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KALUZA-KLEIN COSMOLOGY

R. BHATTACHARJEE[+] AND K. K. BARUAH[++]

[+]*Department of Mathematics, Gauhati University, Guwahati* 781 014

[++]*Department of Mathematics, North Gauhati College, Guwahati* 781 031

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This paper presents (*A*) a study of a non-minimally coupled scalar field in Kaluza-Klein cosmology and (*B*) some models in the same cosmology in which during the process of inflation, anisotropy in three spatial dimensions decays on the one hand and the fifth dimension dies out on the other.

Key Words : Kaluza-Klein Cosmology; Inflation; Anisotropy; Three Spatial Dimension Decays; Scalar Field

INTRODUCTION

The purpose of this paper is to present (*A*) a study of nonminimally coupled scalar field in Kaluza-Klein cosmology and (*B*) some models in the same cosmology in which during the process of inflation, the anisotropy in three spatial dimensions and fifth dimension die out.

(A) Non-Minimally Coupled Scalar Field : There is a widely accepted view that scalar fields play an important role in the investigation of early phases of cosmology. This has been a very interesting field of study over a decade. It would be relevant to mention some recent work. Madsen[1] has derived a class of solutions with conformally coupled scalar field in the presence of a non-zero cosmological constant which turns out to be responsible for inflationary behaviour. Feinstein and Ibanez[2] have studied Bianchi III and VI anisotropic cosmologies with a minimally coupled scalar field. They have worked out two solutions but they have shown that none of the two solutions shows inflation. The same authors[3] have also worked out a family of exact solution which are inhomogeneous generalisations of spatially flat FRW cosmologies. The self interacting scalar field (Φ) with an exponential potential function of the form $V \sim e^{k\Phi}$ plays the role of a source of expansion. However, these inhomogeneous models also do not show inflation. Copozziello and de-Ritis[4] on the other hand, have carried out an investigation for Nöther symmetries in point like ERW Lagrangians derived for a general non-minimally coupled scalar field. They have worked out exact solutions for flat models capable of producing inflation and transforming to the Einstein regime at the present time. Again, Abreu *et al.*[5] have derived some exact solutions of Einstein's gravity coupled to a self interacting conformal scalar field. One class of their solutions leads asymptotically to a power law inflationary character $a(t) \sim t^p$ for $p > 1$.

More recently, Krori *et al.*[6] have derived some new models in cosmology with a non-minimally coupled scalar field. They have shown that all the models inflate under specified conditions. As a follow-up, part I of this paper investigates the behaviour of a non-minimally coupled scalar field in Kaluza-Klein cosmology in respect of inflation on the one hand and compactification of the extra dimension on the other.

(B) *Inflation and Decay of Fifth Dimension and Anisotropy* : A convenient tool has been discovered in inflation for solving some serious problems in standard cosmology. Guth[7] was the first to conceive of inflation through Higgs fields as a possible solution to the horizon and flatness problems. He assumed that the universe undergoes a strongly first order phase transition with the false metastable vacuum dominated by a constant energy density difference between the false and the true vacuum. This constant energy density is responsible for the false vacuum growing exponentially.

Later, a modified version of Guth's concept of inflation, usually called the 'new inflation' was advanced by Albrecht and Steinhardt[8] and Linde[9] independently. In this theory, a scalar field is displaced from the minimum of its potential energy and slowly evolves to the minimum. During this process of slow-rolling evolution, the nearly constant potential energy density of the field causes the expansion of the universe and the scale factor grows exponentially.

Later, La and Steinhardt[10] reconsidered the inflation scenario in the context of the Brans-Dicke Theory of gravity[11]. They showed that if the universe were to be in the false metastable vacuum with a certain energy density and were to undergo a strongly first order transition, the scale factor in the false vacuum would not grow exponentially, but rather as $a(t) \alpha t^{\frac{w+1}{2}}$ (for sufficiently large t) where w is the Brans-Dicke parameter. For the Brans-Dicke theory not to conflict with experiment, W should be larger than 500. Hence, the scale factor must undergo a strong power-law growth. This inflation scenario has been termed as 'extended inflation'. In the meantime a good wealth of literature has grown over this subject of extended inflation [e.g., 12-18].

Now, there is some speculation in cosmology that in the early phase, the universe might have been anisotropic in respect of three spatial dimensions on the one hand and might have had more than four dimensions on the other, and that in the course of evolution, the anisotropy and the extra dimensions had died out and the present universe has turned out to be isotropic and four dimensional.

One might think that inflation might be responsible for the decay of anisotropy and the extra dimensions. The purpose of this paper is to explore this possibility.

Recently, some authors have done a lot of work in five dimensional inhomogeneous Kaluza-Klein cosmology[19-23] with interesting results. But the scalar fields which are believed by some cosmologists to have played significant roles in the early universe, deserve to be considered in the context of Kaluza-Klein cosmology. To the knowledge of the authors of this paper, this aspect has so far received scant attention. The motivation of this paper is to make an attempt to mitigate this deficiency to some extent.

The work is organised as follows :

(A) We present the field equation in § 11.1. We worked out two models in § 11.2. The properties of the models are discussed in § IV.

(B) The field equations in anisotropic Kaluza-Klein theory are presented in § 111.1. Some cosmological models are derived in § 111.2. The applicability of the slow-rolling conditions to these models is treated in details in § IV. Also a concluding remark on the work carried out is made in this section.

(A) NON-MINIMALLY COUPLED SCALAR FIELD

11.1. *Field Equations* :

It is not known what type of coupling operated between the gravitational and scalar field at the plank time; it is reasonable to believe that non-minimal coupling might have played an effective role. So we start with an action of the form

$$A = \int d^5x \sqrt{g} \left[F(\Phi) R - \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} - V(\Phi) \right], \quad \ldots (1)$$

where $F(\Phi) = \xi\Phi^2$, ξ being a coupling-constant, variation w.r.t. Φ gives the Φ equation.

$$\Box \Phi + R F'(\Phi) - V'(\Phi) = 0, \quad \ldots (2)$$

where $$V'(\Phi) = \frac{dv}{d\Phi}.$$

Variations w.r.t. $g_{\mu\nu}$ gives the gravitational field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \quad \ldots (3)$$

where

$$T_{\mu\nu} = F^{-1}(\Phi) \left[-\frac{1}{2} s_{\mu\nu} - g_{\mu\nu} \Box F(\Phi) + F(\Phi)_{,\mu;\nu} \right] \quad \ldots (4)$$

with

$$S_{\mu\nu} = \Phi_{,\mu} \Phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \Phi_{,\alpha} \Phi^{,\alpha} + g_{\mu\nu} V(\Phi). \quad \ldots (5)$$

Let us take FRW metric in 5D

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\Phi^2] - b^2(t) dq^2 \quad \ldots (6)$$

(3), (4) and (6) give the field equations as

$$8\xi \frac{\dot{\Phi}^2}{\Phi^2} + 8\xi \left(\frac{\ddot{\Phi}}{\Phi} + \frac{2\dot{a}\dot{\Phi}}{a\Phi} + \frac{\dot{b}\dot{\Phi}}{b\Phi} \right) + 4\xi \left(2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} + \frac{2\dot{a}\dot{b}}{ab} \right) - \frac{\dot{\Phi}^2}{\Phi^2} + \frac{2v(\Phi)}{\Phi^2} = 0 \quad \ldots (7)$$

$$12\xi \frac{\dot{a}^2}{a^2} + 24\xi \frac{\dot{a}\dot{\Phi}}{a\Phi} + 8\xi \frac{\dot{b}\dot{\Phi}}{b\Phi} + 12\xi \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{\Phi}^2}{\Phi^2} + \frac{2v(\Phi)}{\Phi^2} = 0 \quad \ldots (8)$$

$$8\xi \frac{\dot{\Phi}^2}{\Phi^2} + 8\xi \left(\frac{\ddot{\Phi}}{\Phi} + \frac{3\dot{a}\dot{\Phi}}{a\Phi} \right) + 12\xi \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) - \frac{\dot{\Phi}^2}{\Phi^2} + \frac{2v(\Phi)}{\Phi^2} = 0. \quad \ldots (9)$$

Eq. (2) gives

$$\frac{\dot{\Phi}}{\Phi}+\frac{3\dot{a}\Phi}{a\Phi}-4\xi\left(\frac{3\dot{a}}{a}+\frac{\dot{b}}{b}+\frac{3\dot{a}^2}{a^2}+\frac{3\dot{a}\dot{b}}{ab}\right)-\frac{V'(\Phi)}{\Phi}=0 \qquad \ldots (10)$$

Equations (7) and (8) imply

$$(8\xi-2)\frac{\dot{\Phi}^2}{\Phi^2}+8\xi\frac{\dot{\Phi}}{\Phi}-8\xi\frac{\dot{a}\Phi}{a\Phi}+8\xi\frac{\dot{a}}{a}+4\xi\frac{\dot{b}}{b}-8\xi\frac{\dot{a}^2}{a^2}-4\xi\frac{\dot{a}\dot{b}}{ab}=0. \qquad \ldots (11)$$

Eqs. (7) and (9) give

$$2\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)\frac{\dot{\Phi}}{\Phi}+\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}+2\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)\frac{\dot{a}}{a}=0. \qquad \ldots (12)$$

Eqs. (8) and (9) imply

$$(2-8\xi)\frac{\dot{\Phi}^2}{\Phi^2}-8\xi\frac{\dot{\Phi}}{\Phi}-12\xi\frac{\dot{a}}{a}+8\xi\frac{\dot{b}}{b}\frac{\dot{\Phi}}{\Phi}+12\xi\frac{\dot{a}}{a}\frac{\dot{b}}{b}=0. \qquad \ldots (13)$$

Eq. (12) gives

$$\dot{\Phi}^2=\frac{\lambda^2}{a^2(b\dot{a}-a\dot{b})} \qquad \ldots (14)$$

where λ is a constant.

We are to solve eqs. (10), (11), (13) and (14) to determine a, b, Φ and V. But these nonlinear equations are not easily amenable to solution. So we assume certain expressions for a and b and derive appropriate conditions for their physical validity.

11.2 Models :

Model 1 — Let us choose

$$a=(\alpha+\beta t)^m$$

and

$$b=(\alpha+\beta t)^{-n}, \qquad \ldots (15)$$

where $m > 0$, $n > 0$, α and β are constants.

Using (15) one gets from (14)

$$\Phi=\frac{\lambda}{\sqrt{(m+n)\beta}}(\alpha+\beta t)^{\frac{-3m+n+1}{2}} \qquad \ldots (16)$$

With the help of (15) and (16) we have determined V from (10) as

$$V(\Phi)=\frac{(3m-n-1)\beta^2}{2(3m-n+1)}\left\{\frac{(m+n)\beta}{\lambda^2}\right\}^{\frac{2}{3m-n-1}}$$

$$\times\left[\frac{1}{4}(-9m^2+6m+n^2-1)-4\xi(6m^2-3mn-3m+n^2+n)\right]\Phi^{\frac{2(3m-n+1)}{(3m-n-1)}}. \quad \ldots$$

(17)

Using the solutions (15), (16) and (17) in (11) or (13) one gets

$$\xi=\frac{(3m-n-1)^2}{16(6m^2-3mn-3m+n^2+n)} \quad \ldots (18)$$

which is the relation between m, n and ξ for the solution (15), (16) and (17).

Model 2 — Let us choose

$$a=\alpha e^{mt}$$

and $$b=\beta e^{-nt}, \quad \ldots (19)$$

where α, β are constants and m, n are constants such that $m > 0$, $n > 0$.

Eq. (14) gives

$$\Phi=\frac{\lambda}{\sqrt{\alpha^3\beta(m+n)}}e^{\frac{-3m+n}{2}t}. \quad \ldots (20)$$

Eq. (10) gives

$$V(\Phi)=-\frac{1}{8}[(9m^2-n^2)+16\xi(6m^2-3mn+n^2)]\,\Phi^2. \quad \ldots (21)$$

Using solutions (19), (20) and (21) in (11) or (13) one gets

$$\xi=\frac{(-3m+n)^2}{16(6m^2-3mn+n^2)}, \quad \ldots (22)$$

which is the required relation between m, n and ξ to have the solution (19), (20) and (21).

(B) Inflation and Decay of Fifth Dimension and Anisotropy

111.1 Field Equations :

We start with the following 5D line element

$$ds^2=-dt^2+a_1^2(t)dx^2+a_2^2(t)dy^2+a_3^2(t)dz^2+b^2(t)dq^2. \quad \ldots (23)$$

The energy momentum tensor is taken in the form

$$T_{\mu\nu}=\partial_\mu\Phi\partial_\nu\Phi-g_{\mu\nu}\left\{\frac{1}{2}g^{\alpha\beta}\Phi_\alpha\Phi_\beta+V(\Phi)\right\} \quad \ldots (24)$$

where Φ is a scalar field and $V(\Phi)$ a potential function, Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}. \qquad \text{... (25)}$$

Einstein field eq. (25) for the metric (23) and energy momentum tensor (24) are given by

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{b}}{b} = 8\pi\left(\frac{1}{2}\dot{\Phi}^2 + V\right), \qquad \text{... (26)}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{b}}{b} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_2\dot{b}}{a_2b} + \frac{\dot{a}_3\dot{b}}{a_3b} = -8\pi\left(\frac{1}{2}\dot{\Phi}^2 - V\right), \qquad \text{... (27)}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{b}}{b} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_1\dot{b}}{a_1b} + \frac{\dot{a}_3\dot{b}}{a_3b} = -8\pi\left(\frac{1}{2}\dot{\Phi}^2 - V\right), \qquad \text{... (28)}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{b}}{b} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{b}}{a_1b} + \frac{\dot{a}_2\dot{b}}{a_2b} = -8\pi\left(\frac{1}{2}\dot{\Phi}^2 - V\right), \qquad \text{... (29)}$$

and

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_3\dot{a}_2}{a_3a_2} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} = -8\pi\left(\frac{1}{2}\dot{\Phi}^2 - V\right), \qquad \text{... (30)}$$

where a dot implies differentiation with respect to time.

The scalar field Φ satisfies the equation

$$\nabla^\mu(\partial_\mu \Phi) + V'(\Phi) = 0, \qquad \text{... (31)}$$

where a dash stands for $\frac{d}{d\Phi}$.

Eqs. (23) and (31) yield :

$$\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{\dot{b}}{b}\right)\dot{\Phi} + \ddot{\Phi} - \frac{\dot{V}}{\dot{\Phi}} = 0. \qquad \text{... (32)}$$

Now assuming

$R^3 = a_1a_2a_3$ one gets

and

$$\frac{3\dot{R}}{R} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}.$$

Also let us choose

$$H = \frac{\dot{R}}{R} \text{ and } \sigma^2 = 3H^2 - \left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right)$$

where σ^2 is the anisotropy parameter.

Using these results in (26), (30) and (32) we have the field equations

$$3H^2 - \sigma^2 + 3H\frac{\dot{b}}{b} = 8\pi\left(\frac{1}{2}\dot{\Phi}^2 + V\right), \qquad \ldots (33)$$

$$3H^2 - \sigma^3 - 3\frac{\dot{b}}{b} - 6H\frac{\dot{b}}{b} = 8\pi\left(\frac{1}{2}\dot{\Phi}^2 - V\right) \qquad \ldots$$

(34)

and

$$\left(3H + \frac{\dot{b}}{b}\right)\dot{\Phi}^2 + \Phi\ddot{\Phi} - \dot{V} = 0. \qquad \ldots (35)$$

Eqs. (33) and (34) give

$$9H\frac{\dot{b}}{b} + 3\frac{\ddot{b}}{b} = 16\pi V. \qquad \ldots (36)$$

The three eqs. (33)-(35) contain five unknowns viz, Φ, V, H, σ and b. We shall choose expressions for Φ and $V(\Phi)$ such that in course of evolution b and σ decay with the progress of inflation.

111.2. Cosmological Models

Model 1 — We assume Φ and $V(\Phi)$ as

$$\Phi = \sqrt{\frac{3n}{8\pi}}\ \log\ t \qquad \ldots (37)$$

and

$$V = -\frac{3n}{16\pi}(3m - n - 1)\, e^{-2\sqrt{\frac{8\pi}{3n}}\,\Phi}$$

$$= -\frac{3n}{16\pi}(3m - n - 1)\frac{1}{t^2}. \qquad \ldots (38)$$

Here m and n are positive parameters.

Using the results (37) and (38) in the field equations (33)-(35), one obtains the complete solution as

$$\Phi = \sqrt{\frac{3n}{8\pi}}\ \log t,$$

$$V = -\frac{3n}{16\pi}(3m - n - 1)\, e^{-2\sqrt{\frac{8\pi}{3n}}\,\Phi},$$

$$R = t^m, \qquad \ldots (39)$$

$$b = t^{-n}$$

and

$$\sigma^2 = \frac{3}{2}(2m^2 + mn - n^2 - 2n)\frac{1}{t^2}.$$

For σ^2 to be positive

$$2m^2 + mn > n^2 + 2n. \qquad \ldots (40)$$

The deceleration parameter

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{m-1}{m}, \qquad \ldots (41)$$

q is negative i.e. inflation occurs for $m > 1$.

Model 2 — Now we assume

$$\Phi = \sqrt{\frac{3n}{8\pi}} \log(1 + \alpha t) \qquad \ldots (42)$$

and

$$V = -\frac{3\alpha^2 n}{16\pi}(3m - n - 1)\, e^{-2\sqrt{\frac{8\pi}{3n}}\,\Phi}$$

$$= -\frac{3\alpha^2 n}{16\pi}(3m - n - 1)\frac{1}{(1+\alpha t)^2}. \qquad \ldots (43)$$

Here m, n and α are positive parameters. Using (42) and (43) in the field eqs. (33)-(35), we have the complete solution as

$$\Phi = \sqrt{\frac{3n}{8\pi}} \log(1 + \alpha t),$$

$$V = -\frac{3\alpha^2 n}{16\pi}(3m - n - 1)\, e^{-2\sqrt{\frac{8\pi}{3n}}\,\Phi}, \qquad \ldots (44)$$

$$R = (1 + \alpha t)^m,$$

$$b = (1 + \alpha t)^{-n}$$

and

$$\sigma^2 = \frac{3}{2}(2m^2 + mn - n^2 - 2n)\frac{\alpha^2}{(1+\alpha t)^2}.$$

For a positive σ^2, we must have

$$2m^2 + mn > n^2 + 2n. \qquad \ldots (45)$$

The deceleration parameter q is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{m-1}{m}, \qquad \ldots (46)$$

q is negative i.e., inflation occurs for $m > 1$.

Model 3 — For this model we have

$$\Phi^2 = \frac{3n\alpha}{4\pi}\,\frac{\alpha t^2 - 1}{(1+\alpha t^2)^2}$$

so that

$$\Phi = \sqrt{\frac{3n}{4\pi}}\left[\log\left\{\sqrt{\alpha t^2} + \sqrt{\alpha t^2 - 1}\right\} - \frac{1}{\sqrt{2}}\log\frac{\left\{\sqrt{2\alpha t^2} + \sqrt{\alpha t^2 - 1}\right\}^2}{1+\alpha t^2}\right] \qquad \ldots (47)$$

and

$$V = -\frac{3n\alpha}{8\pi}[(6m - 2n - 1)\alpha t^2 + 1]\frac{1}{(1+\alpha t^2)^2}. \qquad \ldots (48)$$

Here m, n and α are assumed to be positive parameters.

The compiete solution (in terms of t, for convenience) of the field eqs. (33)-(35) is

$$\Phi = \sqrt{\frac{3n}{8\pi}}\left[\log\left\{\sqrt{\alpha t^2} + \sqrt{\alpha t^2 - 1}\right\} - \frac{1}{\sqrt{2}}\log\frac{\left\{\sqrt{2\alpha t^2} + \sqrt{\alpha t^2 - 1}\right\}^2}{1+\alpha t^2}\right], \alpha t^2 > 1 .$$

$$V = -\frac{3n\alpha}{8\pi}[(6m - 2n - 1)\,\alpha t^2 + 1]\frac{1}{(1+\alpha t^2)^2},$$

$$R = (1+\alpha t^2)^m, \qquad \ldots (49)$$

$$b = (1+\alpha t^2)^{-n}$$

and

$$\sigma^2 = \{6\alpha^2 t^2 (2m^2 + mn - n^2 - n) + 6n\alpha\}\,\frac{1}{(1+\alpha t^2)^2}.$$

σ^2 will be positive for

$$2m^2 + mn > n^2 + n. \qquad \ldots (50)$$

The deceleration parameter q is calculated as

$$q = -\frac{\ddot{R}\,R}{\dot{R}^2} = -\frac{(2m-1)\,\alpha t^2 + 1}{2m\alpha t^2}. \qquad \ldots (51)$$

and q is negative i.e., inflation occurs for $2m > 1$.

Model 4 — For this model, we choose

$$\Phi = \sqrt{\frac{3n}{8\pi}}\log\left(\tan h\,\frac{\lambda t}{2}\right) \qquad \ldots (52)$$

$$V = -\frac{3n\lambda^2}{16\pi}\left[(3m-n-1)\cos h^2\left(\sqrt{\frac{8\pi}{3n}}\,\Phi\right)+1\right]$$

$$= -\frac{3n\lambda^2}{16\pi}\left[(3m-n-1)\cot h^2\,\lambda t+1\right] \qquad \ldots (53)$$

and obtain the complete solution as

$$\Phi = \sqrt{\frac{3n}{8\pi}}\log\left(\tan h\frac{\lambda t}{2}\right),$$

$$V = -\frac{3n\lambda^2}{16\pi}\left[(3m-n-1)\cos h^2\left(\sqrt{\frac{8\pi}{3n}}\,\Phi\right)+1\right],$$

$$R = \sec h^m \lambda t, \qquad \ldots (54)$$

$$b = \text{cosec}\, h^n \lambda t$$

and $$\sigma^2 = \frac{3\lambda^2}{2}[2m^2+mn-n^2-2n)\cot h^2\,\lambda t+2n].$$

For σ^2 to be positive

$$2m^2+mn > n^2+2n.$$

At $t\to\infty$, $\sigma^2 \to \frac{3\lambda^2}{2}(2m^2+mn-n^2)$. This limit of σ^2 will be very small if λ^2 is very small.

The deceleration parameter is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{[m-1+\tan h^2\,\lambda t]}{m}$$

and q will be always negative i.e., inflation occurs for $m > 1$.

DISCUSSION

(A) Non-minimally Coupled Scalar Field

Model 1 — From eq. (15), it is evident that m and n must be positive i.e. $m > 0$, $n > 0$. Remembering this we find from eq. (18) that the coupling strength ξ will have to lie between two limits viz.

$$0 < \xi < \frac{3}{32}. \qquad \ldots (55)$$

From Fig. 1(*a*) and eq. (15) it is seen that for a particular value of m, the fifth dimension decays rapidly as the coupling strength ξ decreases. On the other hand it is also evident that for a particular value of n, the 3-space expands rapidly as the coupling strength ξ increases.

Now this coupling strength ξ decides the relationship between $V(\Phi)$ and Φ. Fig. 1(b) shows that as the coupling strength becomes weaker $V(\Phi)$ falls less and less steeply with the increase of Φ.

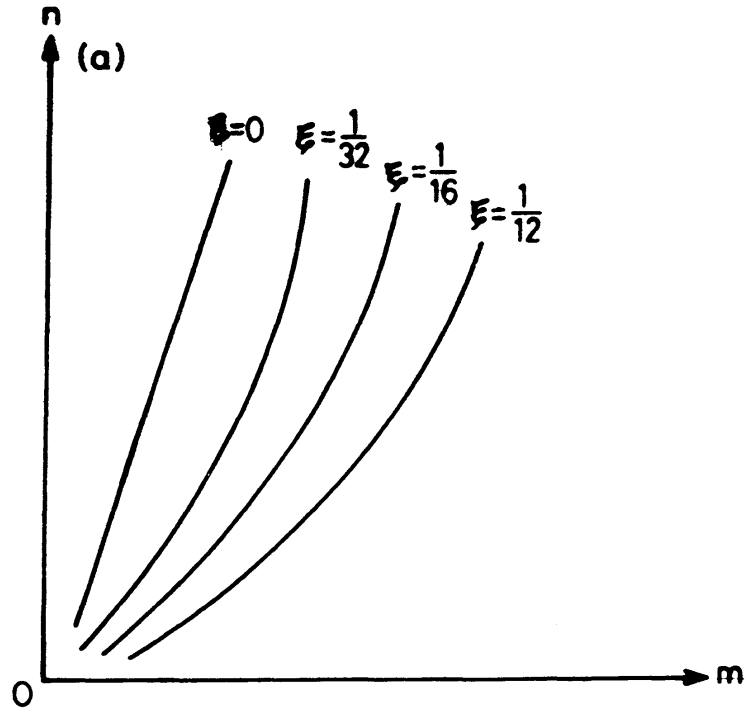

FIG. 1 (a) : n increase very rapidly as m increases for any particular values of ξ given by eq. (55)

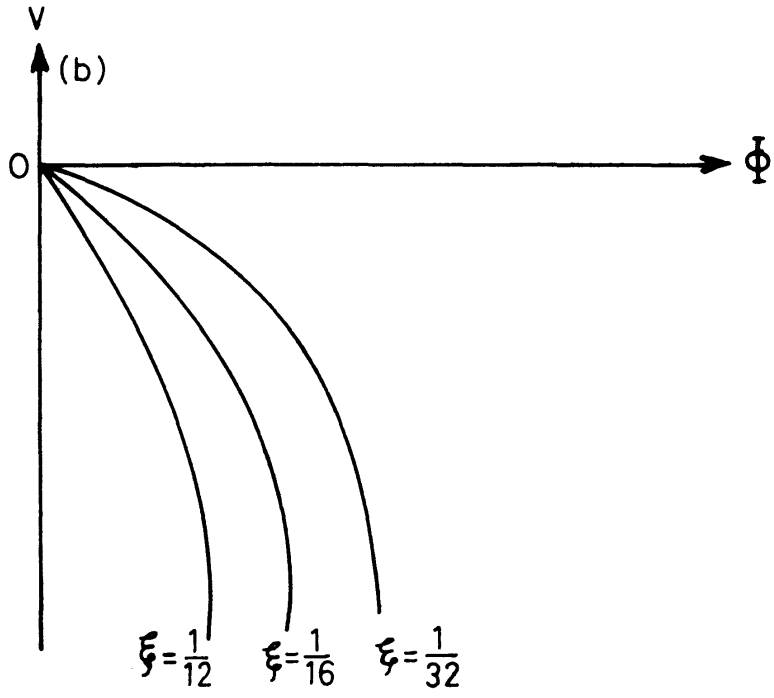

FIG. 1(b) : Φ increases while V decreases for any particular value of ξ given by eq. (55)

Model 2 — By the same reason mentioned in the model 1, one gets from eqs. (19) and (22) that the coupling strength (ξ) will have to lie between the limits viz.

$$0 < \xi < \frac{1}{10} \qquad \ldots (56)$$

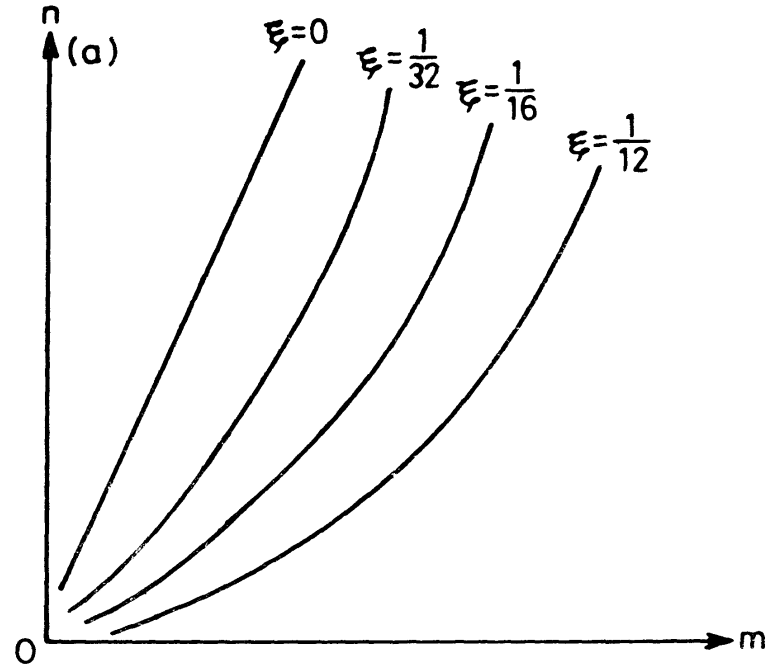

FIG. 2(*a*) : Variation of m, n for particular value of ξ given by (56)

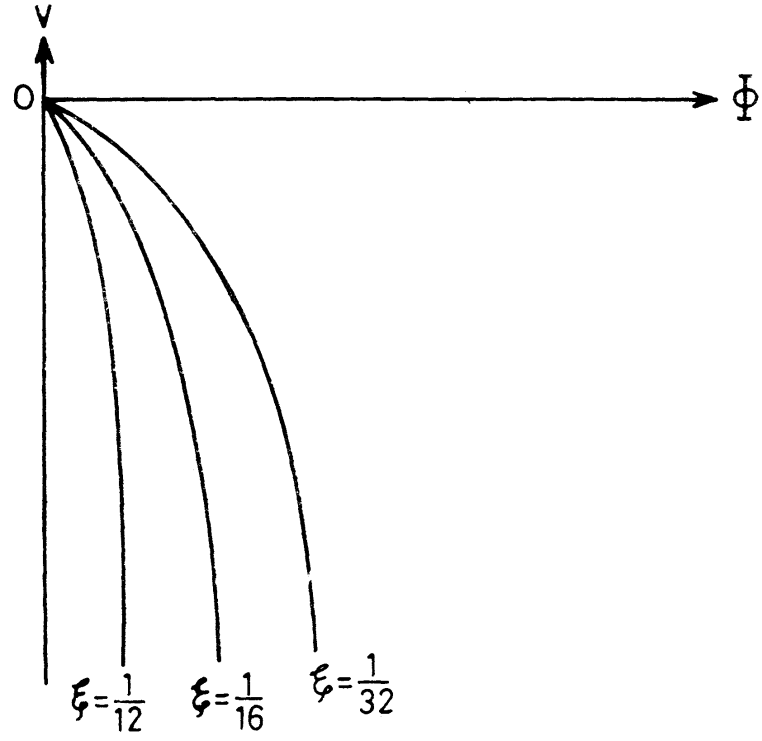

FIG. 2(*b*) Variation of Φ and V for particular value of given by (56)

From Fig. 2(a), it can be observed that the effect of ξ *vis-a-vis* m and n is similar to Fig. 1(a). The effect of ξ *vis-a-vis* $V(\Phi)$ and Φ as shown in Fig. 2(b) is also similar to Fig. 1(b).

In both the models 1 and 2, we have $a(t) \to \infty$ and $b(t) \to 0$ as $t \to \infty$. Finally, there is a possibility for inflation in 3-space in both the models.

(B) Inflation and Decay of Fifth Dimension and Anisotropy

We first find out whether slow rolling is possible in the models presented above. The conditions for slow-rolling[17&18], are given as

$$\frac{\left|\frac{\dot{\Phi}}{\Phi}\right|}{\frac{\dot{R}}{R}} << 1 \qquad \ldots (57)$$

$$\left|\frac{\dot{\Phi}}{\Phi}\right| << H \qquad \ldots (58)$$

$$|\dot{H}| << H^2 \qquad \ldots (59)$$

$$\dot{\Phi}^2 << |V(\Phi)|. \qquad \ldots (60)$$

Now let us study whether these conditions are satisfied for models 1 to 4.

Model 1 — Eqs. (39) and (57) give

$$\frac{1}{|m \log t|} << 1. \qquad \ldots (61)$$

This inequality holds at $t \to 0$ and $t \to \infty$. Since m is assumed to be finite.

Next, eqs. (39) and (58) give

$$\frac{1}{t} << \frac{m}{t}. \qquad \ldots (62)$$

This is valud for $m > > 1$.

Thirdly the eqs. (39) and (59) give

$$\frac{m}{t^2} << \frac{m^2}{t^2}. \qquad \ldots (63)$$

This is also valid for $m > > 1$. Finally the equations (39) and (60) give

$$\frac{3n}{8\pi}\frac{1}{t^2} << \frac{3n}{16\pi}(3m - n - 1)\frac{1}{t^2}. \qquad \ldots (64)$$

This is possible for $3m - n - 1 >> 2$.

The conditions (61) to (64) suggest that the slow rolling occurs at $t \to \infty$ for $m > > 1$ and $3m - n > > 3$.

Model 2 — The solution (44) and the condition (57) give us

$$\left| \frac{1}{m \log (1 + \alpha t)} \right| << 1. \qquad \ldots (65)$$

This is satisfied at $t \to \infty$.

Solutions (44) and (58) give

$$\frac{\alpha}{1 + \alpha t} << \frac{m\alpha}{1 + \alpha t}. \qquad \ldots (66)$$

This requires $m >> 1$.

The solution (44) and (59) give us

$$\frac{m\alpha^2}{(1 + \alpha t)^2} << \frac{m^2\alpha^2}{(1 + \alpha t)^2}. \qquad \ldots (67)$$

This also demands $m > > 1$

The solutions (44) and (60) give

$$\frac{3n}{8\pi} \frac{\alpha^2}{(1 + \alpha t)^2} << \frac{3\alpha^2 n}{16\pi} (3m - n - 1) \frac{1}{(1 + \alpha t)^2}. \qquad \ldots (68)$$

This requires $3m - n > > 3$.

The conditions (65) to (68) hold at $t \to \infty$ for $m >> 1$ and $3m - n >> 3$.

Model 3 — The conditions (57) to (60) subject the solution (49) to the restrictions (69)-(72) respectively. First we have

$$\left| \frac{\sqrt{\alpha t^2 - 1}}{2m\alpha t \left[\log (\sqrt{\alpha t^2} + \sqrt{\alpha t^2 - 1}) - \frac{1}{\sqrt{2}} \log \frac{(\sqrt{2\alpha t^2} + \sqrt{\alpha t^2 - 1})^2}{1 + \alpha t^2} \right]} \right| << 1. \qquad \ldots (69)$$

This condition holds for $t \to \infty$.

Next, we have

$$\left| \frac{\alpha t^2 - 3}{\alpha t^2 + 1} \right| << 2m. \qquad \ldots (70)$$

This condition holds $\alpha t \, t \to \infty$ for $m >> \frac{1}{2}$.

Thirdly we have

$$| \alpha t^2 - 1 | << 2m \, \alpha t^2 \qquad \ldots (71)$$

This condition also holds at $t \to \infty$ for $m >> \frac{1}{2}$.

Finally we have

$$2(\alpha t^2 - 1) << | (6m - 2n - 1)\alpha t^2 + 1 |. \quad \ldots (72)$$

This condition also holds at $t \to \infty$ for $6m - 2n >> 3$.

The conditions (69) to (72) suggests that slow rolling takes place at $t \to \infty$ for $m >> \frac{1}{2}$ and for $6m - 2n >> 3$.

Model 4 — The solution (54) under the condition (57) give

$$\left| \frac{1}{m \cos h\, \lambda t \log\left(\tan h \frac{\lambda t}{2} \right)} \right| << 1. \quad \ldots (73)$$

This inequality holds at $t \to 0$ and $t \to \infty$.

Next, the solution (54) under the condition (58) gives

$$| \lambda \cot h\, \lambda t | << \lambda m \cot h\, \lambda t. \quad \ldots (74)$$

It is satisfied for $m >> 1$.

Thirdly, the solution (54) under the condition (59) gives

$$| m\lambda^2 \operatorname{cosec} h^2 \lambda t | << m^2 \lambda^2 \cot h^2 \lambda t. \quad \ldots (75)$$

It holds at $t \to 0$ and $t \to \infty$ for $m >> 1$.

Finally the solution (54) under the condition (60) gives

$$(3m - n - 1) >> 2 \sec h^2 \lambda t - \tan h^2 \lambda t. \quad \ldots (76)$$

This holds at $t \to 0$ and $t \to \infty$ and $3m - n >> 3$.

The conditions (73)-(76) suggest that the slow rolling occurs at $t \to 0$ and $t \to \infty$ for $m >> 1$ and $3m - n >> 3$.

The above discussion shows that subject to certain restrictions, slow rolling is possible in all the four models presented in this paper.

Further slow rolling occurs in first three models under $3m - n >> 3$ and in the fourth models under $6m - 2n >> 3$.

To bring physical significance, let us put $m = 100$ which means $297 >> n$ for the models 1, 2 and 3 and $299 >> n$ for the model 4. This again indicates n is likely to be a single figure number. From this, we may conclude that slow rolling inflation in 3-space is much faster than the decay of fifth dimension.

In the second places we note that inflation occurs in all the models and at the same time anisotropy and the extra dimension decay in the course of evolution.

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REFERENCES

1. M. S. Madsen, *Gen. Rel. Grav.* **25** (1993) 855.
2. A. Feinstein and J. Ibnez, *Class, Quant. Grav.* **10** (1993), 93.
3. A. Feinstein and J. Ibnez, *Class, Quant. Grav.* **10** (1993) 1227.
4. S. Copozziello and R. de Ritis, *Class, Quant Grav.*, **11** (1994), 107.
5. J. P. Abreu, P. Crawford and J. P. Mimoso, *Class, Quant. Grav.* **11** (1994) 1919.
6. K. D. Krori, P. Borgohain and Surabhi Bhattacharjee. Inflationary coslology with non-minimally coupled scalar field (1995) (Submitted).
7. A. Guth, *Phys Rev. D.*, **23** (1981) 347.
8. A. ALbrecht and P. J. Steinhardt, *Phys. Rev. Lett.*, **48** (1982) 1220.
9. A. Linde., *Phys. Lett.* **108B** (1982) 380.
10. D. La and P. J. Steinhardt, *Phys. Rev. Lett.* **62** (1989) 376.
11. F. S. Accetta and J. J. Trester, *Phys. Rev. D.*, **39** (1989), 2854.
12. E. J. Weinberg *Phys. Rev. D.*, **40** (1989) 3950.
13. A. R. Liddle and D. Wands, *Phys. Rev. D.*, **46** (1992) 3655.
14. A. S. Majumdar and S. K. Sethi, *Phys. Rev. D.* **46** (1992) 5315.
15. N. Deruelle, C. Gundtach and D. Langlois., *Phys. Rev. D.*, **46** (1992) 5337.
16. A. M. Laycock and A. R. Liddle, *Phys. Rev. D.*, **49** (1994) 1827.
17. F. S. Accetta and J. B. Trester, *Phys Rev. D.* **39** (1989) 2854.
18. R. Fakir and W. G. Unruh, *Phys. Rev D.* **41** (1990) 1783.
19. S. Chatterjee, D. Panigrahi and A. Banerjee, *Class, Quant. Grav.* **10** (1993) 371.
20. S. Chatterjee and A. Banerjee, *Class Quant. Grav.* **10** (1993) L1
21. A. Banerjee, D. Panigrahi and S. Chatterjee, *Class. Quant. Grav.* **11** (1994) 1405.
22. S. Chatterjee, B. Bui, M. Barua-Basu and A. Banerjee, *Phys. Rev. D.* **50** (1994) 2924.
23. A. Banerjee, A. Das and D. Panigrahi, *Phys. Rev. D.*, **51** (1995) 6816.