

UNICITY RESULTS FOR MEROMORPHIC FUNCTIONS SHARING SMALL FUNCTIONS*

YUAN WENJUN* AND TIAN HONGGEN**

*Department of Mathematics, Guangzhou Normal University, Guangzhou 510 405,
P. R. China; e-mail: gzywj@163.net

**Department of Mathematics, Xinjiang Normal University, Urumqi 830 053, P. R. China

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In this paper, we first give a different proof of Theorem *D* below. Then prove the following results:

Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions, and a_i ($i = 1, 2, 3, 4, 5$) be five distinct small meromorphic functions of $f(z)$ and $g(z)$.

(1) Suppose that $f(z)$ and $g(z)$ share a_1, a_2, a_3 CM and a_4 IM. If either $\bar{N}(r, f) \leq uT(r, f) + S(r, f)$ or $\bar{N}(r, g) \leq uT(r, g) + S(r, g)$ holds for some $u \in [0, 1)$. Then, $f(z) = g(z)$.

(2) Suppose that $f(z)$ and $g(z)$ share a_1, a_2, a_3 CM and a_4, a_5 IM. Then $f(z) = g(z)$.

(3) Suppose that k is a positive integer, $f^{(k)}(z)$ and $g^{(k)}(z)$ share a_1, a_2, a_3 CM and a_4 IM. Then $f^{(k)}(z) = g^{(k)}(z)$.

Our results are improving or relating results about meromorphic functions sharing small functions in⁷ and¹¹.

Key Words : Meromorphic Function; Unicity Theorem; Sharing Small Function

1. INTRODUCTION AND THE MAIN RESULTS

In this paper, by meromorphic function we always mean a function which is meromorphic in the whole complex plane. We assume that the reader is familiar with the standard notations and basic results of Nevanlinna's value distribution theory (see⁴ or⁵).

Let $f(z)$ be a nonconstant meromorphic function and let $S(f)$ be the set of meromorphic functions $a(z)$ which satisfy

$$T(r, a) = S(r, f),$$

where $S(r, f)$ is any quantity satisfying

$$S(r, f) = o(T(r, f))$$

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for $r \rightarrow \infty$ expecting possibly a set of r of finite linear measure. Such a meromorphic function $a(z)$ is said to be *small* with respect to f . Note that $S(f)$ is a field.

Let $f(z)$ and $g(z)$ be nonconstant meromorphic functions, and let $a(z)$ be meromorphic or constant in C . We say that f and g share the function $a(z)$ CM (IM) if $f(z) - a$ and $g(z) - a$ have the same zeros with the same multiplicities (ignoring multiplicities)¹⁴. Now we generalise the definitions of CM and IM to GCM and GIM, respectively. Denote by $N_c\left(r, \frac{1}{f-a}\right)$ the counting function of those a -point of f where a is taken by f and g with the same multiplicity, counted only once regardless the multiplicity, and $N_i\left(r, \frac{1}{f-a}\right)$ the counting function of those a -point of f where a is taken by f and g regardless the multiplicity, counted only once. We say that f and g share the a GCM, if

$$\bar{N}\left(r, \frac{1}{f-a}\right) - N_c\left(r, \frac{1}{f-a}\right) = S(r, f),$$

and
$$\bar{N}\left(r, \frac{1}{g-a}\right) - N_c\left(r, \frac{1}{g-a}\right) = S(r, g).$$

Similarly, we say that f and g share the a GIM, if

$$\bar{N}\left(r, \frac{1}{f-a}\right) - N_i\left(r, \frac{1}{f-a}\right) = S(r, f),$$

and
$$\bar{N}\left(r, \frac{1}{g-a}\right) - N_i\left(r, \frac{1}{g-a}\right) = S(r, g).$$

We say that f is a quasi-Möbius transformation of g if there exist four small functions $b_j(z)$ ($j = 1, \dots, 4$) of f and g with $b_1 b_4 - b_2 b_3 \neq 0$ such that $f = (b_1 g + b_2)/(b_3 g + b_4)$, and a function $a(z)$ is called a exceptional function of f in $N\left(r, \frac{1}{f-a}\right) = S(r, f)$.

Nevalinna¹⁰ proved the following two well-known results:

Theorem A — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. If $f(z)$ and $g(z)$ share four distinct values CM, then f is a Möbius transformation of g .

Theorem B — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. If $f(z)$ and $g(z)$ share five distinct values IM, then $f = g$.

Many further improvements are given by Gundersen^{2, 3} Mues⁹ and Ueda^{12 & 13} and others.

Recently, Li and Yang⁸ gave a generalization of Theorem A as shown below :

Theorem C — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and a_i ($i = 1, 2, 3, 4$) be four distinct small meromorphic functions of $f(z)$ and $g(z)$. If $f(z)$ and $g(z)$ share a_i

($i = 1, 2, 3$) GC and a_4 GIM, then either $f(z) \equiv g(z)$ or $f(z)$ is a quasi-Möbius transformation of $g(z)$, moreover, two of a_i ($i = 1, 2, 3, 4$) are exceptional functions of $f(z)$ and $g(z)$.

Later, Hua and Gang⁶ used a different method to prove it. They obtained the following results :

Theorem D — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and a_i ($i = 1, 2, 3, 4$) be four distinct small meromorphic functions of $f(z)$ and $g(z)$. If $f(z)$ and $g(z)$ share a_i ($i = 1, 2, 3$) CM and a_4 IM, then f and g satisfy one of the following cases :

- (i) $f \equiv g$;
- (ii) $F = -G, \alpha \equiv -1$ and a_2, a_4 are exceptional functions of f, g ;
- (iii) $F = -G + 2, \alpha \equiv 2$ and a_1, a_4 are exceptional functions of f, g ;
- (iv) $(F - 1/2)(G - 1/2) = 1/4, \alpha \equiv 1/2$ and a_3, a_4 are exceptional functions of f, g ;
- (v) $FG \equiv 1, \alpha \equiv -1$ and a_1, a_3 are exceptional functions of f, g ;
- (vi) $(F - 1)(G - 1) \equiv 1, \alpha \equiv 2$ and a_2, a_3 are exceptional functions of f, g ; and
- (vii) $F = -G + 1, \alpha \equiv 1/2$ and a_2, a_1 are exceptional functions of f, g .

where
$$F := \frac{(f - a_1)(a_2 - a_3)}{(f - a_3)(a_2 - a_1)}, G := \frac{(g - a_1)(a_2 - a_3)}{(g - a_3)(a_2 - a_1)}, \alpha := \frac{(a_4 - a_1)(a_2 - a_3)}{(a_4 - a_3)(a_2 - a_1)}$$

Ishizaki and Toda⁷ proved the following two theorems, which may be thought as relating results of Theorem A and Theorem B, respectively.

Theorem E — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. Suppose that $f(z)$ and $g(z)$ share four distinct values $a_i = (i = 1, 2, 3, 4) \in C \cup \{\infty\}$ IM. If there exists an element a_5 in $C \cup \{\infty\} - \{a_1, a_2, a_3, a_4\}$ satisfying

$$\bar{N}\left(r, \frac{1}{f - a_5}\right) \leq uT(r, f) + S(r, f)$$

and
$$\bar{N}\left(r, \frac{1}{g - a_5}\right) \leq uT(r, g) + S(r, g)$$

for some
$$u \in \left[0, \frac{1}{19}\right].$$
 Then, $f(z) = g(z)$.

Theorem F — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. Suppose that $f(z)$ and $g(z)$ share four distinct small functions a_i ($i = 1, 2, 3, 4$) $\in S(f) \cap S(g)$ IM, and suppose that there exist $a_i, a_j \in \{a_1, a_2, a_3, a_4\}$ ($i \neq j$) such that

$$a = \frac{c - a_i}{a_j - a_i} \text{ or } b = \frac{d - a_i}{a_j - a_i}$$

are constants, where $\{c, d\} = \{a_1, a_2, a_3, a_4\} - \{a_i, a_j\}$.

If either $\bar{N}(r, f) \leq uT(r, f) + S(r, f)$ or $\bar{N}(r, g) \leq uT(r, g) + S(r, g)$

holds for some $u \in [0, 1)$. Then, $f(z) = g(z)$.

For the case of derivatives, Qiu¹¹ gave the following theorem :

Theorem G — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and k is a positive integer. If $f^{(k)}(z)$ and $g^{(k)}(z)$ share four distinct finite values CM, then $f^{(k)}(z) = g^{(k)}(z)$.

In this paper we shall first give a different proof of Theorem C or Theorem D, then by it extend or improve above theorems. Our main results are :

Theorem 1 — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and a_i ($i = 1, 2, 3, 4$) be four distinct small meromorphic functions of $f(z)$ and $g(z)$. Suppose that $f(z)$ and $g(z)$ share a_1, a_2, a_3 GCM (CM) and a_4 GIM (IM). If either

$$\bar{N}(r, f) \leq uT(r, f) + S(r, f) \text{ or } \bar{N}(r, g) \leq uT(r, g) + S(r, g) \quad \dots (1.1)$$

holds for some $u \in [0, 1)$. Then, $f(z) = g(z)$.

Theorem 2 — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and a_i ($i = 1, 2, 3, 4, 5$) be five distinct small meromorphic functions of $f(z)$ and $g(z)$. Suppose that $f(z)$ and $g(z)$ share a_1, a_2, a_3 GCM (CM) and a_4, a_5 GIM (IM). Then, $f(z) = g(z)$.

Theorem 3 — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and a_i ($i = 1, 2, 3, 4$) be four distinct small meromorphic functions of $f(z)$ and $g(z)$. Suppose that k is a positive integer and $f^{(k)}(z)$ and $g^{(k)}(z)$ share a_1, a_2, a_3 GCM(CM) and a_4 GIM(IM). Then, $f^{(k)}(z) = g^{(k)}(z)$.

Remark i : The following example illustrate that the condition (1.1) can not be omitted in our results.

Example — Let α, β and γ be nonconstant entire functions with α having no zeros and $T(r, \alpha) = o(T(r, e^\gamma)), T(r, \beta) = o(T(r, e^\gamma))$. Set $a_1 = \alpha + \beta, a_2 = (2\alpha)/3 + \beta, a_3 = 2\alpha + \beta, a_4 = \beta$, and

$$f = \beta + \frac{2\alpha(e^\gamma - 1)}{e^\gamma - 3}, \quad g = \beta + \frac{2\alpha(e^\gamma - 1)}{3e^\gamma - 1}.$$

It is easy to verify that f and g share a_i ($i = 1, 2, 3, 4$) CM and a_2, a_3 are two exceptional functions of f and g .

Remark ii : From above results we propose a problem that whether or not the condition sharing three values GCM (CM) can be omitted or changed as one or two values GCM (CM).

2. LEMMAS

We shall give some lemmas first.

Lemma 1 — Let $f(z)$ be a nonconstant meromorphic function. Then we have the following inequalities:

(a) For q elements $a_1, \dots, a_q \in \mathbb{C} \cup \{\infty\}$ ($q < \infty$),

$$(q-2) T(r, f) \leq \sum_{j=1}^q \bar{N} \left(r, \frac{1}{f-a_j} \right) + S(r, f).$$

(b) For three distinct elements a_1, a_2 and $a_3 \in S(f) \cup \{\infty\}$,

$$T(r, f) \leq \sum_{j=1}^3 \bar{N} \left(r, \frac{1}{f-a_j} \right) + S(r, f).$$

(c) For four distinct elements $a_1, \dots, a_4 \in S(f) \cup \{\infty\}$,

$$4T(r, f) \leq 3 \sum_{j=1}^4 \bar{N} \left(r, \frac{1}{f-a_j} \right) + S(r, f).$$

PROOF OF LEMMA 1 : (a) and (b) are famous inequalities given in [5] of [6]. From (b) we can easily obtain (c) as the following :

$$T(r, f) \leq \bar{N} \left(r, \frac{1}{f-a_i} \right) + \bar{N} \left(r, \frac{1}{f-a_j} \right) + \bar{N} \left(r, \frac{1}{f-a_k} \right) + S(r, f),$$

where i, j, k are three distinct integers from 1 to 5. By adding them side by side we obtain (c). □

Lemma 2 — Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions and α be a small meromorphic function of $f(z)$ and $g(z)$. Suppose that $f(z)$ and $g(z)$ share $0, 1, \infty$ GCM and α GIM. Then $S(r, f) = S(r, g)$ and $f(z)$ and $g(z)$ share α GCM. Moreover, either $f(z) = g(z)$ or $1, \alpha = -1$ are two exceptional functions of f and g if $0, \infty$ are not exceptional functions of f and g , and

$$N \left(r, \frac{1}{f-a} \right) = \bar{N} \left(r, \frac{1}{f-a} \right) + S(r, f) \text{ and } N \left(r, \frac{1}{g-a} \right) = \bar{N} \left(r, \frac{1}{g-a} \right) + S(r, g), \dots \quad (2.1)$$

where $a = 0, 1, \infty, \alpha$.

PROOF OF LEMMA 2 : By Lemma 1 (c), we have

$$\begin{aligned} 4T(r, f) &\leq 3 \sum_{j=1}^4 \bar{N} \left(r, \frac{1}{f-a_j} \right) + S(r, f) \\ &\leq 3 \bar{N} \left(r, \frac{1}{f-g} \right) + S(r, f) \\ &\leq 3T(r, f) + 3T(r, g) + S(r, f). \end{aligned}$$

In the similar method, we obtain that

$$T(r, f) \leq 3T(r, g) + S(r, f)$$

$$T(r, g) \leq 3T(r, f) + S(r, g).$$

Therefore $S(r, f) = S(r, g)$.

Set
$$\phi := \frac{f'(f-\alpha)}{f(f-1)} - \frac{g'(g-\alpha)}{g(g-1)}.$$

It is easy to see that $T(r, \phi) = S(r, f)$.

If $\phi \equiv 0$, then all common zeros of $f - \alpha$ and $g - \alpha$ other than the zeros and poles of $\alpha(\alpha - 1)$ with the multiplicities great than one are the zeros of one of small functions below :

$$\phi_1 := \frac{f'}{f} - \frac{g'}{g}, \phi_2 := \frac{f'}{f} - \frac{g'}{g} + \frac{\alpha'}{\alpha}, \phi_3 := \frac{f'}{f} - \frac{g'}{g} - \frac{\alpha'}{\alpha}.$$

Obviously, f and g share α GCM.

If $\phi \not\equiv 0$, then we have

$$\bar{N}\left(r, \frac{1}{f-\alpha}\right) \leq N(r, \phi) \leq T(r, \phi) + S(r, f) = S(r, f),$$

$$\bar{N}\left(r, \frac{1}{g-\alpha}\right) \leq N(r, \phi) \leq T(r, \phi) + S(r, f) = S(r, f).$$

Hence f and g still share α GCM. We have obtained that $f(z)$ and $g(z)$ share $0, 1, \infty, \alpha$ GCM.

If (2.1) does not hold, then we can assume that

$$N\left(r, \frac{1}{f-1}\right) \neq \bar{N}\left(r, \frac{1}{f-1}\right) + S(r, f). \tag{2.2}$$

Otherwise, we only need to make a quasi-Möbius transformation.

If $\phi_1 \not\equiv 0$, then we deduce that

$$N_{(2)}\left(r, \frac{1}{f-1}\right) \leq N(r, \phi_1) + S(r, f) \leq T(r, \phi_1) + S(r, f) = S(r, f),$$

which contradicts (2.2).

If $\phi_1 \equiv 0$, then we get $f(z) = cg(z)$ for a constant c . By (2.2), there exists a point z_0 such that $f(z_0) = g(z_0) = 1$, and then $c = 1$. Therefore, we obtain that $f(z) = g(z)$.

Assume that $f(z) \not\equiv g(z)$ and $0, \infty$ are not exceptional functions of f and g . If $\phi_1 \not\equiv 0$, noting that $f(z)$ and $g(z)$ share $0, 1, \infty, \alpha$ GCM, then $\bar{N}(r, f) = S(r, f)$, $\bar{N}\left(r, \frac{1}{f}\right) = S(r, f)$. Combining (2.1), we deduce that $N(r, f) = S(r, f)$ and $N\left(r, \frac{1}{f}\right) = S(r, f)$. This is impossible. Therefore, $\phi_1 \equiv 0$. It implies that 1 and α are two exceptional functions of f and g and $\alpha = -1$. In fact, from $\phi_1 \equiv 0$ we deduce that there is a constant c such that

$$f = cg \tag{2.3}$$

and $f = c(g - 1) + c, f = c(g - c) + c^2. \dots (2.4)$

By (2.3) and $N\left(r, \frac{1}{\alpha}\right) = S(r, f)$, we see that 1 and α are two exceptional functions of f and g . Note that the former equation of (2.4), we know that c is an exceptional function of f . By Lemma 1 (b) and $c \neq 1$, we infer that $c = \alpha$. Substituting it into the later equation of (2.4), we obtain that α^2 is an exceptional function of f . Therefore, $\alpha = -1$. The proof is complete. \square

3. PROOF OF THEOREMS

SIMPLE PROOF OF THEOREM D : We only need to prove that one of later six cases in Theorem D must occur when $f(z) \neq g(z)$. Infact:

If a_1, a_3 are not exceptional functions of f and g , then $F(z) \neq G(z)$ and $0, \infty$ are not exceptional functions of F and G where

$$F := \frac{(f - a_1)(a_2 - a_3)}{(f - a_3)(a_2 - a_1)}, G := \frac{(g - a_1)(a_2 - a_3)}{(g - a_3)(a_2 - a_1)}.$$

Since f and g share $a_i (i = 1, 2, 3)$ GCM and a_4 GIM, F and G share $0, 1, \infty$ GCM and α GIM, where

$$\alpha := \frac{(a_4 - a_1)(a_2 - a_3)}{(a_4 - a_3)(a_2 - a_1)}.$$

Clearly, $f(z) \equiv g(z)$ if and only if $F(z) \equiv G(z)$. By Lemma 2, we know that case (i) occurs.

If a_2, a_3 are not exceptional functions of f and g , Apply Lemma 2 to functions

$$F_{1.4} := \frac{(f - a_2)(a_1 - a_3)}{(f - a_3)(a_1 - a_2)}, G_{1.4} := \frac{(g - a_2)(a_1 - a_3)}{(g - a_3)(a_1 - a_2)},$$

and
$$\alpha_{1.4} := \frac{(a_4 - a_2)(a_1 - a_3)}{(a_4 - a_3)(a_1 - a_2)}.$$

We obtain that $F_{1.4}(z) \neq G_{1.4}(z)$ and $0, \infty$ being not exceptional functions of $F_{1.4}$ and $G_{1.4}$ implies

$$F_{1.4} = -G_{1.4}, \alpha_{1.4} = -1,$$

and a_1, a_4 are two exceptional functions of f and g . It is easy to verify that $F_{1.4} = -G_{1.4}$ and $\alpha_{1.4} = -1$ if and only if $F = -G + 2$ and $\alpha = 2$. (iii) holds.

Others four cases can be obtained in the similar method, perfectly. We omit the detail procedure here. The proof is complete. \square

PROOF OF THEOREM 1 : Suppose that $f \not\equiv g$. By Theorem C, we know that there exist two elements (say a, b) $\in \{a_i, (i = 1, 2, 3, 4)\}$ such that a and b are two exceptional functions of f and g .

If $\bar{N}(r, f) \leq uT(r, f) + S(r, f)$ holds for some $u \in [0, 1)$, then by Lemma 1 (b), we have

$$T(r, f) \leq \bar{N}\left(r, \frac{1}{f-a}\right) + \bar{N}\left(r, \frac{1}{f-b}\right) + \bar{N}(r, f) + S(r, f) \leq uT(r, f) + S(r, f),$$

a contradiction. The other case can be proved similarly. □

PROOF OF THEOREM 2 : Without loss of generalization, we assume that a_1 and a_3 are not exceptional functions of f and g .

$$\text{Set } F := \frac{(f-a_1)(a_2-a_3)}{(f-a_3)(a_2-a_1)}, \quad G := \frac{(g-a_1)(a_2-a_3)}{(g-a_3)(a_2-a_1)}.$$

Since f and g share $a_i (i = 1, 2, 3)$ GCM and a_4, a_5 GIM, F and G share $0, 1, \infty$ GCM and α, β GIM, where

$$\alpha := \frac{(a_4-a_1)(a_2-a_3)}{(a_4-a_3)(a_2-a_1)}, \quad \beta := \frac{(a_5-a_1)(a_2-a_3)}{(a_5-a_3)(a_2-a_1)}.$$

Suppose that $f(z) \not\equiv g(z)$. Then $F(z) \not\equiv G(z)$ and 0 and ∞ are not exceptional functions of F and G . By Lemma 2, we see that $1, \alpha$ and β are three distinct exceptional small functions of $F(z)$ and $G(z)$. From Lemma 1 (b) we have

$$T(r, F) \leq \bar{N}\left(r, \frac{1}{F-\alpha}\right) + \bar{N}\left(r, \frac{1}{F-\beta}\right) + \bar{N}\left(r, \frac{1}{F}\right) + S(r, F) \leq S(r, F),$$

a contradiction. Hence $f \equiv g$. The proof is complete. □

PROOF OF THEOREM 3 : Suppose that $f^{(k)} \not\equiv g^{(k)}$. By Theorem C, we know that there exist two elements (say a, b) $\in \{a_i (i = 1, 2, 3, 4)\}$ such that a and b are two exceptional small functions of $f^{(k)}$ and $g^{(k)}$.

Note that $\bar{N}(r, f^{(k)}) \leq \frac{1}{2} N(r, f^{(k)}) \leq \frac{1}{2} T(r, f^{(k)})$, by Lemma 1 (b), we have

$$\begin{aligned} T(r, f^{(k)}) &\leq \bar{N}\left(r, \frac{1}{f^{(k)}-a}\right) + \bar{N}\left(r, \frac{1}{f^{(k)}-b}\right) \\ &\quad + \bar{N}(r, f^{(k)}) + S(r, f^{(k)}) \leq \frac{1}{2} T(r, f^{(k)}) + S(r, f^{(k)}), \end{aligned}$$

a contradiction. Therefore, $f^{(k)} \equiv g^{(k)}$. The proof is complete. □

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