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ASYMMETRIC VIBRATIONS OF POLAR ORTHOTROPIC CIRCULAR PLATES OF PARABOLICALLY VARYING PROFILE WITH RESTRAINED ELASTIC EDGE

U. S. GUPTA AND A. H. ANSARI

Department of Mathematics, University of Roorkee, Roorkee - 247 667, India

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Asymmetric vibrations of polar orthotropic circular plates of parabolically varying thickness with restrained elastic edge are discussed on the basis of the classical plate theory. Ritz method has been employed to obtain the approximate solution of the problem. Frequency parameter of the plate has been obtained for the first three modes of vibrations for different values of taper parameter, flexibility parameter and rigidity ratio. Results have been compared with those available in literature for special cases of isotropy, uniform thickness and of axisymmetric vibrations obtained by other techniques which show an excellent agreement.

Key Words : Asymmetric Vibration; Plates - Polar Orhotropic Circular; Elastic Edge; Parabolic; Ritz Method; Isotropy

Nomenclature

a	radius of the circular plate
n	nodal diameter
α	taper parameter
β_i, γ_i	are constants
$h = h(r)$	thickness of the plate at any point
h_0	thickness of the plate at the centre
r, θ	polar coordinates of any point in the plate
$w(r,\theta, t)$	transverse displacement
t	time
ρ	mass density of the material
E_r, E_θ	moduli of the plate material in r and θ directions respectively
ν_r, ν_θ	Poisson's ratios

$G_{r\theta}$ shear rigidity

U, T strain and kinetic energies respectively

ω circular frequency in radians per second

Ω frequency parameter

(K, K_ϕ) translational and rotational flexibility parameters

(D_r, D_θ) flexural rigidities in r and θ directions respectively

$D_{r\theta}$ shear modulus

Cl, SS, Fr denote the clamped, simply supported and free boundary edge respectively

1. INTRODUCTION

The dynamic behaviour of isotropic homogeneous plates of various geometries e.g., triangular, rectangular, circular and annular are broadly analysed by a number of workers and reported[1-17]. The structural components used in telephone industry, machine design, naval structures and optical systems are of non-uniform thickness[10-22]. This has necessitated the study of plates of variable thickness. Due to the development of fibre-reinforced materials, and its increasing use in the fabrication of components used in aerospace, ocean engineering, electronic components etc. the study of vibrational characteristics of such plates[19-29] has acquired great importance. The consideration of thickness variation together with orthotropy (an special case of anisotropy) not only ensures the reduction in size and weight while maintaining high strength but also meets the desirability of economy. Keeping this in view fairly recent researches[19-29] dealing with free vibration of circular and annular orthotropic plates have been reported. Most of the studies are concerned with axisymmetric case with classical boundary conditions i.e., clamped, simply supported and free. A survey of literature shows that little work has been done to study asymmetric vibrations of circular plates of variable thickness[14, 15, 29].

In practice classical boundary conditions correspond more closely to some form of elastic constraint. Therefore, a number of workers such as Azimi[8, 9] have analysed the dynamic response of plates with restrained elastic edge conditions. Gupta *et al.*[19,20] studied the axisymmetric vibrations of polar orthotropic circular plates of variable thickness with elastically restrained edge. In a recent paper Laura[13, 14, 23] studied the vibration of isotropic and orthotropic nonuniform circular plate with restrained elastic edge. The present paper presents an analysis of free vibration of elastically restrained polar orthotropic circular plate with parabolically varying profile employing Ritz method as no closed form solution is possible.

2. ANALYSIS

Consider a circular plate of radis a, variable thickness $h = h(r)$, elastically restrained against rotation and translation at the edge, (r, θ) being the polar coordinates of any point in the neutral surface of the plate shown in Fig. 1. The maximum kinetic energy of the plate is given by

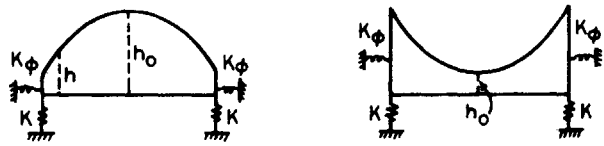

FIG. 1

$$T = \frac{1}{2}\rho\omega^2 \int_0^a \int_0^{2\pi} h\, w^2\, r d\theta dr, \qquad \ldots (1)$$

where w is the transverse delfection, ρ the mass density, ω the frequency in radians per second.

The maximum strain energy of the plate is given by

$$U = \frac{1}{2}\int_0^a \int_0^{2\pi} \left\{ D_r \left\{ \left(\frac{\partial^2 w}{\partial r^2} \right)^2 + 2\nu_\theta \frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right) \right\} \right.$$

$$\left. + D_\theta \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right)^2 + 4 D_k \left\{ \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta} \right) \right\}^2 \right\} r d\theta\, dr$$

$$+ \frac{1}{2} ak \int_0^{2\pi} w^2(a, \theta)\, d\theta + \frac{1}{2} ak_\phi \int_0^{2\pi} \left(\frac{\partial w(a, \theta)}{\partial r} \right)^2 d\theta, \qquad \ldots (2)$$

where

$$D_r = \frac{E_r h^3}{12(1 - \nu_r \nu_\theta)}, \quad D_\theta = \frac{E_\theta h^3}{12(1 - \nu_r \nu_\theta)}, \quad D_k = \frac{G_{r\theta} h^3}{12}.$$

The constants k and k_ϕ express the translation and rotational stiffness of the springs at the edge.

Ritz method requires that the Lagrangian functional

$$J(w) = T - U = \frac{1}{2}\rho\omega^2 \int_0^a \int_0^{2\pi} h\, w^2\, r d\theta dr$$

$$-\frac{1}{2}\int_0^a\int_0^{2\pi}\left\{D_r\left\{\left(\frac{\partial^2 w}{\partial r^2}\right)^2+2\nu_\theta\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)\right\}\right.$$

$$\left.+D_\theta\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)^2+4D_k\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial\theta}\right)\right\}^2\right\}rd\theta dr$$

$$+\frac{1}{2}ak\int_0^{2\pi}w^2(a,\theta)\,d\theta+\frac{1}{2}ak_\phi\int_0^{2\pi}\left(\frac{\partial w(a,\theta)}{\partial r}\right)^2 d\theta, \qquad \ldots (3)$$

be minimised.

The thickness vâriation can be of any type : Exponential[11], linear[12-15, 20-22], parabolic[17-19], stepped[16]. The discussion here is confined to the parabolic thickness variation given by $h = h_0\left(1-\alpha\left(\frac{r}{a}\right)^2\right)$, h_0 denotes the thickness of the plate at the centre $r = 0$.

Introducing the non-dimensional variables $\overline{W} = \frac{w}{a}$, $R = \frac{r}{a}$, we assume the deflection function as

$$\overline{W} = W(R)\cos n\theta = \cos n\theta \sum_{i=0}^{N} A_i F_i, \text{ where } A_i \text{ are unknown constants} \qquad \ldots (4)$$

and

$$F_i = (1+\beta_i R^4+\gamma_i R^{1+p})R^{2i+n}. \qquad \ldots (5)$$

$p^2 = E_\theta/E_r$, β_i and γ_i

are unknown constants and n is a non-negative integer. Using non-dimensional variables $\overline{W}$ and R along with the relation (4), the functional $J(W)$ given by (3) becomes

$$J(W) = \frac{\pi}{2}b_{r0}\left\{\Omega^2\int_0^1(1-\alpha R^2)W^2\,RdR-\int_0^1(1-\alpha R^2)^3\right.$$

$$\left\{\left(\frac{d^2W}{dR^2}\right)^2+2\nu_\theta\frac{d^2W}{dR^2}\left(\frac{1}{R}\frac{dW}{dR}-\frac{n^2W}{R^2}\right)\right.$$

$$\left\{ + p^2 \left(\frac{1}{R}\frac{dW}{dR} - \frac{n^2 W}{R^2} \right)^2 + n^2 D_{k0} \left(\frac{d}{dR}\left(\frac{W}{R} \right) \right) \right\} RdR$$

$$\left\{ + W^2(1) + K_\phi \left(\frac{dW(1)}{dR} \right)^2 \right\}, \qquad \ldots (6)$$

where $D_{r0} = \dfrac{E_r h_0^3}{12(1 - \nu_r \nu_\theta)}$, $D_{k0} = \dfrac{G_{r\theta} h_0^3}{3D_{r0}}$, $K = \dfrac{a^3 k}{D_{r0}}$, $K_\phi = \dfrac{ak_\phi}{D_{r0}}$, $\Omega^2 = \dfrac{a^4 \rho h_0 \omega^2}{D_{r0}}$, α the taper parameter.

The minimization of the functionl $J(W)$ given by (6) requires

$$\frac{\partial}{\partial A_i} J(W) = 0, \qquad i = 0\ (1)\ N, \qquad \ldots (7)$$

which leads to a system of homogeneous equations in A_i, $i = 1\ (1)\ N$ and its non-trivial solution leads to the frequency equation.

$$| A - \Omega^2 B | = 0, \qquad \ldots (8)$$

where $A = [a_{ij}]$ and $B = [b_{ij}]$ are square matrices of order N and

$$a_{ij} = \int_0^1 (1 - \alpha R^2) \left\{ F_i'' F_j'' + 2\nu_\theta F_i'' \left(\frac{F_j'}{R} - \frac{n^2}{R^2} F_j \right) \right.$$

$$\left. + p^2 \left(\frac{F_i'}{R} - \frac{n^2}{R^2} F_i \right)\left(\frac{F_j'}{R} - \frac{n^2}{R^2} F_j \right) + n^2 D_{k0} \left(\frac{F_i'}{R} - \frac{1}{R^2} F_i \right)\left(\frac{F_j'}{R} - \frac{1}{R^2} F_j \right) \right\} RdR$$

$$+ K F_i(1) F_j(1) + K_\phi F_i'(1) F_j'(1)$$

and

$$b_{ij} = \int_0^1 (1 - \alpha R^2) F_i F_j RdR.$$

As each co-ordinate function (6) has to satisfy the boundary conditions [Leissa (1), pp. 14], we have

$$K_\phi \frac{dW(1)}{dR} = -(1 - \alpha)^3 \left[\frac{d^2 W}{dR^2} + \nu_\theta \left(\frac{1}{R}\frac{dW}{dR} - \frac{n^2 W}{R^2} \right) \right]_{R=1} \qquad \ldots (9a)$$

and

$$K\,W(1) = (1-\alpha)^3\left[\cdot\frac{d}{dR} - \left(\frac{d^2W}{dR^2} + \frac{1}{R}\frac{dW}{dR} - \frac{n^2W}{R^2}\right) - \frac{n^2 D_{k0}}{2}\left(\frac{1}{R}\frac{dW}{dR} - \frac{W}{R^2}\right)\right]_{R=1} \quad \ldots (9b)$$

The unknown constants β_i and γ_i are determined using these boundary conditions.

3. Numerical Results

The frequency eq. (8) is solved by the hybrid-secant method in such a way that to retain the advantages of each, namely certainty of bisection and speed of secant. The method consists in finding two values of Ω^2 for which the frequency determinant changes its sign. These values are obtained by giving small increment in the values of Ω^2. A new small interval containing the root of Ω^2 is then obtained by secant method, if the generated secant iterated falls inside the initial interval, otherwise by bisection method. The process is repeated till desired accuracy is obtained. The frequency equation has been solved for various values of plate parameters such as taper parameter α, rigidity ratio E_θ/E_r, flexibility parameter K_ϕ and translational stiffness parameter K for parabolic thickness variation. The numerical results have been calculated for α (= − 0.5(0.2)0.5; 0.0), E_θ/E_r (= 0.5, 0.75, 1.0, 2.0, 5.0) and K_ϕ = (0, 1, 10, 10^2, 10^3, $10^{20} \cong \infty$) for first three modes of vibration for lower axisymmetric and antisymmetric modes i.e., n = 0, 1, 2. The Poisson's ratio ν_θ and rigidity modulus D_k/D_{r0} have been taken as 0.3 and 0.35 respectively. Special cases of classical boundary conditions i.e., simply supported, clamped and free have been considered by taking appropriate values of flexibility parameters.

4. Discussion

The results are presented in tables (I-XII) and Figs. (2 and 3). During computations it was found that the present choice of functions based on the static deflection [Ref. 30, p. 370] approximating the transverse displacement has faster rate of convergence as compared to one taken in [14, 20-23]. Although the number of such functions have to be increased up to ten with the increase in orthotropy, nodal diameter and taper parameter, but these remain smaller than those employed by[14, 20-23], which takes sixteen to twenty terms. The present method ~~thus has the~~ advantage of handling smaller order of matrices in evaluating the natural frequencies.

Tables (I-III) show the effect of E_θ/E_r and of the taper parameter α on the frequency parameter Ω for the first three modes of axisymmetric vibrations when the plate is free, simply supported or clamped. The frequency parameter is found to increase with the increasing values of E_θ/E_r and nodal diameter n whatever be the boundary conditions. The frequency parameter Ω is also found to be greater for

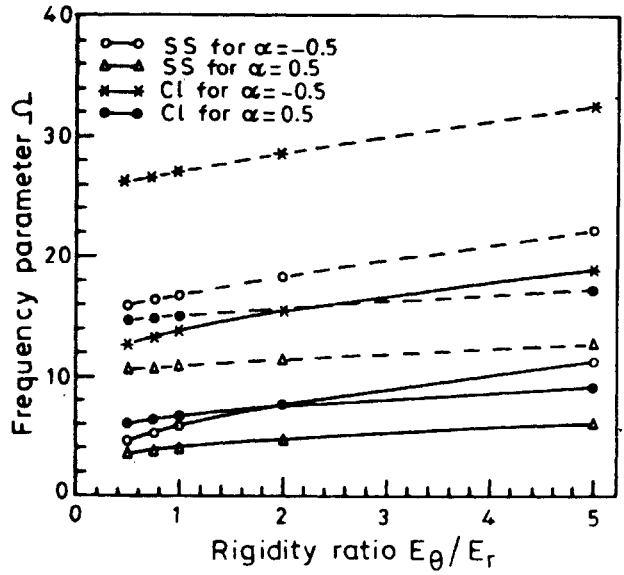

FIG. 2

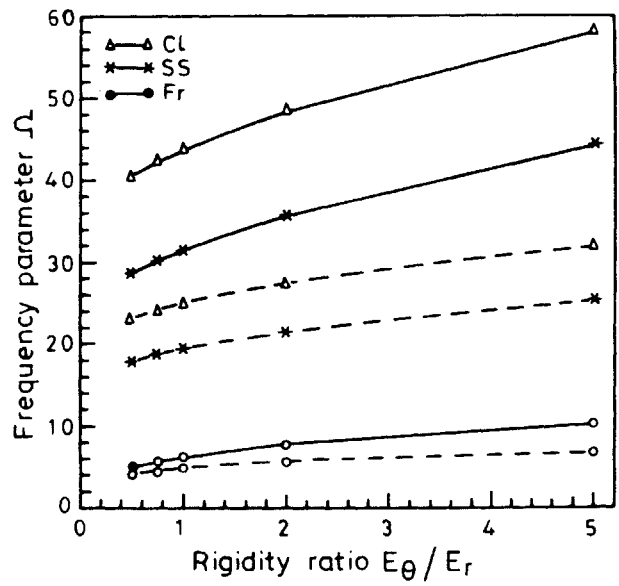

FIG. 3

negative values of the taper parameter α than those for positive values of α. This may be attributed to the greater thickness in the case of negative taper. The frequency parameter Ω_{SS}, for simply supported edge is lesser than Ω_{Cl}, for clamped edge but greater than Ω_{Fr}, for free edge keeping all other plate parameters fixed. Fig. 2 shows the plot of frequency parameter Ω for various values of E_θ/E_r when the plate is vibrating in fundamental mode. The plot shows that for negative values of taper parameter α, the rate of increase of Ω_{SS} with the increase of E_θ/E_r is less than Ω_{Cl} but greater than Ω_{Fr}, while the rate of increase is small for positive values of

α. Tables (IV-IX) give the values of frequency parameter in antisymmetric modes of vibration for $n = 1$ and $n = 2$ respectively. These tabular values show that Ω increases with the increase in E_θ/E_r as in the case of axisymmetric vibration. However, the rate of increase of Ω is higher for antisymmetric modes of vibration [Figs. 2-3].

TABLE I

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic free circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 0$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{01}		
− 0.5	7.7945	8.9850	10.0019	13.1730	19.5720
− 0.3	7.5747	8.6484	9.5660	12.4379	18.2342
− 0.1	7.3920	8.3545	9.1768	11.7485	16.9481
0.0	7.3181	8.2275	9.0031	11.4353	16.3295
0.1	7.2566	8.1149	8.8461	11.1260	15.7322
0.3	7.1849	7.9465	8.5932	10.5989	14.6197
0.5	7.2035	7.8821	8.4538	10.2055	13.6689
			Ω_{02}		
− 0.5	40.3945	42.0526	43.5210	48.1650	58.2407
− 0.3	38.5897	40.1514	41.5278	45.9579	55.2077
− 0.1	36.7372	38.1999	39.4853	43.5959	52.1048
0.0	35.7958	37.2082	38.4432	42.4898	50.5094
0.1	34.8367	36.1973	37.3860	41.1550	48.9009
0.3	32.8872	34.1350	35.2234	38.6710	45.6153
0.5	30.8817	32.0168	33:0007	36.0794	42.2199
			Ω_{03}		
− 0.5	95.6748	98.1162	100.3099	106.9411	121.8355
− 0.3	91.0788	93.3904	95.4541	102.1124	115.6593
− 0.1	86.2688	88.4434	90.3823	96.6054	109.2192
0.0	83.7833	85.8881	87.7502	95.3392	105.7660
0.1	81.2153	83.2454	85.0435	90.7468	102.2924
0.3	75.8556	77.7148	79.3634	84.6470	95.0496
0.5	70.0515	71.7397	73.2292	77.9372	87.2085

The effect of flexibility parameter K_ϕ on frequency parameter is investigated and is reported in Tables (X-XII) for the first three modes of vibration. These tables show that as the flexibility parameter K_ϕ increases, Ω increases keeping other plate parameters fixed.

TABLE II

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic simply supported circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 0$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{00}		
− 0.5	4.6056	5.2827	5.8537	7.6231	11.1625
− 0.3	4.3882	4.9802	5.4787	7.0216	10.1049
− 0.1	4.1767	4.6863	5.1142	6.4343	9.0640
0.0	4.0723	4.5418	4.9351	6.1456	8.5501
0.1	3.9680	4.3982	4.7576	5.8598	8.0405
0.3	3.7562	4.1101	4.4034	5.2936	7.0320
0.5	3.5298	3.8098	4.0392	4.7237	6.0293
			Ω_{01}		
− 0.5	32.1657	33.5404	34.7290	38.5200	46.2729
− 0.3	30.3647	31.6641	32.7863	36.3364	43.5711
− 0.1	28.4917	29.7132	30.7664	34.0739	40.7316
0.0	27.5204	28.7032	29.7200	32.9122	39.3294
0.1	26.5199	27.6648	28.6437	31.7081	37.8492
0.3	24.3410	25.4710	26.3765	29.1770	34.7539
0.5	22.0881	23.0627	23.8870	26.4289	31.4254
			Ω_{02}		
− 0.5	81.9556	84.1731	86.1120	92.3922	104.8697
− 0.3	77.5738	79.6692	81.5074	87.3725	99.1346
− 0.1	72.9818	74.9468	76.6758	82.1359	92.7464
0.0	70.5860	72.4892	74.1561	79.4616	89.9636
0.1	68.1086	69.9516	71.5534	76.6296	86.7122
0.3	62.5320	64.5593	66.0394	70.3776	79.8563
0.5	57.0722	58.6199	59.9533	64.1479	72.6474

TABLE III

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic clamped circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 0$, $\nu_\theta = 0.3$

	VALUES OF RIGIDITY RATIO E_θ/E_r				
α	0.50	0.75	1.00	2.00	5.00
			Ω_{00}		
− 0.5	12.5988	13.2086	13.7310	15.3818	18.7620
− 0.3	11.2905	11.8526	12.3317	13.8342	16.8795
− 0.2	9.9775	10.4898	10.9235	12.2723	14.9731
0.0	9.3194	9.8057	10.2158	11.4852	14.0090
0.1	8.6602	9.1195	9.5055	10.6932	13.0363
0.3	7.3394	7.7414	8.0759	9.0923	11.0598
0.5	6.0167	6.3543	6.6320	7.4614	9.0266
			Ω_{01}		
− 0.5	45.2532	46.8259	48.1833	52.4664	61.0809
− 0.3	42.1512	43.6319	44.9242	48.9576	57.0031
− 0.1	38.9196	40.3268	41.5301	45.3017	52.8031
0.0	37.2451	38.6210	39.7711	43.4074	50.6339
0.1	35.5260	36.8349	37.9627	41.4591	48.3663
0.3	31.9041	33.1246	34.1610	37.3636	43.6545
0.5	27.9680	29.0773	30.0152	32.9868	38.4482
			Ω_{02}		
− 0.5	100.7135	103.1079	105.2133	111.9793	125.5748
− 0.3	94.7598	96.9872	99.0172	105.3942	117.9040
− 0.1	88.4939	90.6423	92.5050	98.4669	110.2809
0.0	85.2211	87.3622	89.1041	94.8535	106.3664
0.1	81.8454	83.8287	85.5877	91.1141	101.9682
0.3	74.6434	76.5114	78.1241	83.1842	93.1695
0.5	66.7226	68.4082	69.8624	74.4100	82.8134

Our results have been compared with Azimi[5,6] who used Receptance method to obtain the natural frequencies for isotropic circular plates of uniform thickness with elastically restrained edge shown in Table XIII. Table XIV gives the comparison of axisymmetrically vibrating orthotropic circular plate by Gupta *et al.*[20]. The results show an excellent agreement.

TABLE IV

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic free circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 1$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{11}		
− 0.5	22.1100	22.7412	23.3520	25.6251	31.3496
− 0.3	21.0899	21.6464	22.1856	24.1969	29.2900
− 0.1	20.0858	20.5695	21.0385	22.7924	27.2561
0.0	19.5919	20.0399	20.4746	22.1016	26.2527
0.1	19.1052	19.5182	19.9190	21.4210	25.2629
0.3	18.1633	18.5086	18.8439	20.1020	23.3291
0.5	17.2966	17.5782	17.8517	18.8779	21.5119
			Ω_{12}		
− 0.5	66.7432	67.6516	68.5336	71.8425	80.3203
− 0.3	63.4886	64.3228	65.1327	68.1660	75.9430
− 0.1	60.1210	60.8817	61.6184	64.3786	71.4324
0.0	58.3912	59.1120	59.8116	62.4310	69.1227
0.1	56.6225	57.3049	57.9669	60.4444	66.7924
0.3	52.9572	53.5609	54.1463	56.3346	61.8998
0.5	49.0887	49.6109	50.1169	52.0054	56.7930
			Ω_{13}		
− 0.5	134.1990	135.4550	136.6739	141.2502	152.9019
− 0.3	127.5358	128.7044	129.8381	134.0855	144.9202
− 0.1	120.5565	121.6340	122.6805	126.6007	136.5575
0.0	116.9244	117.9563	118.9573	122.7036	132.2427
0.1	113.1815	114.1664	115.1213	118.6917	127.9531
0.3	105.2970	106.1828	107.0415	110.2493	118.3569
0.5	96.7144	97.4941	98.2495	101.0654	108.1913

TABLE V

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic simply supported circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 1$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{10}		
− 0.5	15.8478	16.2762	16.6903	18.2288	22.0912
− 0.3	14.8696	15.2365	15.5915	16.9133	20.2465
− 0.1	13.8663	14.1733	14.4706	15.5798	18.3887
0.0	13.3514	13.6291	13.8982	14.9029	17.4523
0.1	12.8247	13.0736	13.3149	14.2165	16.5087
0.3	11.7218	11.9148	12.1021	12.8027	14.5886
0.5	10.5135	10.6532	10.7889	11.2963	12.5903
			Ω_{11}		
− 0.5	55.3120	56.0652	56.7951	59.5215	66.4492
− 0.3	52.2126	52.9038	53.5737	56.0747	62.4232
− 0.1	48.9813	49.6096	50.2184	52.4900	58.2478
0.0	47.3048	47.9012	48.4789	50.6341	56.0923
0.1	45.5793	46.1434	46.6898	48.7271	53.8822
0.3	41.9439	42.4420	42.9243	44.7210	49.2574
0.5	37.9610	38.3906	38.8062	40.3532	44.2473
			Ω_{12}		
− 0.5	117.2978	118.4158	119.4987	123.5433	133.8139
− 0.3	111.0134	112.0542	113.0626	116.8279	126.3796
− 0.1	104.4097	105.3710	106.3019	109.7761	118.5665
0.0	100.9621	101.8823	102.7733	106.0966	114.4974
0.1	97.3985	98.2766	99.1267	102.2958	110.2979
0.3	89.8396	90.6299	91.3947	94.2431	101.4188
0.5	81.4846	82.1804	82.8533	85.3569	91.6440

TABLE VI

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic clamped circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 1$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{10}		
− 0.5	26.1499	26.5805	26.9970	28.5460	32.4390
− 0.3	23.9978	24.3791	24.7480	26.1196	29.5666
− 0.1	21.7893	22.1204	22.4408	23.6322	26.6254
0.0	20.6589	20.9646	21.2604	22.3602	25.1234
0.1	19.5075	19.7875	20.0583	21.0656	23.5958
0.3	17.1251	17.3525	17.5726	18.3910	20.4461
0.5	14.5912	14.7643	14.9318	15.5548	17.1182
			Ω_{11}		
− 0.5	71.2404	72.0583	72.8504	75.8033	83.2651
− 0.3	66.7174	67.4740	68.2064	70.9356	77.8309
− 0.1	61.9880	62.6812	63.3521	65.8506	72.1509
0.0	59.5288	60.1894	60.8287	63.2086	69.2182
0.1	56.9940	57.6213	58.2282	60.4870	66.1734
0.3	51.6427	52.2006	52.7402	54.7472	59.7888
0.5	45.7684	46.2518	46.7193	48.4564	52.8105
			Ω_{12}		
− 0.5	138.9388	140.1298	141.2833	145.5941	156.5104
− 0.3	130.9464	132.0591	133.1364	137.1586	147.3910
− 0.1	122.5331	123.5637	124.5621	128.2845	137.7079
0.0	118.1337	119.1222	120.0792	123.6466	132.7338
0.1	113.5805	114.5254	115.4400	118.8479	127.4516
0.3	103.9002	104.7536	105.5791	108.6526	116.3784
0.5	93.1569	93.9101	94.6383	97.3460	104.1312

TABLE VII

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic Free circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 2$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{20}		
− 0.5	4.8810	5.6401	6.2307	7.8251	10.2671
− 0.3	4.6446	5.3191	5.8512	7.2657	9.4206
− 0.1	4.4391	5.0438	5.5102	6.7502	8.6207
0.0	4.3520	4.9211	5.3583	6.5139	8.2440
0.1	4.2772	4.8127	5.2219	6.2950	7.8887
0.3	4.1767	4.6500	5.0078	5.9284	7.2560
0.5	4.1655	4.5879	4.9027	5.6858	6.7829
			Ω_{21}		
− 0.5	38.5141	40.1436	41.5838	46.2686	56.1281
− 0.3	36.2298	37.7006	39.0555	43.3194	52.2177
− 0.1	33.9390	35.3179	36.5250	40.3735	48.2990
0.0	32.7962	34.1119	35.2601	38.9000	46.3270
0.1	31.6547	32.9084	33.9967	37.4200	44.3825
0.3	29.3912	30.5143	31.4820	34.4874	40.4300
0.5	27.1814	28.1688	29.0118	31.5485	36.5493
			Ω_{22}		
− 0.5	93.2899	95.7925	98.0193	105.2726	119.9351
− 0.3	88.3073	90.5383	92.7013	99.3760	112.8828
− 0.1	83.1258	85.2842	87.1990	93.3256	105.6440
0.0	80.4603	82.5314	84.3662	90.2148	101.8539
0.1	77.7321	79.7191	81.4702	87.0012	98.2251
0.3	72.0763	73.8740	75.4544	80.4307	90.1283
0.5	66.0756	67.6685	69.0628	73.1766	81.8587

TABLE VIII

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic simply supported circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 2$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{20}		
− 0.5	28.6476	30.1203	31.4158	35.5883	44.3000
− 0.3	26.6095	27.9449	29.1215	32.9388	40.6131
− 0.1	24.5150	25.7377	26.7940	30.1302	36.8839
0.0	23.4585	24.6158	25.6133	28.7377	35.0547
0.1	22.3942	23.4805	24.4177	27.3288	33.1091
0.3	20.1884	21.1492	21.9650	24.4519	29.2490
0.5	17.8584	18.6874	19.3801	21.4324	25.2493
			Ω_{21}		
− 0.5	78.5169	80.8116	82.8465	89.3604	102.7180
− 0.3	73.9257	76.0217	77.9089	84.2767	96.1315
− 0.1	69.0575	71.0417	72.7745	78.2931	89.2828
0.0	66.5637	68.4578	70.1170	75.3678	86.5104
0.1	64.0200	65.8073	67.3874	72.3611	82.2264
0.3	58.6208	60.2423	61.6581	66.0848	74.7351
0.5	52.7539	54.1846	55.4257	59.2413	66.6487
			Ω_{22}		
− 0.5	151.1290	154.1665	156.9131	165.8152	184.3520
− 0.3	142.8688	145.6315	148.2026	157.3737	173.5258
− 0.1	133.9884	136.6781	138.1467	146.8176	162.1120
0.0	129.4254	131.9988	134.2978	141.7123	156.2511
0.1	124.7430	127.1743	129.3765	136.4356	150.4545
0.3	114.7154	116.9562	118.9547	125.3368	137.8529
0.5	103.6885	105.6889	107.4655	113.0196	123.9919

TABLE IX

Frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for polar orthotropic clamped circular plate of parabolically varying thickness : $D_{k0} = 1.4$, $n = 2$, $\nu_\theta = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r

α	0.50	0.75	1.00	2.00	5.00
			Ω_{20}		
− 0.5	40.5157	42.2324	43.7285	48.4815	58.1481
− 0.3	37.3133	38.8940	40.2669	44.6351	53.2766
− 0.1	34.0193	35.4613	36.7046	40.5907	48.2659
0.0	32.3298	33.7001	34.8770	38.5332	45.7062
0.1	30.6087	31.9026	33.0122	36.4362	43.1009
0.3	27.0385	28.1723	29.1426	32.0941	37.6626
0.5	23.2049	24.1869	25.0103	27.4589	31.9982
			Ω_{21}		
− 0.5	96.5637	99.0388	101.2238	108.2268	122.6172
− 0.3	90.4341	92.7487	94.7953	101.6234	114.5774
− 0.1	84.0348	86.1848	88.0757	94.1120	106.0739
0.0	80.7088	82.7726	84.5826	90.3291	101.7028
0.1	77.2902	79.2556	80.9830	86.4394	97.2153
0.3	70.0817	71.8331	73.3861	78.2567	87.0865
0.5	62.1182	63.6879	65.0513	69.2598	77.4222
			Ω_{22}		
− 0.5	175.0714	178.2775	181.1594	190.4574	210.4032
− 0.3	164.8949	167.9206	170.6490	179.6091	198.1073
− 0.1	154.2162	157.0486	159.5929	167.9382	184.5072
0.0	148.6315	151.3678	153.8151	161.7755	177.6245
0.1	142.8756	145.4900	147.8375	155.4308	170.5571
0.3	107.2193	133.0033	135.1380	141.9932	155.4239
0.5	117.0212	119.1595	121.0579	127.0301	138.8035

TABLE X

Frequency parameter as a function of taper parameter α, flexibility parameter K_ϕ and rigidity ratio E_θ/E_r for polar orthotropic circular plate of parabolically varying thickness : $n = 0$, $D_{k0} = 1.4$, $\nu_\theta = 0.3$, for $\alpha = 0.3$

VALUES OF RIGIDITY RATIO E_θ/E_r					
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{00}		
1	5.0728	5.4084	5.6884	6.5442	8.2277
10	6.7993	7.1757	7.4888	8.4400	10.2824
10^2	7.2766	7.6753	8.0071	9.0149	10.9656
10^3	7.3387	7.7407	8.0752	9.0915	11.0589
			Ω_{01}		
1	26.0187	27.1038	28.0201	30.8591	36.4767
10	29.9651	31.1176	32.0993	35.1475	41.1008
10^2	31.6607	32.8673	33.8947	37.0749	43.3168
10^3	31.9011	33.1230	34.1583	37.3612	43.7315
			Ω_{02}		
1	64.6381	66.3824	67.8677	72.5292	81.7450
10	70.6341	72.3990	73.9363	78.8453	88.2844
10^2	74.1059	75.9392	77.5346	82.5663	92.4680
10^3	74.6383	76.5097	78.1180	83.1812	93.9146
			for $\alpha = -0.3$		
VALUE OF RIGIDITY RATIO E_θ/E_r					
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{00}		
1	5.4716	5.9918	6.4386	7.8563	10.7786
10	8.7544	9.2305	9.6397	10.9412	13.6441
10^2	10.9042	11.4473	11.9102	13.3635	16.3133
10^3	11.2866	11.8484	12.3273	13.8269	16.8737
			Ω_{01}		
1	31.0683	32.3688	33.4900	37.0377	44.2586
10	35.0712	36.3939	37.5308	41.1200	48.3941
10^2	40.6840	42.1276	43.3687	47.2705	55.0794
10^3	42.1450	43.6254	44.9067	48.6962	57.0236
			Ω_{02}		
1	78.2098	80.3130	82.1498	88.0204	99.7598
10	82.5384	84.6482	86.4832	92.3509	104.1645
10^2	91.5820	93.7918	95.7251	101.9134	114.2457
10^3	94.7485	96.9929	98.9779	103.4998	118.2358

TABLE XI

Frequency parameter as a function of taper parameter α, flexibility parameter K_ϕ and rigidity ratio E_θ/E_r for polar orthotropic circular plate of parabolically varying thickness : $n = 1$, $D_{k0} = 1.4$, $\nu_\theta = 0.3$, for $\alpha = 0.3$

	VALUES OF RIGIDITY RATIO E_θ/E_r				
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{10}		
1	13.1952	13.3884	13.5757	14.2755	16.0352
10	15.9920	16.2047	16.4105	17.1763	19.1020
10^2	16.9860	17.2114	17.4295	18.2407	20.2775
10^3	17.1239	17.3510	17.5711	18.3895	20.4443
			Ω_{11}		
1	43.6851	44.1869	44.6726	46.4817	51.0452
10	48.6957	49.2252	49.7374	51.6427	56.4327
10^2	51.2463	51.7999	52.3354	54.3268	59.3305
10^3	51.6386	52.1964	52.7360	54.7430	59.7872
			Ω_{12}		
1	91.7243	92.5165	93.2832	96.1379	103.3332
10	98.5800	99.3962	100.1859	103.1255	110.5207
10^2	103.1231	103.9700	104.7893	107.8406	115.5209
10^3	103.8927	104.7454	105.5708	108.6461	116.4041
			for $\alpha = -0.3$		
	VALUES OF RIGIDITY RATIO E_θ/E_r				
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{10}		
1	15.6846	16.0422	16.3885	17.6799	20.9482
10	19.3923	19.7352	20.0674	21.3069	24.4475
10^2	23.1656	23.5355	23.8934	25.2245	28.5709
10^3	23.9888	24.3699	24.7386	26.1098	29.5556
			Ω_{11}		
1	52.8810	53.5713	54.2403	56.7380	63.0788
10	57.0698	57.7619	58.4324	60.9350	67.2828
10^2	64.4509	65.1846	65.8948	68.5418	75.2207
10^3	66.6912	67.4475	68.1796	70.9082	77.8065
			Ω_{12}		
1	111.6409	112.6811	113.6889	117.4529	127.0077
10	116.0451	117.0861	118.0946	121.8614	131.4384
10^2	126.6950	127.7763	128.8231	132.7309	142.5647
10^3	130.8954	132.0062	133.0836	137.1077	147.3659

TABLE XII

Frequency parameter as a function of taper parameter α, flexibility parameter K_ϕ and rigidity ratio E_θ/E_r for polar orthotropic circular plate of parabolically varying thickness : $n = 2$, $D_{k0} = 1.4$, $\nu_\theta = 0.3$, for $\alpha = 0.3$

	VALUES OF RIGIDITY RATIO E_θ/E_r				
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{20}		
1	21.7576	22.7476	23.5814	26.1211	31.0074
10	25.3346	26.4110	24.2272	30.1036	35.3906
10^2	26.8260	27.9473	28.9093	31.8392	37.4085
10^3	27.0317	28.1724	29.1402	32.1027	37.7261
			Ω_{21}		
1	60.4179	62.0659	63.4858	67.9351	76.7279
10	66.2234	67.9142	27.3255	74.0302	83.0584
10^2	69.5558	71.2871	72.8264	77.6713	87.0832
10^3	70.0620	71.8341	73.3803	78.3012	87.8721
			Ω_{22}		
1	116.6056	118.8933	120.8922	127.2984	140.4753
10	124.1759	126.4657	128.5163	135.0776	147.9179
10^2	129.7070	132.0173	134.1323	140.9677	154.3038
10^3	130.6227	133.0229	135.1273	142.1283	156.0593

for $\alpha = -0.3$

	VALUES OF RIGIDITY RATIO E_θ/E_r				
K_ϕ	0.50	0.75	1.00	2.00	5.00
			Ω_{20}		
1	27.3182	28.6644	29.8423	33.3731	41.3222
10	31.1075	32.5004	33.7110	37.5555	45.3689
10^2	36.0553	37.5812	38.9050	43.1052	51.5291
10^3	37.3145	38.8906	40.2516	44.5932	53.2652
			Ω_{21}		
1	74.5359	76.6641	78.5516	80.1250	96.7704
10	78.7851	80.9319	82.8196	88.8809	101.0794
10^2	87.4684	89.7047	91.6720	98.0663	110.9579
10^3	90.4435	92.7508	94.7580	101.3856	114.6067
			Ω_{22}		
1	143.4017	146.2416	148.8167	157.2130	174.1810
10	147.8171	150.6955	153.2620	161.6992	178.6474
10^2	159.7396	162.6691	165.2757	174.0760	192.3656
10^3	164.9304	168.0038	170.5773	179.7499	198.4559

TABLE XIII

Comparison of frequency parameter Ω for uniform isotropic circular plate for $\nu_\theta = 0.3$ and different values of K_Φ

$K = 0.00$

K_Φ	Ω_{00}	Ω_{01}	Ω_{02}	Ω_{10}	Ω_{11}	Ω_{12}	Ω_{20}	Ω_{21}	Ω_{22}
	-	+9.003	+38.443	-	+20.475	+59.812	+5.358	+35.260	+84.368
0	-	9.0031	38.4432	-	20.4746	59.8116	5.3583	35.2601	84.3662
	-	+13.513	+45.764	-	+26.298	+68.068	+8.050	+41.844	+93.256
10	-	13.5129	45.7643	2.7735	26.2982	68.0681	8.0497	41.8438	93.2569
	-	+14.539	+48.746	-	+28.126	+72.168	+8.693	+44.483	+98.431
100	-	14.5388	48.7457	-	28.1262	72.1675	8.6933	44.4832	98.4336
	-	+14.682	+49.218	-	+28.399	+72.860	+8.785	+44.904	+99.359
∞	-	14.6820	49.2159	3.0948	28.3900	72.8781	8.7849	44.9031	99.3466

$K = \infty$

	+4.935	+29.720	+74.156	+13.898	+48.479	+102.772	+25.613	+70.117	+134.290
0	4.9351	29.7200	74.1560	13.8982	48.4789	102.7733	25.6133	70.1170	134.2978
	+8.752	+35.218	+80.685	+18.543	+54.516	+109.650	+30.848	+76.5442	+141.441
10	9.7519	35.2190	80.6869	18.5438	54.5164	109.6529	30.8481	76.5442	141.4431
	+10.019	+39.029	+87.488	+20.858	+59.710	+117.925	+34.226	+83.050	+151.126
100	10.0192	39.0288	87.4900	20.8587	59.7102	117.9353	34.2259	83.0501	151.1094
	+10.216	+39.771	+89.103	+21.260	+60.829	+120.077	+34.877	+84.584	+153.830
∞	10.2158	39.7711	89.1041	21.2604	60.8287	120.0792	34.8770	84.5826	153.8151

+ values are from references [8, 9]

5. CONCLUSION

Asymmetric vibrations of parabolically tapered circular plates have been studied on the basis of classical plate theory. The transverse displacement is assumed in terms of static deflection which has faster rate of convergence than that of polynomial coordinate functions used by earlier workers[14,15, 19,20]. The problem dealt with here is more general. The special cases of isotropic/orthotropic, uniform/non-uniform, classical boundary conditions and axisymmetric can be obtained by giving suitable values to

TABLE XIV

Comparison of frequency parameter Ω as a function of taper parameter α and rigidity ratio E_θ/E_r for orthotropic plate of parabolically varying thickness in axisymmetric fundamental mode for $\nu_\theta = 0.3$

α	VALUES OF RIGIDITY RATIO E_θ/E_r 0.50	0.75	1.00	2.00	5.00
			Simply Supported Edge		
	$^{\oplus}4.6418$	$^{\oplus}5.2894$	$^{\oplus}5.8538$	$^{\oplus}7.6412$	$^{\oplus}11.2136$
− 0.5	4.6056	5.2827	5.8537	7.6231	11.1625
	$^{\oplus}4.4195$	$^{\oplus}4.9854$	$^{\oplus}5.4787$	$^{\oplus}7.0382$	$^{\oplus}10.1438$
− 0.3	4.3882	4.9802	5.4787	7.0216	10.1049
	$^{\oplus}4.2041$	$^{\oplus}4.6905$	$^{\oplus}5.1142$	$^{\oplus}6.4483$	$^{\oplus}9.0910$
− 0.1	4.1767	4.6863	5.1142	6.4343	9.0640
	$^{\oplus}4.0979$	$^{\oplus}4.5457$	$^{\oplus}4.9351$	$^{\oplus}6.1581$	$^{\oplus}8.5717$
0.0	4.0723	4.5418	4.9351	6.1456	8.5501
	$^{\oplus}3.9920$	$^{\oplus}4.4018$	$^{\oplus}4.7576$	$^{\oplus}5.8708$	$^{\oplus}8.0575$
0.1	3.9680	4.3982	4.7576	5.3598	8.0405
	$^{\oplus}3.7766$	$^{\oplus}4.1131$	$^{\oplus}4.4034$	$^{\oplus}5.3022$	$^{\oplus}7.0430$
0.3	3.7562	4.1101	4.4034	5.2936	7.0320
	$^{\oplus}3.5452$	$^{\oplus}3.8118$	$^{\oplus}4.0395$	$^{\oplus}4.7322$	$^{\oplus}6.0401$
0.5	3.5298	3.8098	4.0392	4.7237	6.0293
			Clamped Edge		
	$^{\oplus}12.6895$	$^{\oplus}13.2334$	$^{\oplus}11.7347$	$^{\oplus}15.4277$	$^{\oplus}18.9497$
− 0.5	12.5988	13.2086	11.7310	15.3818	18.7620
	$^{\oplus}11.3572$	$^{\oplus}11.8670$	$^{\oplus}12.3323$	$^{\oplus}13.8782$	$^{\oplus}13.0231$
− 0.3	11.2905	11.8526	12.3317	11.8342	16.8795
	$^{\oplus}10.0289$	$^{\oplus}10.4990$	$^{\oplus}10.9236$	$^{\oplus}12.3094$	$^{\oplus}16.0698$
− 0.1	9.9775	10.4898	10.9235	12.2723	14.9731
	$^{\oplus}9.3652$	$^{\oplus}9.8135$	$^{\oplus}10.2158$	$^{\oplus}11.5174$	$^{\oplus}14.0843$
0.0	9.3194	9.8057	10.2158	11.4852	14.0090
	$^{\oplus}8.7013$	$^{\oplus}9.1263$	$^{\oplus}9.5055$	$^{\oplus}10.7205$	$^{\oplus}14.0932$
0.1	8.6602	9.1195	9.5055	10.6932	13.0263
	$^{\oplus}7.3715$	$^{\oplus}7.7462$	$^{\oplus}8.0760$	$^{\oplus}9.1118$	$^{\oplus}11.0927$
0.3	7.3394	7.7414	8.0759	9.0923	11.0598
	$^{\oplus}6.0381$	$^{\oplus}6.3578$	$^{\oplus}6.6244$	$^{\oplus}7.4828$	$^{\oplus}9.0597$
0.5	6.0167	6.3543	6.6320	7.4614	9.0266

⊕ values are from references [20]

various parameters. The frequency parameter Ω is found to increase with the increase in the values of rigidity ratio E_θ/E_r, nodal diameter n, taper parameter α and flexibility parameters K and K_ϕ.

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