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COMPLETELY GENERALIZED STRONGLY NONLINEAR QUASI-COMPLEMENTARITY PROBLEMS FOR FUZZY MAPPINGS

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In this paper, we introduce and study a new class of completely generalized strongly non-linear quasi-complementarity problems for fuzzy mappings. We also discuss the existence of solutions for this kind of quasi-complementarity problems without compactness and the convergence of iterative sequences generated by the algorithms.

1. INTRODUCTION

Complementarity theory introduced by Lemke[17] and Cottle and Dantzig[8] in the early 1960s and later developed by others plays an important and fundamental role in the study of a wide class of problems arising in mechanics, physics, nonlinear programming, optimization and control, economics and transportation equilibrium, contact problems in elasticity, fluid flow through porous media, and many other branches of mathematical and engineering sciences (see Lin-Cryer[18], Cottle[7], Cottle *et al.*[9], Crank[10], Chang[1], Isac[14], Mosco[19] and the references therein). Among these generalizations of the complementarity problems, the quasi-(implicit) complementarity problems considered and studied by Pang[30, 31], Noor[21, 22, 23] and the class of mildly nonlinear complementarity problems introduced and studied by Noor[24, 25, 26] are important and useful generalizations in which the constraint set depend on the solution.

Chang and Huang[3, 4] introduced and studied some new class of quasi-(implicit) complementarity problems for set-valued mappings with compact values in Hilbert spaces, which included many complementarity and quasi-complementarity problems studied by Noor[21, 22, 23], Isac[13] and Pang[30, 31] as special cases.

On the other hand, Chang and Zhu[6] first introduced and studied a class of variational inequalities for fuzzy mappings. Recently, several kinds of variational inequalities and complementarity problems were considered by Chang[1, 2], Chang and Huang[5], Noor[28] and Lee *et al.*[15, 16]. These works may lead to new and significant results in these areas (see Noor *et al.*[29]).

Motivated and inspired by recent research work in this field, in this paper, we introduce and study a new class of completely generalized strongly nonlinear quasi-complementarity problems for fuzzy mappings which includes many known classes as special cases. We also discuss the existence of solutions for this kind of quasi-complementarity problems without compactness and the convergence of iterative sequences generated by the algorithms.

2. Preliminaries and Formulations

Let H be a real Hilbert space endowed with a norm $\| \cdot \|$ and inner product $(\cdot, \cdot)$. If $K \subset H$ is a closed convex cone, we denote by K^* the polar cone of K, i.e.,

$$K^* = \{u \in H : (u, v) \geq 0, \quad \forall \quad v \in K\}.$$

Let $\mathcal{F}(H)$ be a collection of all fuzzy sets over H. A mapping F from H into $\mathcal{F}(H)$ is called a fuzzy mapping on H. If F is a fuzzy mapping on H then $F(x)$ (denote it by F_x, in the sequel) is a fuzzy set on H and $F_x(y)$ is the membership function of y in F_x.

Let $M \in \mathcal{F}(H)$, $q \in [0, 1]$. Then the set

$$(M)_q = \{x \in H : M(x) \geq q\}$$

is called a q-cut set of M.

Let $F, G : H \to \mathcal{F}(H)$ be two fuzzy mappings satisfying the following condition (I) :

(I) There exist two functions $a, b : H \to [0, 1]$ such that for all $x \in H$ the set $(F_x)_{a(x)} \in CB(H)$ and $(G_x)_{b(x)} \in CB(H)$, where $CB(H)$ denotes the family of all nonempty bounded closed subsets of H.

By using the fuzzy mappings F and G, we can define two set-valued mapping $\tilde{F}$ and $\tilde{G}$ as follows :

$$\tilde{F} : H \to CB(H), x \mapsto (F_x)_{a(x)},$$

$$\tilde{G} : H \to CB(H), x \mapsto (G_x)_{b(x)}.$$

In the sequel, $\tilde{F}$ and $\tilde{G}$ are called the set-valued mappings induced by the fuzzy mappings F and G respectively.

Given mappings $m, S, T, g : H \to H$, functions $a, b : H \to [0, 1]$ and fuzzy mappings $F, G : H \to \mathcal{F}(H)$, we consider the following problem :

Find $u, x, y \in H$, such that

$$\left.\begin{aligned} &F_u(x) \geq a(u), G_u(y) \geq b(u), g(u) \in K(u), \\ &Sx + Ty \in K^*(u), (g(u) - m(u), Sx + Ty) = 0 \end{aligned}\right\} \quad \ldots (2.1)$$

where $K(u) = m(u) + K$, and $K^*(u) = (m(u) + K)^*$. If m, S, T, g are all nonlinear mappings, then the problem (2.1) is called the completely generalized strongly nonlinear quasi-complementarity problem for fuzzy mappings.

If $F, G : H \to 2^H$ (where 2^H denotes all the nonempty subsets of H) are classical set-valued mappings, then the problem (2.1) is equivalent to finding $u, x, y \in H$, such that

$$\left\{ \begin{array}{c} x \in Fu, y \in Gu, g(u) \in K(u), \\ Sx + Ty \in K^*(u), (g(u) - m(u), Sx + Ty) = 0, \end{array} \right\} \quad \dots (2.2)$$

which is called the completely generalized strongly set-valued nonlinear quasi-complementarity problem.

Remark 2.1 : For appropriate and suitable choice of the mappings m, S, T, g, F, G, the functions a, b and the convex cone K, a number of known classes of complementarity and quasi-complementarity problems can be obtained as special cases studied previously by many authors including Chang and Huang[3,4], Ding[11], Huang and Hu[12], Siddiqi and Ansari[32, 33], Noor[21-26] and Zeng[34].

3. Iterative Algorithm

We first give the following results.

Lemma 3.1[3] — If $K(u) = m(u) + K$, then

$$K^*(u) = m^*(u) + K^*.$$

Lemma 3.2[1, 27] — If $K \subset H$ is a closed convex set and $z \in H$ is a given point, then $u \in K$ satisfies the inequality

$$(u - z, v - u) \geq 0, \quad \forall \quad v \in K$$

if and only if

$$u = P_K z, \quad \dots (3.1)$$

where P_K is the projection of H onto K.

Lemma 3.3[1, 27] — The mapping P_K defined by (3.1) is nonexpansive, that is,

$$\| P_K u - P_K v \| \leq \| u - v \|, \ \forall u, v \in H .$$

Lemma 3.4[1,27] — If $K(u) = m(u) + K$ and $K \subset H$ is a closed convex set, then for any $u, v \in H$, we have

$$P_{K(u)} v = m(u) + P_K (v - m(u)). \quad \dots (3.2)$$

Lemma 3.5 — If $K \subset H$ is a closed convex cone and $K(u) = m(u) + K$, then $u, x, y \in H$ are a solution of the completely generalized strongly nonlinear quasi-complementarity problem (2.1), if and only if $u \in H, x \in \tilde{F} u, y \in \tilde{G} u$ satisfy $g(u) \in K(u)$ and

$$(v - g(u), Sx + Ty) \geq 0, \quad \forall \ v \in K(u), \quad \dots (3.3)$$

which is called the completely generalized strongly nonlinear quasi-variational inequality for fuzzy mappings.

PROOF : Let $u \in H, x \in \widetilde{F}u, y \in \widetilde{G}u$ be a solution of the problem (2.1). Since $K(u) = m(u) + K$, if $v \in K(u)$, it can be written as $v = m(u) + z$ for some $z \in K$. Using the assumptions we have

$$\begin{aligned}(Sx + Ty, v - g(u)) &= (Sx + Ty, m(u) + z - g(u)) \\ &= (Sx + Ty, m(u) - g(u)) + (Sx + Ty, z) \\ &= (Sx + Ty, z) \geq 0.\end{aligned}$$

This implies that $u \in H, x \in \widetilde{F}u, y \in \widetilde{G}u$ are a solution of the problem (3.3).

Conversely, suppose that $u \in H, x \in \widetilde{F}u, y \in \widetilde{G}u$ satisfy (3.3). Since $g(u) \in K(u)$, we know that $g(u) - m(u) \in K$ and hence, $2g(u) - m(u) \in K(u)$. From $0 \in K$, we get $m(u) \in K(u)$. Taking $v = 2g(u) - m(u)$ and $v = m(u)$ in (3.3), in turn, we obtain

$$(Sx + Ty, g(u) - m(u)) \geq 0, (Sx + Ty, m(u) - g(u)) \geq 0.$$

It follows from the above inequalities that

$$(Sx + Ty, g(u) - m(u)) = 0. \qquad \dots (3.4)$$

It remains to prove that $Sx + Ty \in K^*(u)$. Taking $v = m(u) + z$ in (3.3), we have

$$\begin{aligned}0 \leq (Sx + Ty, v - g(u)) &= (Sx + Ty, m(u) + z - g(u)) \\ &= (Sx + Ty, z).\end{aligned}$$

This shows that $Sx + Ty \in K^*$. On the other hand, taking $v = m(u) + g(u)$ in (3.3), we have $v \in K(u)$ and hence

$$0 \leq (Sx + Ty, m(u) + g(u) - g(u)) = (Sx + Ty, m(u)).$$

This implies that $Sx + Ty \in m^*(u)$. It follows from the Lemma 3.1 that $Sx + Ty \in K^*(u)$. □

From Lemmas 3.2 and 3.4, we have the following lemma.

Lemma 3.6—Let $K \subset H$ be a closed convex cone and $K(u) = m(u) + K$. Then $u \in H, x \in \widetilde{F}u, y \in \widetilde{G}u$ satisfy $g(u) \in K(u)$ and (3.3), if and only if $u \in H$, $x \in \widetilde{F}u, y \in \widetilde{G}u$ satisfy $g(u) \in K(u)$ and the relation

$$u = (1 - \lambda)\, u + \lambda\, [u - g(u) + m(u) + P_K\, (g(u) - \rho\, (Sx + Ty) - m(u))], \dots (3.5)$$

where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

Based on Lemmas 3.5 and 3.6, we are now in a position to propose the following general and unified algorithms for the problem (2.1).

Algorithm 3.1 — Suppose that $K \subset H$ is a closed cone and $m, S, T, g : H \to H$. Let $F, G : H \to \mathcal{F}(H)$ be two fuzzy mappings satisfying the condition (I) and $\widetilde{F}, \widetilde{G} : H \to CB(H)$ be set-valued mappings induced by F, G respectively. For given $u_0 \in H$, we take $x_0 \in \widetilde{F}u_0$ and $y_0 \in \widetilde{G}u_0$, and let

$$u_1 = (1-\lambda)\, u_0 + \lambda\, [u_0 - g(u_0) + m(u_0) + P_K\,(g(u_0) - \rho(Sx_0 + Ty_0) - m(u_0))].$$

Since $x_0 \in \widetilde{F}\, u_0 \in CB(H), y_0 \in \widetilde{G}\, u_0 \in CB(H)$, by Nadler[20] there exist $x_1 \in \widetilde{F}\, u_1$, $y_1 \in \widetilde{G}\, u_1$ such that

$$\| x_0 - x_1 \| \leq (1+1)\, H\,(\widetilde{F}\, u_0, \widetilde{F}\, u_1),$$

$$\| y_0 - y_1 \| \leq (1+1)\, H\,(\widetilde{G}\, u_0, \widetilde{G}\, u_1),$$

where H (., .) is the Hausdorff metric on $CB(H)$. Let

$$u_2 = (1-\lambda)\, u_1 + \lambda\, [u_1 - g(u_1) + m(u_1) + P_K\,(g(u_1) - \rho(Sx_1 + Ty_1) - m(u_1))].$$

By induction, we can obtain three sequences $\{x_n\}$, $\{y_n\}$ and $\{u_n\}$ as following :

$$\left.\begin{array}{r} x_n \in \widetilde{F}\, u_n, \| x_n - x_{n+1} \| \leq (1 + 1/(1+n))\, H\,(\widetilde{F}\, u_n, \widetilde{F}\, u_{n+1}), \\ y_n \in \widetilde{G}\, u_n, \| y_n - y_{n+1} \| \leq (1 + 1/(1+n))\, H\,(\widetilde{G}\, u_n, \widetilde{G}\, u_{n+1}), \\ u_{n+1} = (1-\lambda)\, u_n + \lambda\, [u_n - g(u_n) + m(u_n) + P_K\,(g(u_n) - \rho\,(Sx_n + Ty_n) - m(u_n))], \\ n = 0, 1, 2, \ldots, \end{array}\right\} \quad \ldots (3.6)$$

where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

If $F, G : H \to CB(H)$ are two classical set-valued mappings, then from the algorithm 3.1 we can obtain the following :

Algorithm 3.2 — For given $u_0 \in H, x_0 \in Fu_0, y_0 \in Gu_0$, we can obtain three sequences $\{x_n\}, \{y_n\}$ and $\{u_n\}$ as following:

$$\left.\begin{array}{r} x_n \in Fu_n, \| x_n - x_{n+1} \| \leq (1 + 1/(1+n))\, H\,(Fu_n, Fu_{n+1}), \\ y_n \in Gu_n, \| y_n - y_{n+1} \| \leq (1 + 1/(1+n))\, H\,(Gu_n, Gu_{n+1}), \\ u_{n+1} = (1-\lambda)\, u_n + \lambda\, [u_n - g(u_n) + m(u_n) + P_K\,(g(u_n) - \rho(Sx_n + Ty_n) - m(u_n))], \\ n = 0, 1, 2, \ldots, \end{array}\right\} \quad \ldots (3.7)$$

where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

If $F, G : H \to H$ are identity mappings, then from the algorithm 3.2 we can obtain the following:

Algorithm 3.3 — For given $u_0 \in H$, compute

$$u_{n+1} = (1-\lambda)\, u_n + \lambda\, [u_n - g(u_n) + m(u_n) + P_K\,(g(u_n) - \rho\,(Su_n + Tu_n) - m(u_n))], \quad n = 0, 1, 2 \ldots, \quad \ldots (3.8)$$

where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

Furthermore, if $m = 0$, then from the algorithms 3.3 we can get the following :

Algorithm 3.4 — For given $u_0 \in H$, compute

$$u_{n+1} = (1-\lambda)\, u_n + \lambda\, [u_n - g(u_n) + P_K\, (g(u_n) - \rho\, (Su_n + Tu_n))],$$
$$n = 0, 1, 2, \ldots, \quad \ldots (3.9)$$

where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

4. Existence and Convergence

In this section, we study the existence of solutions for the completely generalized strongly nonlinear quasi-complementarity problems (2.1), (2.2) without compactness and the convergence of the iterative sequences generated by the Algorithms 3.1 and 3.2. We first give the following definitions.

Definition 4.1 — A mapping $g: H \to H$ is said to be

(i) strongly monotone if there exists some $\alpha > 0$ such that

$$(g(u_1) - g(u_2), u_1 - u_2) \geq \alpha \left|\left| u_1 - u_2 \right|\right|^2, \quad \forall\ u_i \in H, i = 1, 2.$$

(ii) Lipschitz continuous if there exists some $\beta > 0$ such that

$$\left|\left| g(u_1) - g(u_2) \right|\right| \leq \beta \left|\left| u_1 - u_2 \right|\right|, \quad \forall\ u_i \in H, i = 1, 2.$$

Definition 4.2 — A set-valued mapping $F: H \to 2^H$ is said to be

(i) strongly monotone with respect to a mapping $S: H \to H$ if there exists some $\gamma > 0$ such that

$$(Sw_1 - Sw_2, u_1 - u_2) > \gamma \left|\left| u_1 - u_2 \right|\right|^2, \quad \forall\ u_i \in H, w_i \in Fu_i, i = 1, 2.$$

(ii) H-Lipschitz continuous if there exists some $\delta > 0$ such that

$$H\,(Fu_1, Fu_2) \leq \delta \left|\left| u_1 - u_2 \right|\right|, \quad \forall\ \ u_i \in H, i = 1, 2.$$

Theorem 4.1 — Suppose that $K \subset H$ is a closed convex cone, $F, G: H \to \mathcal{F}(H)$ are two fuzzy mappings satisfying the condition (1) and $\tilde{F}, \tilde{G}: H \to CB(H)$ are set-valued mappings induced by F, G respectively. Let $S, g: H \to H$ be Lipschitz continuous with Lipschitz constants β, σ respectively, and g strongly monotone with constant δ. Let $\tilde{F}$, $\tilde{G}$ be H-Lipschitz continuous with H-Lipschitz constants ε, η respectively, $\tilde{F}$ strongly monotone with respect to mapping S with constant α, and $T, m: H \to H$ Lipschitz continuous with Lipschitz constants ξ, μ respectively. If the following conditions hold :

$$\left| \rho - \frac{\alpha - (1-2k)\,\xi\eta}{\beta^2\,\varepsilon^2 - \xi^2\,\eta^2} \right| < \frac{\sqrt{[\alpha - (1-2k)\,\xi\eta]^2 - 4\,(\beta^2\,\varepsilon^2 - \xi^2\,\eta^2)\,k\,(1-k)}}{\beta^2\,\varepsilon^2 - \xi^2\,\eta^2}, \quad \ldots (4.1)$$

$$\alpha > (1-2k)\xi\eta + 2\sqrt{(\beta^2\varepsilon^2 - \xi^2\eta^2)k(1-k)}, \quad \ldots (4.2)$$

$$\rho\xi\eta < 1-2k, \quad k = \mu + \sqrt{1-2\delta+\sigma^2} < 1/2, \quad \xi\eta < \beta\varepsilon, \quad \ldots (4.3)$$

then there exist $u, x, y \in H$ which are a solution of the problem (2.1), and

$$u_n \to u, x_n \to x, y_n \to y, n \to \infty,$$

where $\{u_n\}$, $\{x_n\}$ and $\{y_n\}$ are defined in the Algorithm 3.1.

PROOF : From the Algorithm 3.1 and Lemma 3.3, we have

$$\begin{aligned}\| u_{n+1} - u_n \| \le \lambda \| & u_n - u_{n-1} - (g(u_n) - g(u_{n-1})) + m(u_n) - m(u_{n-1}) \| \\ & + (1-\lambda) \| u_n - u_{n-1} \| + \lambda \| g(u_n) - g(u_{n-1}) - \rho (Sx_n - Sx_{n-1}) \\ & - \rho (Ty_n - Ty_{n-1}) - m(u_n) + m(u_{n-1}) \| \\ & \le 2\lambda \| u_n - u_{n-1} - (g(u_n) - g(u_{n-1})) \| + 2\lambda \| m(u_n) - m(u_{n-1}) \| \\ & \quad + (1-\lambda) \| u_n - u_{n-1} \| + \lambda \| u_n - u_{n-1} \\ & \quad - \rho (Sx_n - Sx_{n-1}) \| \\ & \quad + \lambda\rho \| Ty_n - Ty_{n-1} \|. \end{aligned} \quad \ldots (4.4)$$

By the Lipschitz continuity and strongly monotonicity of g, we obtain

$$\| u_n - u_{n-1} - (g(u_n) - g(u_{n-1})) \|^2 \le (1-2\delta+\sigma^2) \| u_n - u_{n-1} \|^2. \quad \ldots (4.5)$$

Since $\tilde{F}$ is strongly monotone with respect to S and H-Lipschitz continuous, and S is Lipschitz continuous, we have

$$\begin{aligned}& \| u_n - u_{n-1} - \rho (Sx_n - Sx_{n-1}) \|^2 \\ & \qquad \le (1 - 2\rho\alpha + \rho^2\beta^2(1+1/n)^2\varepsilon^2) \| u_n - u_{n-1} \|^2. \end{aligned} \quad \ldots (4.6)$$

Further, since $\tilde{G}$ is H-Lipschitz continuous and T, m are Lipschitz continuous, we get

$$\begin{aligned}\| T(y_n) - T(y_{n-1}) \| & \le \xi \| y_n - y_{n-1} \| \\ & \le \xi\eta(1+1/n) \| u_n - u_{n-1} \|. \end{aligned} \quad \ldots (4.7)$$

$$\| m(u_n) - m(u_{n-1}) \| \le \mu \| u_n - u_{n-1} \|. \quad \ldots (4.8)$$

From (4.4)-(4.8), it follows that

$$\| u_n - u_{n-1} \| \le \theta_n \| u_n - u_{n-1} \|, \quad \ldots (4.9)$$

where

$$\theta_n = 2\lambda k + (1-\lambda) + \lambda\sqrt{1-2\rho\alpha+\rho^2\beta^2(1+1/n)^2} + \lambda\rho\xi\eta(1+1/n)$$

and $$k = \mu + \sqrt{1-2\delta+\sigma^2}.$$

Letting

$$\theta = 2\lambda k + (1-\lambda) + \lambda\sqrt{1-2\rho\alpha+\rho^2\beta^2} + \lambda\rho\xi\eta$$

we know that $\theta_n \searrow \theta$. It follows from (4.1)-(4.3) that $\theta < 1$. Hence $\theta_n < 1$, for n sufficiently large. Therefore, the (4.9) implies that $\{u_n\}$ is a Cauchy sequence in H and we can suppose that $u_n \to u \in H$.

Now we prove that $x_n \to x \in \tilde{F}u, y_n \to y \in \tilde{G}u$. In fact, it follows from the Algorithm 3.1 that

$$\|x_n - x_{n-1}\| \le (1+1/n)\,\varepsilon\,\|u_n - u_{n-1}\|,$$

$$\|y_n - y_{n-1}\| \le (1+1/n)\,\eta\,\|u_n - u_{n-1}\|,$$

that is $\{x_n\}$ and $\{y_n\}$ are also Cauchy sequences in H. Let $x_n \to x, y_n \to y$. Further we have

$$\begin{aligned} d(x, \tilde{F}u) &= \inf\{\|x - z\| : z \in \tilde{F}u\} \\ &\le \|x - x_n\| + d(x_n, \tilde{F}u) \\ &\le \|x - x_n\| + H(\tilde{F}u_n, \tilde{F}u) \\ &\le \|x - x_n\| + \varepsilon\|u_n - u\| \to 0. \end{aligned}$$

Hence, $x \in \tilde{F}u$. Similarly, $y \in \tilde{G}u$.

By using the continuity of P_K, S, T, g, m and the Algorithm 3.1, we have

$$u = (1-\lambda)u + \lambda[u - g(u) + m(u) + P_K(g(u) - \rho(Sx + Ty) - m(u))],$$

that is

$$g(u) = m(u) + P_K(g(u) - \rho(Sx + Ty) - m(u)) \in K(u).$$

It follows from the lemmas 3.5 and 3.6 that $u \in H, x \in \tilde{F}u, y \in \tilde{G}u$ are a solution of the problem (2.1) and

$$u_n \to u, x_n \to x, y_n \to y, n \to \infty,$$

where $\{u_n\}$, $\{x_n\}$ and $\{y_n\}$ are defined in the Algorithm 3.1. □

From the theorem 4.1, we can get following result.

Theorem 4.2 — Let $S, g : H \to H$ be Lipschitz continuous with Lipschitz constants β, σ respectively, and g strongly monotone with constant δ. Let $F, G : H \to CB(H)$ be H-Lipschitz continuous with H-Lipschitz constants ε, η respectively, F strongly monotone with respect to S with constant α, and $T, m : H \to H$ Lipschitz continuous with Lipschitz constants ξ, μ respectively. If the conditions (4.1)-(4.3) of Theorem 4.1 hold, then there exist $u \in H, x \in Fu, y \in Gu$ which are a solution of the completely generalized strongly set-valued nonlinear quasi-complementarity problem (2.2), and

$$u_n \to u, \quad x_n \to x, \quad y_n \to y, \quad n \to \infty,$$

where $\{u_n\}$, $\{x_n\}$ and $\{y_n\}$ are defined in the algorithm 3.2.

Remark 4.1 : For a suitable choice of the operators g, S, T, F, G and m, we can obtain several known results[1, 3, 4, 11, 21-26, 34] as special cases of the main result of this paper.

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