is sometimes referred to as the *dṛkkṣepavṛtta*, or the great circle corresponding to the *dṛkkṣepa*.

[21] Shown in our discussion of verse 31.



(a) Visualising the *dṛkkṣepa* from the point of view of the ecliptic plane.



(b) Planar triangles used to determine the *dṛkkṣepajyā* and the *dṛkkṣepalagna*.

Figure 6 Determining the *dṛkkṣepajyā* and the *dṛkkṣepalagna*.

or

$$bāhu = R \sin z_m \sin \mu. \tag{10}$$

The expression for $\sin \mu$ can be determined from the spherical triangle ΓER in Figure 5, where we have $ER = \mu$, $R\hat{\Gamma}E = \epsilon$, and $\Gamma\hat{E}R = 90 - \delta_m$.[22] Applying the sine rule in this triangle, we have

$$\sin \mu = \frac{\sin \Gamma R \times \sin \epsilon}{\sin(90 - \delta_m)}. \tag{11}$$

However, as the *rāśikūṭalagna* is ninety degrees from the *madhyalagna*, we have

$$\Gamma R = 90 - M\Gamma \ = 90 - (360 - \lambda_m) = \lambda_m - 270.$$

Using the above expression for ΓR in (11), we have

$$\sin \mu = \frac{\cos \lambda_m \sin \epsilon}{\cos \delta_m} = \frac{R \cos \lambda_m \sin \epsilon}{R \cos \delta_m}.$$

Substituting the above expression in (10), we have

$$bāhu = \frac{R \sin z_m \times R \cos \lambda_m \sin \epsilon}{R \cos \delta_m},$$

which is the same as (8).

2.6 Determining *paraśaṅku*, *dṛkkṣepalagna*, and *udayalagna*

दृक्क्षेपवर्गे त्रिगुणस्य वर्गात्
त्यक्तेऽस्य मूलं परशङ्कुमाहुः ।
त्रिज्याहतं बाहुमनेन भक्तं
चापीकृतं मध्यविलग्नकेऽस्मिन् ॥३७॥
क्रमाद्धनर्णं मृगकर्कटाद्योः
व्यस्तं च तन्मध्यगुणे तु सौम्ये ।
तदा तु दृक्क्षेपविलग्नकं स्यात्
तत्सत्रिभं तूदयलग्नमाहुः ॥३८॥

dṛkkṣepavarge triguṇasya vargāt
tyakte'sya mūlaṃ paraśaṅkumāhuḥ |
trijyāhataṃ bāhumanena bhaktaṃ
cāpīkṛtaṃ madhyavilagnake'smin ||37||
kramāddhanarṇaṃ mṛgakarkaṭādyoḥ
vyastaṃ ca tanmadhyaguṇe tu saumye |
tadā tu dṛkkṣepavilagnakaṃ syāt
tatsatribhaṃ tūdayalagnamāhuḥ ||38||

When the square of the *dṛkkṣepa*[*jyā*] is subtracted from the square of the radius (*trijyā*), [people] state its (the difference's) square-root to be the *paraśaṅku*. The *bāhu* [stated in the previous verse] multiplied by the radius (*trijyā*) divided by this (*paraśaṅku*) is converted to arc and applied positively and negatively in order to the meridian ecliptic point (*madhyalagna*) depending on [whether the *madhyalagna* is in the six signs] Capricorn (*mṛgādi*) etc., or Cancer (*karkaṭādi*) etc. It (the positive or negative application of the arc to the *madhyalagna*) is reversed when the *madhyajyā* is northward. Then, the nonagesimal (*dṛkkṣepa-vilagna*) would be [obtained]. That added by three signs is stated to be the rising ecliptic point (*udaya-lagna*).

The two verses above (in the *indravajrā* and *upajāti* metres respectively) are essentially meant for providing an expression for the ascendant or the *udayalagna*. The expression for the *udayalagna* is given in terms of the *dṛkkṣepalagna*, which in turn is defined in terms of the *paraśaṅku*. Hence, the set of verses above first give a relation for the *paraśaṅku* or the gnomon corresponding to the nonagesimal, then for the *dṛkkṣepalagna* or the longitude of the nonagesimal, and finally for the *udayalagna* or the rising ecliptic point. The relation for the *paraśaṅku* is given as:

$$paraśaṅku = \sqrt{(triguṇa)^2 - (dṛkkṣepajyā)^2}$$

$$\text{or,} \qquad R \cos z_d = \sqrt{R^2 - (R \sin z_d)^2}. \tag{12}$$

The expression for *dṛkkṣepalagna* is given to be:

$$dṛkkṣepalagna = madhyalagna \pm cāpa\left(\frac{bāhu \times trijyā}{paraśaṅku}\right)$$

or,

$$\lambda_d = \lambda_m \pm R \sin^{-1}\left(\frac{bāhu \times R}{R \cos z_d}\right), \tag{13}$$

where *bāhu* is given by (8), and *paraśaṅku* by (12).

Now, the expression for the *udayalagna* stated in terms of the *dṛkkṣepalagna* in the last quarter of verse 38 is:

$$udayalagna = dṛkkṣepalagna + tribha$$

$$\text{or,} \qquad \lambda_l = \lambda_d + 90. \tag{14}$$

We now provide the rationale behind the above expressions.

[22] As P is the pole of the equator, and M is the pole of the great circle arc $KJRE$, we have $PT = JM = 90$, and $PJ = TM = \delta_m$. Therefore, $\Gamma\hat{E}R = T\hat{E}P - J\hat{E}P = 90 - \delta_m$.

Expression for the *paraśaṅku*

The *paraśaṅku* is the gnomon dropped from the *dṛkkṣepa-lagna* (point D in Figure 5) to the horizon. In later verses, this quantity is also referred to as the *dṛkkṣepakoṭikā* or the *rāśikūṭaprabhā*. The length of this gnomon would be equal to the Rsine of the arc CD. As $CD = 90 - z_d$, we have

$$paraśaṅku = R\sin(90 - z_d) = R\cos z_d,$$

which is equivalent to (12).

Expression for the *dṛkkṣepalagna*

For observers in the northern hemisphere, the *dṛkkṣepa-lagna* is generally south of the zenith, and can be either in the eastern hemisphere or the western hemisphere, depending upon the longitude of the *madhyalagna*. When the longitude of the *madhyalagna* is in the range of 270 degrees to 90 degrees (*mṛgādi*), the *dṛkkṣepalagna* is in the eastern hemisphere, and when the longitude of the *madhyalagna* is in the range of 90 degrees to 270 degrees (*karkaṭādi*), the *dṛkkṣepalagna* is in the western hemisphere. In Figure 5, where the longitude of the *madhya-lagna* is *mṛgādi*, it can be seen that the *dṛkkṣepalagna* is in the eastern hemisphere, and that its longitude is equal to the sum of the longitude of the *madhyalagna* (M) and the arc MD. Alternatively, when the *dṛkkṣepalagna* is in the western hemisphere, this arc would have to be subtracted from the longitude of the *madhyalagna* to obtain the *dṛkkṣepalagna*. Thus, we have

$$\lambda_d = \lambda_m \pm MD. \tag{15}$$

In some cases, for observers at lower latitudes ($\phi < \epsilon$), the *dṛkkṣepalagna* and the *dṛkkṣepajyā* can be north of the zenith (for instance, see Figure 7b). In these cases, the *dṛkkṣepalagna* is in the eastern hemisphere when the longitude of the *madhyalagna* is in the range of 90 degrees to 270 degrees (*karkaṭādi*), and in the western hemisphere when the longitude of the *madhyalagna* is in the range of 270 degrees to 90 degrees (*mṛgādi*). Therefore, for obtaining the *dṛkkṣepalagna* in this case, the arc MD has to be subtracted from the longitude of the *madhyalagna* when it is *mṛgādi*, and added to it when the *madhyalagna* is *karkaṭādi*. Therefore, when compared to the situation where the *dṛkkṣepa* is to the south of the zenith, the procedure of applying the arc MD to the *madhyalagna* is reverse in the case when the *dṛkkṣepajyā* is northwards.

The length of the arc MD can be determined by considering the triangles DFO and $D'M'O$ shown in Figure 6b. The triangle DFO lies on the plane of the ecliptic, in which $OD = R$, and DF is the perpendicular dropped on the radius OM from D. Thus,

$$DF = R\sin MD, \quad \text{and} \quad OF = R\cos MD.$$

The triangle $D'M'O$ also lies on the plane of the ecliptic, where we have already shown in our discussion of the previous verse that $M'D' = bāhu$, $OM' = R\cos z_m$, and $OD' = R\cos z_d$. Using (8), it can be shown that

$$(OD')^2 = (OM')^2 + (M'D')^2,$$

which implies that triangle $D'M'O$ is right-angled at M'.

Thus, the triangles DFO and $D'M'O$ are similar, as they are both right-angled, and also share the common angle at O. Applying the rule of proportionality of the sides of similar triangles, we have

$$R\sin MD = \frac{bāhu \times R}{R\cos z_d}. \tag{16}$$

Substituting this result in (15), we have

$$\lambda_d = \lambda_m \pm R\sin^{-1}\left(\frac{bāhu \times R}{R\cos z_d}\right),$$

which is the same as (13).

Expression for the *udayalagna*

In Figure 5, one observes that the *udayalagna* (L) is 90 degrees from the pole of the ecliptic (K), as well as the zenith (Z). Therefore, L is the pole of the great circle $KZDC$, which means $LD = 90$. Therefore,

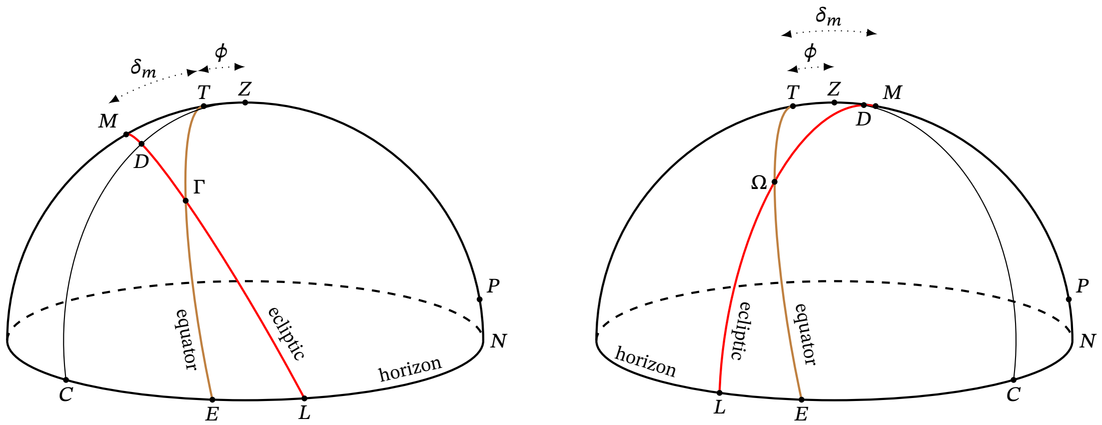
$$\lambda_l = \lambda_d + 90,$$

which is the same as (14).

From the fact that L is at 90 degrees from both C and D, the angle $C\hat{L}D$ between the ecliptic and the horizon will be equal to the measure of the arc CD, and therefore

$$C\hat{L}D = z'_d = 90 - z_d. \tag{17}$$

Though the above result is not stated in the above verses, we make a note of it here as it is essential for later discussions which will be brought out as a sequel to this article.



(a) Southern hemisphere. **(b)** Northern hemisphere.

Figure 7 Direction of the *dṛkkṣepa* at lower latitudes.

3 Conclusion

The *Lagnaprakaraṇa* is a unique text focusing on a single problem in astronomy, namely the determination of the ascendant. This is indeed a non-trivial problem that seems to have attracted the attention of astronomers in India and around the world. While the problem has been attempted to be solved using a variety of approximations by various civilisations at different points of time, in our understanding, precise formulations appear in this work of Mādhava for the first time in the annals of Indian and world astronomy. Indeed, Mādhava seems to have approached this problem from the viewpoint of a pure mathematician and employs a variety of techniques to present multiple precise relations for the computation of the ascendant.

In this paper, we have discussed the first technique described by Mādhava in the *Lagnaprakaraṇa*. From our discussion it is evident that Mādhava seems to have been exceptionally good at visualising the motion of celestial objects in the celestial sphere and the projection of various points on it in several planes. This mastery enabled him to precisely derive the *udayalagna* through a series of fairly complex mathematical steps involving the determination of several quantities such as *rāśikūṭalagna*, *madhyalagna*, *madhyakāla*, *madhyajyā*, *dṛkkṣepajyā*, *paraśaṅku*, *dṛkkṣepalagna*, etc. This complexity perhaps explains why previous astronomers did not give precise relations for the ascendant, and attests to Mādhava's reputation as the *Golavid*, or the knower of the celestial sphere, in the Kerala astronomical tradition.

In subsequent papers, we plan to present other techniques of determining the ascendant described by Mādhava in the *Lagnaprakaraṇa*.

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