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EFFECT OF ROTATION ON THERMAL CONVECTION IN MICROPOLAR FLUIDS IN POROUS MEDIUM

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The thermal instability of micropolar fluids heated from below in the presence of uniform rotation in porous medium has been considered. It is found that the presence of coupling between thermal and micropolar effects, uniform rotation and permeability may introduce oscillatory motions in the system. The increase in Rayleigh number for the stationary convection and decrease in Rayleigh number for overstability with the increase in permeability is depicted graphically. Also the Rayleigh number is found to increase with the increase in rotation.

Key Words : Rotation Effect; Thermal Convection; Micropolar Fluids; Porous Medium; Rayleigh Numbers

1. INTRODUCTION

Micropolar fluid theory was introduced by Eringen[1] in order to describe some physical systems which do not satisfy the Navier-Stokes equation. To explain the kinematics of such media two new variables should be added to the velocity. These variables are the spin, responsible for microrotations, and the microinertia tensor which accounts for the atoms and molecules inside the macroscopic fluid particles. These fluids are able to describe the behaviour of suspensions, liquid crystals, animal blood etc[2, 3]. A generalization of these theories including thermal effects has been developed by Eringen[4].

Now-a-days, the stability of these kinds of fluids has become an important field of research. An important stability problem is the thermal convection in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics has been given by Chandrasekhar[5]. Ahmadi[6] and Pérez-Garcia *et al.*[7] have studied the effects of the microstructures on the thermal convection and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities holds. Pérez-Garcia and Rubi[8] have shown that when the coupling between thermal and micropolar effects is present, the principle of exchange of stabilities may not be fulfilled and hence that oscillatory motions are present in micropolar fluids.

The existence of oscillatory motions in micropolar fluids has been depicted by Lekkerkerker[9] (in liquid crystals), Bradley[10] (in dielectric fluids) and Laidlaw[11] (in binary mixtures). Qin and Kaloni[12] have studied the thermal instability problem in a rotating micropolar fluid. Sharma and Kumar[13] have studied the effect of rotation on thermal convection in micropolar fluids and found that the presence of coupling between thermal and micropolar effects, and uniform rotation may introduce oscillatory motions in the system. In another study, Sharma and Kumar[14] have considered

the effect of magnetic field on the micropolar fluids heated from below. The medium has been considered to be non-porous in all the above studies.

The problem of stability of a fluid layer in a porous medium subjected to a temperature gradient is of importance in geophysics, ground water hydrology, petroleum engineering, chemical engineering etc. Keeping in mind the importance and applications in chemical engineering and geophysics, the effect of uniform rotation on thermal convection in micropolar fluids in porous medium has been considered in the present paper.

2. Formulation of the Problem and Disturbance Equations

Consider the stability of a horizontal layer of an incompressible micropolar fluid of thickness d in porous medium acted on by a uniform rotation $\boldsymbol{\Omega}$ (0, 0, Ω). This layer is heated from below but convection sets in when the temperature gradient between the lower and upper boundaries exceeds a certain critical value. This is the thermal convection problem in mircropolar fluids in the presence of uniform rotation.

The mass, momentum, internal angular momentum and internal energy balance equations, following Boussinesq approximation, are

$$\nabla \cdot \boldsymbol{v} = 0, \qquad \ldots (1)$$

$$\frac{\rho_0}{\varepsilon}\frac{d\boldsymbol{v}}{dt} = -\nabla\left(p - \frac{\rho_0}{2}|\boldsymbol{\Omega}\times\boldsymbol{r}|^2\right) - \frac{1}{k_1}(\mu + x)\boldsymbol{v} + x\,\nabla\times\boldsymbol{v} - \rho g\,\hat{e}_z + \frac{2\rho_0}{\varepsilon}(\boldsymbol{v}\times\boldsymbol{\Omega}), \qquad \ldots (2)$$

$$\rho_0 j\frac{d\boldsymbol{v}}{dt} = (\varepsilon' + \beta')\,\nabla(\nabla\cdot\boldsymbol{v}) + \gamma\nabla^2\boldsymbol{v} + \frac{x}{\varepsilon}\nabla\times\boldsymbol{v} - 2x\boldsymbol{v} \qquad \ldots (3)$$

and

$$[\rho_0 C_v \varepsilon + \rho_s C_s (1-\varepsilon)]\frac{dT}{dt} + \rho_0 C_v \boldsymbol{v}\cdot\nabla T = k_T\nabla^2 T + \delta(\nabla\times\boldsymbol{v})\cdot\nabla T, \qquad \ldots (4)$$

and the equation of state is given by

$$\rho = \rho_0[1 - \alpha(T - T_0)], \qquad \ldots (5)$$

where ρ_0, T_0 are reference density and reference temperature at the lower boundary and α is the coefficient of thermal expansion. Here $\boldsymbol{v}, \boldsymbol{v}, \mu, \rho, p, T, \boldsymbol{g}, \hat{e}_z, j, C_v, k_T, \delta, \rho_s, C_s', \varepsilon$ and k_1 denote respectively, the velocity, spin, coefficient of viscosity, density, pressure, temperature, gravitational acceleration, unit vector in the z-direction, microinertia constant, specific heat at constant volume, thermal conductivity, coefficient giving account of the coupling between the spin flux and the heat flux, density of the solid matrix, heat capacity of the solid matrix, medium porosity, and medium permeability. We have assumed that external couples and heat sources are not present. ε', x, β' and γ stand for micropolar coefficients of viscosity and $\boldsymbol{r} = (x, y, z)$.

Let us now consider the stability of the system in the usual way by giving small perturbations on the initial (rest) state and seeing the reaction of the perturbations on the system. The initial state is

$$\boldsymbol{v} = 0,\ \boldsymbol{v} = 0,\ p = p(z),\ \rho = \rho(z),\quad T = T(z).$$

Let $\boldsymbol{u}(u_x, u_y, u_z)$, $\boldsymbol{\varpi}$, $\delta\rho$, δp and θ denote respectively the perturbations in velocity $\boldsymbol{v}$, spin $\boldsymbol{v}$, density ρ, pressure p and temperature T. Then eqs. (1)-(5) yield the perturbation equations

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \ldots (6)$$

$$\frac{\rho_0}{\varepsilon}\frac{d\boldsymbol{u}}{dt} = -\nabla\delta p - \frac{1}{k_1}(\mu + x)\boldsymbol{u} + x\,\nabla\times\boldsymbol{\varpi} + g\,\alpha\rho_0\,\theta\hat{e}_z + \frac{2\rho_0}{\varepsilon}(\boldsymbol{u}\times\boldsymbol{\Omega}), \qquad \ldots (7)$$

$$\rho_0 j\frac{d\boldsymbol{\varpi}}{dt} = (\varepsilon' + \beta')\nabla(\nabla\cdot\boldsymbol{\varpi}) + \gamma\nabla^2\boldsymbol{\varpi} + \frac{x}{\varepsilon}\nabla\times\boldsymbol{u} - 2x\boldsymbol{\varpi}, \qquad \ldots (8)$$

and

$$[\rho_0 C_v\varepsilon + \rho_s C_s(1-\varepsilon)]\frac{d\theta}{dt} = k_T\nabla^2\theta - \delta(\nabla\times\boldsymbol{\varpi})_z\beta + \delta(\nabla\times\boldsymbol{\varpi})\cdot\nabla\theta + \rho_0 C_v\beta U_z, \qquad \ldots (9)$$

where $\beta = -dT/dz$ is the uniform adverse temperature gradient.

Using

$$z = z^* d, \quad t = \frac{\rho_0 d^2}{\mu}t^*, \quad \theta = \beta d\theta^*, \quad \boldsymbol{u} = \frac{x_T}{d}\boldsymbol{u}^*,$$

$$p = \frac{\mu x_T}{d^2}p^*, \quad \boldsymbol{\varpi} = \frac{x_T}{d^2}\boldsymbol{\varpi}^*, \quad \boldsymbol{\Omega} = \frac{\mu}{\rho_0 d^2}\boldsymbol{\Omega}^*,$$

and then removing the stars for convenience, the non-dimensional forms of eqs. (6)-(9) become

$$\nabla\cdot\boldsymbol{u} = 0, \qquad \ldots (10)$$

$$\frac{1}{\varepsilon}\frac{d\boldsymbol{u}}{dt} = -\nabla\delta p - \frac{1}{K_1}(1+K)\boldsymbol{u} + K(\nabla\times\boldsymbol{\varpi}) + R\theta\hat{e}_z + \frac{2}{\varepsilon}(\boldsymbol{u}\times\boldsymbol{\Omega}), \qquad \ldots$$

(11)

$$\bar{j}\frac{d\boldsymbol{\varpi}}{dt} = C_1\nabla(\nabla\cdot\boldsymbol{\varpi}) - c_0\nabla\times\nabla\times\boldsymbol{\varpi} + K\left(\frac{1}{\varepsilon}\nabla\times\boldsymbol{u} - 2\boldsymbol{\varpi}\right), \qquad \ldots (12)$$

and

$$Ep\frac{d\theta}{dt} = \nabla^2\theta + u_z + \bar{\delta}[\nabla\theta\cdot\nabla\times\boldsymbol{\varpi} - (\nabla\times\boldsymbol{\varpi})_z], \qquad \ldots (13)$$

where new dimensionless coefficients are

$$\bar{j} = \frac{j}{d^2}, \quad \bar{\delta} = \frac{\delta}{\rho_0 c_v d^2}, \quad c_0 = \frac{\gamma}{\mu d^2}, \quad c_1 = \frac{(\varepsilon' + \beta' + \gamma)}{\mu d^2},$$

$$K = \frac{x}{\mu}, \quad \bar{k}_1 = \frac{k_1}{d^2}, \quad E = \varepsilon + \frac{(1-\varepsilon)\rho_s c_s}{\rho_0 c_v}, \qquad \ldots (14)$$

and the dimensionless Rayleigh number R and Prandtl number p are

$$R=\frac{g\alpha\beta\rho_0 d^4}{\mu x_T}, \quad p=\frac{\mu}{\rho_0 x_T}, \qquad \text{... (15)}$$

and we have put $x_T=\dfrac{k_T}{\rho_0 c_v}$ for thermal diffusivity.

Here we consider both the boundaries to be free and perfectly heat conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The dimensionless boundary conditions are

$$u_z=0, \quad \frac{\partial^2 u_z}{\partial z^2}=0, \quad \varpi = 0, \quad \theta = 0, \quad \zeta_z = 0 \text{ at } z = 0 \text{ and } z = 1, \qquad \text{.... (16)}$$

where $\zeta_z=(\nabla\times\boldsymbol{u})_z$ is the z-component of vorticity.

3. Linear Theory : Dispersion Relation

As the perturbations applied on the system are small, the second and higher order perturbations are negligibly small. In the linear stability analysis, second and higher order perturbations are neglected and only linear terms are retained. Thus, the non-linear terms $(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}$, $(\boldsymbol{u}\cdot\nabla)\theta$, $\nabla\theta\cdot\nabla\times\boldsymbol{\varpi}$ and $(\boldsymbol{u}\cdot\nabla)\boldsymbol{\varpi}$ are neglected.

Apply the curl operator twice to eq. (11) and taking the z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 U_z)=R\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\frac{1}{K_1}(1+K)\,\nabla^2 u_z+K\,\nabla^2\xi-\frac{2}{\varepsilon}\Omega\frac{\partial\zeta_z}{\partial z}. \qquad \text{... (17)}$$

Apply the curl operator once to eqs. (11) and (12) and taking the z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial\zeta_z}{\partial t}=-\frac{1}{k_1}(1+K)\,\zeta_z+\frac{2}{\varepsilon}\Omega\frac{\partial u_z}{\partial z}, \qquad \text{... (18)}$$

and

$$\bar{j}\frac{\partial\xi_z}{\partial t}=C_0\,\nabla^2\xi_z-\dot{K}\left[\frac{1}{\varepsilon}\nabla^2 u_z+2\,\xi_z\right]. \qquad \text{... (19)}$$

The linearized form of eq. (13) is

$$Ep\frac{\partial\theta}{\partial t}=\nabla^2\theta+u_z-\bar{\delta}\xi_z. \qquad \text{... (20)}$$

Here

$$\xi_z=(\nabla\times\boldsymbol{\varpi})_z.$$

The boundary conditions are

$$u_z=0, \quad \frac{\partial^2 u_z}{\partial z^2}=0, \quad \frac{\partial\zeta_z}{\partial z}=0, \quad \xi_z = 0, \quad \theta = 0 \text{ at } z = 0 \text{ and } z = 1. \qquad \text{... (21)}$$

In the equation for spin (19), the coefficient K accounts for coupling between vorticity and spin effects whereas coefficient C_0 accounts for spin diffusion.

Following the normal mode analysis we assume that the solutions of eqs. (17)-(20) are given by

$$[u_z, \zeta_z, \xi_z, \theta] = [U(z), Z(z), G(z), \Theta(z)] \exp(ik_x x + ik_y y + \sigma t), \quad \ldots (22)$$

where $k = (k_x^2 + k_y^2)^{1/2}$ is the wave number and σ is the stability parameter which is, in general, a complex constant. For solutions having the dependence of the form (22), eqs. (17)-(20) yield

$$(D^2 - k^2)\left[\frac{1}{\varepsilon}\sigma + \frac{1}{\bar{k}_1}(1+K)\right]U = -Rk^2\Theta + K(D^2 - k^2)G - \frac{2}{\varepsilon}\Omega DZ, \quad \ldots (23)$$

$$\left[\frac{1}{\varepsilon}\sigma + \frac{1}{\bar{k}_1}(1+K)\right]Z = \frac{2}{\varepsilon}\Omega DU, \quad \ldots (24)$$

$$[l\sigma + 2A - (D^2 - k^2)]G = -A\varepsilon^{-1}(D^2 - k^2)U, \quad \ldots (25)$$

$$[Ep\sigma - (D^2 - k^2)]\Theta = U - \bar{\delta}G, \quad \ldots (26)$$

where $l = \bar{j}\frac{A}{K}$, $A = \frac{K}{C_0}$ and $D = \frac{d}{dz}$.

Eliminating Z, G and Θ from equations (23)-(26), we obtain

$$(D^2 - k^2)\left[\sigma\varepsilon^{-1} + \frac{1}{\bar{k}_1}(1+K)\right]^2 [Ep\sigma - (D^2 - k^2)]\,[l\sigma + 2A - (D^2 - k^2)]U$$

$$= -Rk^2\left[\sigma\varepsilon^{-1} + \frac{1}{\bar{k}_1}(1+K)\right][l\sigma + 2A - (D^2 - k^2) + \bar{\delta}\varepsilon^{-1}A(D^2 - k^2)]U$$

$$- KA\varepsilon^{-1}(D^2 - k^2)^2 [Ep\sigma - (D^2 - k^2)]\left[\sigma\varepsilon^{-1} + \frac{1}{\bar{k}_1}(1+K)\right]U$$

$$- 4\Omega^2\varepsilon^{-2}[Ep\sigma - (D^2 - k^2)]\,[l\sigma + 2A - (D^2 - k^2)]D^2U. \quad \ldots (27)$$

The boundary conditions (21) transform to

$$U = D^2U = 0, \quad DZ = 0, \quad G = 0, \quad \Theta = 0 \text{ at } z = 0 \text{ and } z = 1. \quad \ldots (28)$$

Using (20), equations (23)-(26) give

$$D^2\Theta = 0, \quad D^2G = 0 \quad \text{at} \quad z = 0 \text{ and } z = 1. \quad \ldots (29)$$

The proper solution for U characterizing the lowest mode is

$$U = U_0 \sin \pi z, \quad \ldots (30)$$

where U_0 is a constant. Substituting (30) in (27) and putting $b = \pi^2 + k^2$, we obtain

$$b\left[\sigma\varepsilon^{-1}+\frac{1}{\bar{k}_1}(1+K)\right]^2[Ep\sigma+b]\,[l\sigma+2A+b]$$

$$=Rk^2\left[\sigma\varepsilon^{-1}+\frac{1}{\bar{k}_1}(1+K)\right][l\sigma+2A+b-\bar{\delta}A\varepsilon^{-1}b]$$

$$+KA\varepsilon^{-1}b^2[Ep\sigma+b]\left[\sigma\varepsilon^{-1}+\frac{1}{\bar{k}_1}(1+K)\right]$$

$$-4\Omega^2\varepsilon^{-2}\pi^2[Ep\sigma+b]\,[l\sigma+2A+b]. \quad \ldots (31)$$

In the absence of rotation ($\Omega=0$), eq. (31) reduces to

$$b\left[\varepsilon^{-1}b+\frac{1}{\bar{k}_1}(1+K)\right][Ep\sigma+b]\,[l\sigma+2A+b]$$

$$=Rk^2[l\sigma+2A+b-\bar{\delta}A\varepsilon^{-1}b]+KA\varepsilon^{-1}b^2[Ep\sigma+b], \quad \ldots (32)$$

a result derived by [Sharma and Gupta[15], eq. (28)].

4. Case of Overstability and Conclusions

Since σ is, in general, a complex constant, so we put $\sigma=\sigma_r+i\sigma_i$, where σ_r, σ_i are real. The marginal state is reached when $\sigma_r = 0$; if $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stabilities is valid otherwise we have overstability and then $\sigma=i\sigma_i$ at marginal stability.

Putting $\sigma=i\sigma_i$ in equation (31), the real and imaginary parts of eq. (31) yield

$$R=\frac{1}{k^2}\left[\left[-\sigma_i^2\varepsilon^{-2}+\frac{1}{\bar{k}_1}(1+K)\left\{\frac{1}{\bar{k}_1}(1+K)-KA\varepsilon^{-1}\right\}\right]b^3\right.$$

$$+\left[\sigma_i^2\left\{EpKA\varepsilon^{-2}-2A\varepsilon^{-2}-\frac{2}{\bar{k}_1}(1+K)\varepsilon^{-1}(Ep+l)\right\}\right.$$

$$\left.+\frac{2A}{\bar{k}_1^2}(1+K)^2+4\Omega^2\varepsilon^{-2}\pi^2\right]b^2+\left[Ep\varepsilon^{-2}l\sigma_i^4\right.$$

$$-\frac{1}{\bar{k}_1}(1+K)Ep\left\{\frac{1}{\bar{k}_1}(1+K)l+4\varepsilon^{-1}A\right\}\sigma_i^2+8\Omega^2\varepsilon^{-2}\pi^2A\Big]b$$

$$\left.-4\Omega^2\varepsilon^{-2}\pi^2Ep\sigma_i^2l\right]\left[\frac{1}{\bar{k}_1}(1+K)\{b(1-\bar{\delta}A\varepsilon^{-1})+2A\}-\sigma_i^2\varepsilon^{-1}l\right]^{-1}, \quad \ldots (33)$$

and

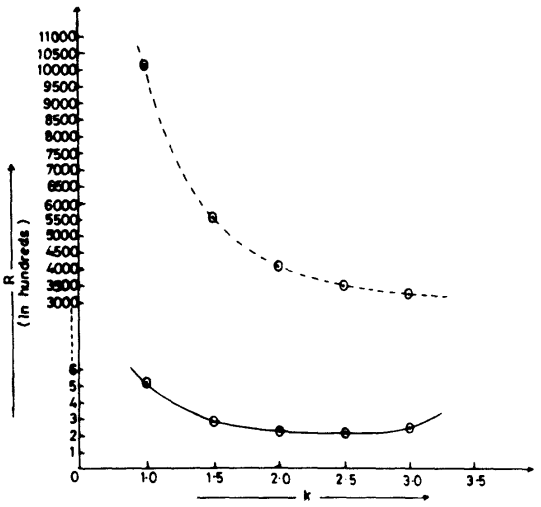

FIG. 1. The variation of Rayleigh number with wave numbers for $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\Omega = 10$ rev. min^{-1}, $\bar{k}_1 = 1$.

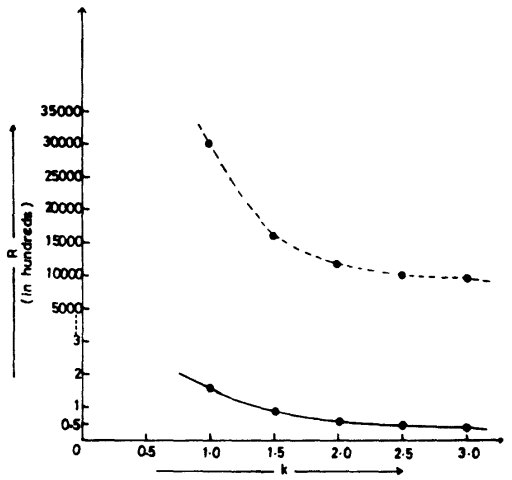

FIG. 2. The variation of Rayleigh number with wave numbers for $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\Omega = 10$ rev. min^{-1}, $\bar{k}_1 = 3$.

$$\left[-l\varepsilon^{-3}(l+\bar{\delta}A\varepsilon^{-1}Ep)b^2 - \left\{ \frac{1}{\bar{k}_1}(1+K)\varepsilon^{-2}Epl^2 \right\} b \right] \sigma_i^4$$

$$+ \left[[-(1-\bar{\delta}A\varepsilon^{-1})\varepsilon^{-3}]b^4 + \varepsilon^{-2}\left[-\frac{1}{\bar{k}_1}(1+K)(1-\bar{\delta}A\varepsilon^{-1})(Ep+l) \right.\right.$$

$$\left.\left. - 2A\varepsilon^{-1}(2-\bar{\delta}A\varepsilon^{-1}) + AK\varepsilon^{-1}\{Ep(1-\bar{\delta}A\varepsilon^{-1}) - 2l\} + l \right] b^3 \right.$$

$$+ \left[-2\varepsilon^{-2}Ep\frac{A}{\bar{k}_1}(1+K)(2-\bar{\delta}A\varepsilon^{-1}) - \frac{l\varepsilon^{-1}}{\bar{k}_1^2}(1+K)^2(Ep\,\bar{\delta}A+l) \right.$$

$$\left. + 2A^2\varepsilon^{-3} \right] b^2 + \left[4\Omega^2\varepsilon^{-3}l\pi^2(l+\varepsilon^{-1}Ep\,\bar{\delta}A) - \frac{Ep(1+K)}{\bar{k}_1} \right.$$

$$\left\{ 2A\left(\frac{1}{\bar{k}_1}l\,\overline{1+K} + 2A\varepsilon^{-1} \right) + \frac{l^2(1+K)^2}{\bar{k}_1^2} \right\} + \frac{2A}{\bar{k}_1}\varepsilon^{-1}Ep(1+K)$$

$$\left. \left\{ 2A\varepsilon^{-1}\frac{l}{\bar{k}_1}(1+K) \right\} \right] b + \left[-4\Omega^2\varepsilon^{-2}\pi^2 Ep\frac{l^2}{\bar{k}_1}(1+K) \right] \left. \right] \sigma_i^2$$

$$+ \left[\left[-(1-\bar{\delta}A\varepsilon^{-1})(1+K)^2\frac{\varepsilon^{-1}}{\bar{k}_1^2} \right] b^4 + \left[4(1-\bar{\delta}A\varepsilon^{-1})\varepsilon^{-3}\Omega^2\pi^2 \right.\right.$$

$$\left.\left. + \frac{(1+K)^3}{\bar{k}_1^3}\{l\bar{\delta}A\varepsilon^{-1} - (1-\bar{\delta}A\varepsilon^{-1})Ep\} - \frac{A\varepsilon^{-1}}{\bar{k}_1^2}(1+K)^2\{K(l-Ep \right. \right.$$

$$1-\bar{\delta}A\varepsilon^{-1}) + 2(2-\bar{\delta}A\varepsilon^{-1})\}\Big] b^3 + \Big[2A\Big\{ 4\Omega^2\varepsilon^{-3}\pi^2 - \frac{(1+K)^3}{\bar{k}_1^3} Ep \Big\}$$

$$(2-\bar{\delta}A\varepsilon^{-1}) + \frac{2A^2\varepsilon^{-1}}{\bar{k}_1^2}(1+K)^2(KEp-2) + \frac{4(1+K)}{\bar{k}_1}\Omega^2\varepsilon^{-2}\pi^2$$

$$\{l\bar{\delta}A\varepsilon^{-1} - Ep(1-\bar{\delta}A\varepsilon^{-1})\}\Big] b^2 + \Big[16\Omega^2\varepsilon^{-3}\pi^2A^2 - 8A\Omega^2\varepsilon^{-2}\pi^2\frac{Ep}{\bar{k}_1}$$

$$(1+K)(2-\bar{\delta}A) - 4A^2Ep\frac{(1+K)^3}{\bar{k}_1^3}\Big] b - \Big[16\Omega^2\varepsilon^{-2}\pi^2A^2\frac{(1+K)}{\bar{k}_1}Ep\Big]\Big] = 0. \quad \ldots (34)$$

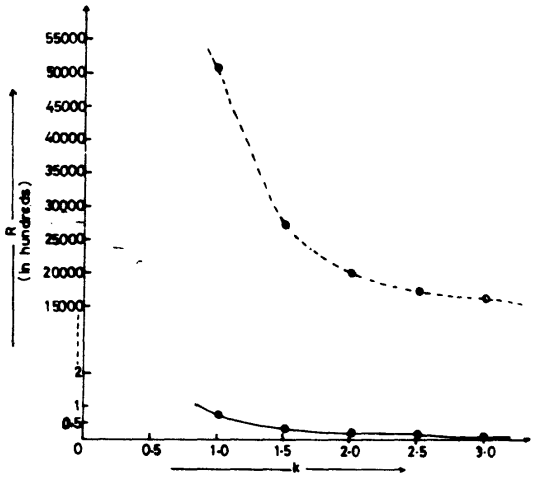

FIG. 3. The variation of Rayleigh number with wave numbers for $A = 0.5$, $K = 1$, $l = 1$, $\bar{\delta} = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\Omega = 10$ rev.min^{-1}, $\bar{k}_1 = 5$.

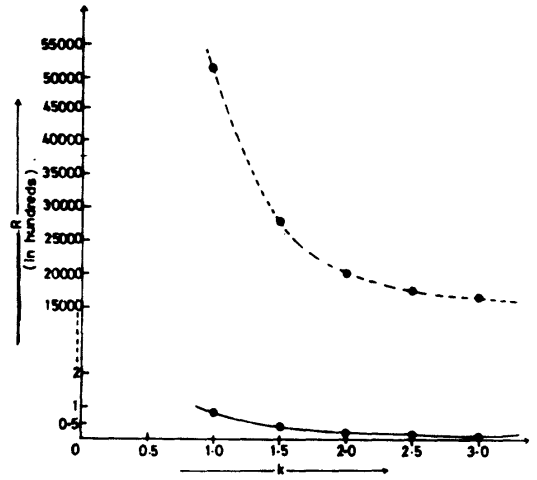

FIG. 4. The variation of Rayleigh number with wave numbers for $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\bar{k}_1 = 5$, $\Omega = 10$ rev.min^{-1}.

It is evident from eq. (34) that oscillatory modes will not be present for all values of the parameters. For example, in the absence of coupling between spin and heat fluxes ($\bar{\delta} = 0$), rotation ($\Omega = 0$) and permeability ($\bar{k}_1 \to \infty$), overstable solutions will not take place if

$$AKEp + l\varepsilon < 4A, \quad \ldots (35)$$

because then the only possible solutions of eq. (34) are those with $\sigma_i = 0$ and so overstable solutions will not exists. Thus the presence of rotation and coupling between spin and heat fluxes may bring overstability in the system which agrees with the earlier result for non-porous medium (Sharma and Kumar[13]). The medium permeability also brings in overstability in the system.

In the absence of coupling between spin and heat fluxes ($\bar{\delta} = 0$) and $\sigma_i = 0$, equation (33) reduces to

$$R = \frac{b^3\frac{(1+K)}{\bar{k}_1}\Big[\frac{(1+K)}{\bar{k}_1} - KA\varepsilon^{-1}\Big] + b^2\Big[\frac{2A(1+K)^2}{\bar{k}_1^2} + 4\Omega^2\varepsilon^{-2}\pi^2\Big] + b[8\Omega^2\varepsilon^{-2}\pi^2A]}{\frac{k^2(1+K)}{\bar{k}_1}[b+2A]}. \quad \ldots (36)$$

For stationary convection i.e., $\sigma_i = 0$ and in the presence of coupling between spin and heat fluxes ($\bar{\delta} \neq 0$), eq. (33) gives

$$R = \frac{b^3 \frac{(1+K)}{\bar{k}_1}\left[\frac{(1+K)}{\bar{k}_1} - KA\varepsilon^{-1}\right] + b^2\left[\frac{2A(1+K)^2}{\bar{k}_1^2} + 4\Omega^2\varepsilon^{-2}\pi^2\right] + b[8\Omega^2\varepsilon^{-2}\pi^2 A]}{\frac{k^2(1+K)}{\bar{k}_1}[b(1-\bar{\delta}A\varepsilon^{-1}) + 2A]} \quad \ldots (37)$$

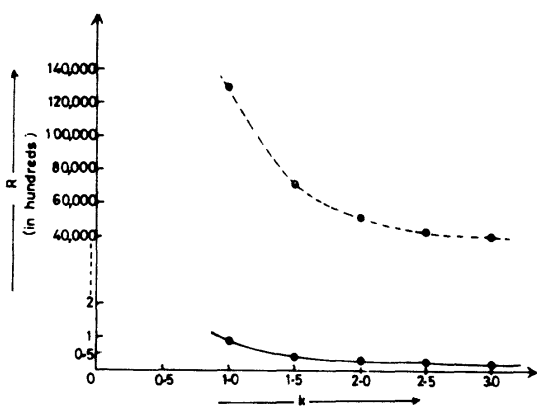

FIG. 5. The variation of Rayleigh number with wave numbers for $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\bar{k}_1 = 5$, $\Omega = 16$ rev.min^{-1}.

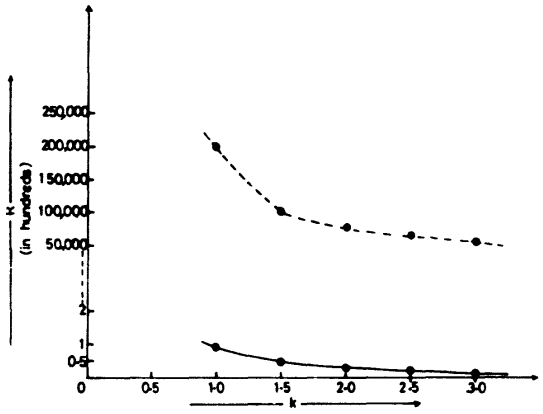

FIG. 6. The variation of Rayleigh number with wave numbers for $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $\varepsilon = 0.5$, $p = 0$, $E = 1$, $\bar{k}_1 = 5$, $\Omega = 20$ rev.min^{-1}.

We have plotted the variation of the Rayleigh number with the wave numbers using eq. (33) satisfying (34) for both stationary and overstable cases for values of the dimensionless parameters $A = 0.5$, $\bar{\delta} = 1$, $K = 1$, $l = 1$, $p = 0$, $E = 1$ and $\varepsilon = 0.5$. Figures 1, 2 and 3 correspond to three values of the permeability $\bar{k}_1 = 1$, 3 and 5 respectively and Figs. 4, 5 and 6 correspond to three values of the angular velocity of rotation $\Omega = 10$, 16 and 20 rev. min^{-1} respectively.

Figs. 1-3 show that the Rayleigh number for the stationary convection increases with the increase in permeability whereas the Rayleigh number for the overstable case decreases with the increase in permeability. Therefore permeability has a stabilizing effect on stationary convection. It is also noted from the figures that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave number.

It is also evident from the Figures 4-6 that the Rayleigh number increases with the increase in rotation parameter depicting the stabilizing effect of rotation which is in agreement with the earlier result obtained by Sharma and Kumar[13] for non-porous media. Moreover, rotation introduces oscillatory modes in the system. The presence of coupling between thermal and micropolar effects may bring overstability in the system. It is also noted from Figures 4-6 that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave number.

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