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ON GENERALIZED PREREGULAR CLOSED SETS IN TOPOLOGICAL SPACES

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In this paper a new class of sets called generalized preregular closed sets is introduced and its properties are studied. Further the notion of preregular $T_{1/2}$ space and generalized preregular continuity are introduced.

1. INTRODUCTION

One of the generalization of closed sets is α-generalized closed sets which was defined by Maki *et al.*[15]. Recently Balachandran and Rani[4] defined the notion of generalized preclosed sets (briefly gp-closed). It follows easily from definitions that every αg-closed set is a gp-closed set. Palaniappan and Rao[18] defined a set A to be regular generalized closed (briefly rg-closed) if cl $A \subset U$ whenever $A \subset U$ and U is regular open. In this paper we define and study the properties of generalized preregular closed sets (briefly gpr-closed) which is the weaker form of the above mentioned generalizations. Moreover, in this paper we define preregular $T_{1/2}$ spaces as the spaces in which every generalized preregular closed set is preclosed. We also introduce the notion of generalized preregular continuity and study its properties.

2. PRELIMINARIES

Throughout this paper $(X, \tau), (Y, \sigma)$ and (Z, η) represent nonempty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) cl A and int A denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1 — A subset A of a space (X, τ) is called

(1) a preopen set[13] if $A \subset$ int cl A and a preclosed set if cl int $A \subset A$,

(2) a semi-open set[11] if $A \subset$ cl int A and a semi-closed set if int cl $A \subset A$,

(3) a α-open set[16] if $A \subset$ int cl int A and a α-closed set if cl int cl $A \subset A$,

(4) a semi-preopen set[1] if $A \subset$ cl int cl A and a semi-preclosed set if int cl int $A \subset A$,

(5) a regular open set if $A =$ int cl A and a regular closed set if $A =$ cl int A.

The pre-interior (pint A) of a subset $A \subset X$ is the union of all preopen sets contained in A. The pre-closure (pcl A) of A is the intersection of all preclosed sets that contain A. Note that pcl $A = A \cup$ cl int A and pint $A = A \cap$ int cl A Andrijevic[1]. The semi-preclosure (spcl A) of A is analogously defined. The family of all preopen (resp. preclosed, regular open) subsets of (X, τ) will be denoted by $PO(\tau)$ (resp. $PC(\tau), RO(\tau)$).

Definition 2.2 — A subset A of a space (X, τ) is called

(1) a generalized closed set (briefly g-closed)[12] if cl $A \subset U$ whenever $A \subset U$ and U is open,

(2) a semi-generalized closed set (briefly sg-closed)[3] if scl $A \subset U$ whenever $A \subset U$ and U is semi-open,

(3) a generalized α-closed set (briefly $g\alpha$-closed)[14] if α cl $A \subset U$ whenever $A \subset U$ and U is α-open,

(4) a generalized semi-closed set (briefly gs-closed)[2] if scl $A \subset U$ whenever $A \subset U$ and U is open,

(5) a α-generalized closed set (briefly αg-closed)[15] if α cl $A \subset U$ whenever $A \subset U$ and U is open,

(6) a regular generalized closed set (briefly rg-closed)[18] if cl $A \subset U$ whenever $A \subset U$ and U is regular open.

(7) a generalized preclosed set (briefly gp-closed)[4] if pcl $A \subset U$ whenever $A \subset U$ and U is open.

(8) a generalized semi-preclosed set (briefly gsp-closed)[6] if spcl $A \subset U$ whenever $A \subset U$ and U is open.

Definition 2.3 — A function $f : (X, \tau) \to (Y, \sigma)$ is called

(1) αg-continuous[15] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V of (Y, σ),

(2) rg-continuous[18] if $f^{-1}(V)$ is regular generalized closed in (X, τ) for every closed set V of (Y, σ),

(3) p-open[9] if $f(U)$ is preopen in (Y, σ) for every preopen set U of (X, τ),

(4) pre-continuous[13] if $f^{-1}(V)$ is preclosed in (X, τ) for every closed set V of (Y, σ).

(5) regular irresolute[18] if $f^{-1}(V)$ is regular closed in (X, τ) for every regular closed set V of (Y, σ).

Definition 2.4 — A space (X, τ) is called

(1) a $T_{1/2}$ space[5] if every g-closed set is closed,

(2) a semi-pre $T_{1/2}$ space[6] if every generalized semi-preclosed set is semi-preclosed.

3. Basic Properties of Generalized Preregular Closed Sets

We introduce the following definition :

Definition 3.1 — A subset A of (X, τ) is called generalized preregular closed (briefly gpr-closed) if pcl $A \subset U$ whenever $A \subset U$ and U is regular-open in (X, τ).

By GPRC (τ) we mean the family of all gpr-closed subsets of the space (X, τ).

Theorem 3.2 — Every regular generalized closed set is gpr-closed.

PROOF : Let $A \subset X$ be regular generalized closed. Let $A \subset U$ and U be regular open. Then cl $A \subset U$ since A is rg-closed. Since every closed set is preclosed, pcl $A \subset$ cl A. Therefore pcl $A \subset U$. Hence A is gpr-closed.

The following example shows that the converse of the above theorem is not true.

Example 3.3 — Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Consider $A = \{a\}$. $A \subset \{a, b\}$ where $\{a, b\}$ is regular open. Cl $A = \{a, b, e\}$ and pcl $A = \{a\}$. Therefore A is gpr-closed but not rg-closed.

Theorem 3.4 — Every gp-closed set is gpr-closed.

PROOF : Let A be gp-closed in (X, τ) and $A \subset U$ where U is regular open. Since every regular open set is open and A is gp-closed, pcl $A \subset U$. Hence A is gpr-closed.

Converse of this theorem is not true and is proved by the following example.

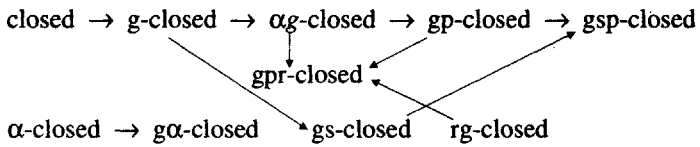
Example 3.5 — Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Let $A = \{a, b\}$. The only regular open set containing A is X and so A is gpr-closed $A \subset A$-open, but pcl $A = X \not\subset A$. Hence A is not gp-closed.

Remark 3.6 : gpr-closed sets and gsp-closed sets are independent of each other. It is shown by the following examples.

Example 3.7 — In Example 3.5, A is gpr-closed. But spcl $A = X \not\subset A$. So A is not gsp-closed.

Example 3.8 — Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $U = \{a, b\}$. U is gsp-closed since spcl $U = U$. But pcl $U = \{a, b, e\}$ which is not contained in the regular open set $\{a, b\}$. So U is not gpr-closed.

We have the following diagram,



Also, regular closed → preclosed → gpr-closed which easily follows from definitions. However,

gpr-closed ↛ preclosed ↛ regular closed.

Example 3.9 — Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Let $A = \{a, b\}$. Then A is gpr-closed but not preclosed in (X, τ). Consider $B = \{c\}$. Then B is preclosed but not regular closed.

Theorem 3.10 — If A is regular open and gpr-closed, then A is pre-closed and hence clopen.

PROOF : If A is regular open and gpr-closed, then pcl $A \subset A$. This implies A is preclosed. Hence A is clopen, since every preclosed (regular) open set is (regular) closed.

Remark 3.11 : Union of two gpr-closed sets need not be gpr-closed.

Example 3.12 — Consider Example 3.3. Let $A = \{a\}$ and $B = \{b\}$. Here A and B are gpr-closed but $A \cup B = \{a, b\}$ is not gpr-closed, since $\{a, b\}$ is regular open and pcl $\{a, b\} = \{a, b, e\}$.

Remark 3.13 : Intersection of two gpr-closed sets need not be gpr-closed.

Example 3.14 — Consider Example 3.5. Let $A = \{a, b\}$ and $B = \{a, c\}$. A and B are gpr-closed in (X, τ). But $A \cap B = \{a\}$ is not gpr-closed in (X, τ).

Theorem 3.15 — Let A be gpr-closed in (X, τ). Then pcl $A - A$ does not contain any non-empty regular closed set.

PROOF : Let F be a regular closed set such that $F \subset$ pcl $A - A$. Then $F \subset X - A$ implies $A \subset X - F$. A is gpr-closed and $X - F$ is regular open. Therefore pcl $A \subset X - F$. That is $F \subset X$-pcl A. Hence $F \subset$ pcl $A \cap (X$-pcl $A) = \phi$. This shows $F = \phi$.

Reverse implication does not hold.

Example 3.16 — If pcl $A - A$ contains no non-empty regular closed set, then A need not be gpr-closed. In Example 3.3, let $A = \{a, b\}$. Then pcl $A = \{a, b, e\}$. pcl $A - A = \{e\}$ which does not contain any non-empty regular closed set. Also A is not gpr-closed.

Corollary 3.17 — Let A be gpr-closed in (X, τ). Then A is pre-closed if and only if pcl $A - A$ is regular closed.

PROOF : *Necessity* : Let A be preclosed. Then pcl $A = A$ and so pcl $A - A = \phi$ which is regular closed.

Sufficiency : Suppose pcl $A - A$ is regular closed. Then pcl $A - A = \phi$ since A is gpr-closed. That is pcl $A = A$ or A is preclosed.

Definition 3.18 — Let (X, τ) be a topological space, $A \subset X$ and $x \in X$. x is said to be a pre-limit point of A if and only if every preopen set containing x contains a point of A different from x.

Definition 3.19 — Let (X, τ) be a topological space and $A \subset X$. The set of all pre-limit points of A is said to be the pre-derived set of A and is denoted by $D_P[A]$.

Theorem 3.20 — Let A and B be gpr-closed sets in (X, τ) such that $D[A] \subset D_P[A]$ and $D[B] \subset D_P[B]$. Then $A \cup B$ is gpr-closed.

PROOF : For any set $E \subset (X, \tau)$, $D_P[E] \subset D[E]$. Therefore $D_P[A] = D[A]$ and $D_P[B] = D[B]$. That is cl A = pcl A and cl B = pcl B.

Let $A \cup B \subset U$ where U is regular open. Then $A \subset U$ and $B \subset U$. Since A and B are gpr-closed pcl $A \subset U$ and pcl $B \subset U$. Now, cl $(A \cup B) =$ cl $A \cup$ cl $B =$ pcl $A \cup$ pcl $B \subset U$. But pcl $(A \cup B) \subset$ cl $(A \cup B)$. So pcl $(A \cup B) \subset U$ and hence $A \cup B$ is gpr-closed.

Theorem 3.21 — If A is gpr-closed and $A \subset B \subset$ pcl A, then B is gpr-closed.

PROOF : Let $B \subset U$ where U is regular open. Then $A \subset B$ implies $A \subset U$. Since A is gpr-closed pcl $A \subset U$. $B \subset$ pcl A implies pcl $B \subset$ pcl A. Thus pcl $B \subset U$ and shows that B is gpr-closed.

Recall that a space (X, τ) is called irreducible or hyperconnected if every open subset of X is dense. The following result characterizes hyperconnected spaces in terms of gpr-closed sets :

Theorem 3.22 — For a space (X, τ) the following conditions are equivalent :

(i) X is hyperconnected.

(ii) Every subset of X is gpr-closed and X is connected.

PROOF : (i) $\rightarrow$ (ii) : As mentioned in Jankovic[7], the only regular open subsets of X are trivial ones. Hence every subset of X is gpr-closed. On the other hand, every hyperconnected space is trivially connected.

(ii) $\rightarrow$ (i) : Let A be a non-void proper regular open subset of X. By (ii), A is gpr-closed. From Theorem 3.10, it follows that A is clopen. Contradiction, since X is connected.

4. GENERALIZED PREREGULAR OPEN SETS

Definition 4.1 — A set $A \subset X$ is called generalized preregular open if and only if its complement is generalized preregular closed.

Remark 4.2 : pcl $(X - A) = X -$ pint A

Theorem 4.3 — $A \subset X$ is gpr-open if and only if $F \subset$ pint A whenever F is regular closed and $F \subset A$.

PROOF : *Necessity* : Let A be gpr-open. Let F be regular closed and $F \subset A$. Then

$X - A \subset X - F$ where $X - F$ is regular open. Gpr-closedness of $X - A$ implies pcl $(X - A) \subset X - F$. By Remark 4.2, X-pint $A \subset X - F$. That is $F \subset$ pint A.

Sufficiency : Suppose F is regular closed and $F \subset A$ implies $F \subset$ pint A.

Let $X - A \subset U$ where U is regular open. Then $X - U \subset A$ where $X - U$ is regular closed. By hypothesis $X - U \subset$ pint A. That is X-pint $A \subset U$. By Remark 4.2, pcl $(X - A) \subset U$. This implies $X - A$ is gpr-closed and A is gpr-open.

Theorem 4.4 — If pint $A \subset B \subset A$ and A is gpr-open, then B is gpr-open.

PROOF : pint $A \subset B \subset A$ implies $X - A \subset X - B \subset X -$ pint A. That is, $X - A \subset X - B \subset$ pcl $(X - A)$ by Remark 4.2. Since $X - A$ is gpr-closed, by Theorem 3.21, $X - B$ is gpr-closed and B is gpr-open.

Remark 4.5 : For any $A \subset X$, pint (pcl $A - A$) = ϕ.

Theorem 4.6 — If $A \subset X$ is gpr-closed, then pcl $A - A$ is gpr-open.

PROOF : Let A be gpr-closed. Let F be a regular closed set such that $F \subset$ pcl $A - A$. Then by Theorem 3.15, $F = \phi$. So $F \subset$ pint (pcl $A - A$). This shows pcl $A - A$ is gpr-open.

The reverse implication does not hold.

Example 4.7 — Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{b\}$. Then pcl $A = \{b, c\}$ and (pcl A) $- A = \{c\}$ which is gpr-open in (X, τ). But A is not gpr-closed in (X, τ).

5. PREREGULAR $T_{1/2}$ SPACES

Definition 5.1 — A space (X, τ) is called a preregular $T_{1/2}$ space if every generalized preregular closed set is preclosed.

Example 5.2 — Any set with indiscrete topology is an example for a preregular $T_{1/2}$ space.

The notions preregular $T_{1/2}$ and $T_{1/2}$ are independent of each other.

Example 5.3 — Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\phi, X, \{a, b\}, \{c\}\}$. (X, τ) is $T_{1/2}$ but not preregular $T_{1/2}$ whereas (X, σ) is preregular $T_{1/2}$ but not $T_{1/2}$.

Recall first that a subset A of a space (X, τ) is called θ-semi-closed (Joseph and Kwack[10]) if θ-scl $A = A$, where θ-scl A is the θ-semi-closure of A, that is the set $\{x \in X : A \cap \text{cl } U \neq \phi$ for every semi-open set U containing $x\}$.

Theorem 5.4 — For a topological space (X, τ) the following conditions are equivalent :

(i) X is preregular $T_{1/2}$.

(ii) Every singleton of X is either regular closed or preopen.

(iii) Every singleton of X is either θ-semi-closed or semi-preopen.

PROOF : (i) $\rightarrow$ (ii) : Let $x \in X$ and assume that $\{x\}$ is not regular closed. Then clearly $X - \{x\}$ is not regular open and $X - \{x\}$ is trivially generalized preregular

closed. By (i), it is preclosed and thus $\{x\}$ is preopen.

(ii) $\rightarrow$ (i) : Let $A \subset X$ be generalized preregular closed. Let $x \in$ pcl A. We will show that $x \in A$. For, consider the following two cases :

Case (i) — The set $\{x\}$ is regular closed. Then, if $x \notin A$, there exists a regular closed set in pcl $A - A$. By Theorem 3.15, $x \in A$.

Case (ii) — The set $\{x\}$ is preopen. Since $x \in$ pcl A, then $\{x\} \cap$ pcl $A \neq \phi$. Thus $x \in A$.

So, in both cases, $x \in A$. This shows that pcl $A \subset A$ or equivalently A is preclosed.

(ii) $\rightarrow$ (iii) is obvious, since every regular closed set is θ-semi-closed and every preopen set is semi-preopen.

(iii) $\rightarrow$ (ii) If a singleton $\{x\}$ is θ-semi-closed, then by Lemma 2.3 from Noiri[17], it is the union of regular closed sets. Now, clearly $\{x\}$ is regular closed. Assume next that $\{x\}$ is semi-preopen. Then if it is not preopen, it must be nowhere dense[8] and hence α-closed. Since, semi-preopen, α-closed sets are even regular open, then $\{x\}$ is again preopen.

Remark 5.5 : The theorem above shows that the class of preregular $T_{1/2}$ spaces is rather pathological. Even a metric space need not be preregular $T_{1/2}$ as the real line with the usual topology shows.

Definition 5.6 (Janković[7]) — A space (X, τ) is called locally indiscrete (=partition space) if every closed set is regular closed.

Corollary 5.7 — Every locally indiscrete, semi-pre-$T_{1/2}$ space (X, τ) is preregular $T_{1/2}$.

PROOF : Since the space X is semi-pre-$T_{1/2}$, every singleton is closed or preopen[6]. Since X is locally indiscrete, then by Theorem 3.3 from Janković[7], all closed singletons are regular closed. By Theorem 5.4, the space X is preregular $T_{1/2}$.

Corollary 5.8 — Every preregular $T_{1/2}$ space is semi-pre $T_{1/2}$.

PROOF : Let (X, τ) be a preregular $T_{1/2}$ space. Then, by Theorem 5.4, every singleton of (X, τ) is closed or preopen. This implies (X, τ) is semi-pre $T_{1/2}$ (Dontchev[6]).

Converse does not hold.

Example 5.9 — Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then every singleton is closed or preopen. Therefore (X, τ) is semi-pre $T_{1/2}$ but not preregular $T_{1/2}$ since the gpr-closed set $\{a, b\}$ is not preclosed.

Our next theorem shows that the class of preregular $T_{1/2}$ spaces has already been under investigation.

Theorem 5.10 — For a topological space (X, τ) the following conditions are equivalent :

(i) X is preregular $T_{1/2}$.

(ii) Every nowhere dense singleton of X is regular closed.

(iii) The only nowhere dense subset of X is the void set.

(iv) Every subset of X is preopen.

(v) X is locally indiscrete.

PROOF : (i) $\rightarrow$ (ii) : By Theorem 5.4, every singleton is preopen or regular closed. Since non-void nowhere dense sets cannot be preopen at the same time, then every nowhere dense singleton of X is regular closed.

(ii) $\rightarrow$ (i) : Since every singleton is either preopen or nowhere dense[8], then by (ii) every singleton of X is either preopen or regular closed. Hence, in the notion of Theorem 5.4, X is preregular $T_{1/2}$.

(ii) $\rightarrow$ (iii) : Let $A \subset X$ be nowhere dense and non-void. Then, there exists a nowhere dense singleton in X, since the nowhere dense sets of every space form a subideal (that is, subsets of members of the family are members of the family). By (ii), this singleton is regular closed. If the nowhere dense (and hence a codense) singleton $\{x\}$ is regular closed, then $\{x\} = \text{cl}(\text{int}\{x\}) = \text{cl}(\phi) = \phi$. Contradiction.

(iii) $\rightarrow$ (iv) : By (iii) and the fact that singletons are either preopen or nowhere dense, it follows that every singleton of X is preopen. Since union of preopen sets is a preopen set, then every set in X is preopen.

(iv) $\rightarrow$ (i) : Follows from the definition of a preregular $T_{1/2}$ space, since by (iv) every set in X is preopen and hence preclosed.

(iii) $\leftrightarrow$ (v) : is proved in Theorem 3.3 in Janković[7].

GPRO (τ) denotes the collection of all generalized preregular open sets.

Theorem 5.11 — (i) $\text{PO}(\tau) \subset \text{GPRO}(\tau)$.

(ii) A space (X, τ) is preregular $T_{1/2}$ if and only if $\text{PO}(\tau) = \text{GPRO}(\tau)$.

PROOF : (i) Let A be preopen. Then $X - A$ is preclosed and so gpr-closed. This implies A is gpr-open. Hence $\text{PO}(\tau) \subset \text{GPRO}(\tau)$.

(ii) *Necessity* : Let (X, τ) be preregular $T_{1/2}$. Let $A \in \text{GPRO}(\tau)$. Then $X - A$ is gpr-closed. By hypothesis, $X - A$ is preclosed and thus $A \in \text{PO}(\tau)$. Hence $\text{GPRO}(\tau) = \text{PO}(\tau)$.

Sufficiency : Let $\text{PO}(\tau) = \text{GPRO}(\tau)$. Let A be gpr-closed. Then $X - A$ is gpr-open. Hence $X - A \in \text{PO}(\tau)$. Thus A is preclosed thereby implying (X, τ) is preregular $T_{1/2}$.

6. gpr-CONTINUOUS AND gpr-IRRESOLUTE FUNCTIONS

Definition 6.1 — A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called gpr-continuous if $f^{-1}(V)$ is gpr-closed in (X, τ) for every closed set V of (Y, σ).

Example 6.2 — Let $X = \{a, b\}$, $\tau = \{\phi, X, \{a\}, \{b\}\}$ and $Y = \{a, b, c\}$, $\sigma = \{\phi, X, \{a, b\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$ and $f(b) = c$. Since every subset of (X, τ) is gpr-closed, $f: (X, \tau) \rightarrow (Y, \sigma)$ is gpr-continuous.

Definition 6.3 — A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called gpr-irresolute if $f^{-1}(V)$ is gpr-closed in (X, τ) for every gpr-closed set V in (Y, σ).

Example 6.4 — Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$ and $\sigma = \{\phi, X, \{a, b\}, \{c\}\}$. Define $f: (X, \tau) \to (X, \sigma)$ by $f(a) = c$, $f(c) = a$ and $f(b) = b$. Then inverse image of every gpr-closed set is gpr-closed under f. Hence f is gpr-irresolute.

Remark 6.5 : Every gpr-irresolute function is gpr-continuous, but not conversely. The proof easily follows from definitions. Consider $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\phi, X, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \to (X, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is gpr-continuous but not gpr-irresolute.

Theorem 6.6 — (i) Let $f: (X, \tau) \to (Y, \sigma)$ be rg-continuous. Then f is gpr-continuous.

(ii) Let $f: (X, \tau) \to (Y, \sigma)$ be αg-continuous. Then f is gpr-continuous.

PROOF : Let V be closed in (Y, σ). Then $f^{-1}(V)$ is regular generalized closed in (X, τ) as f is rg-continuous. By Theorem 3.2, $f^{-1}(V)$ is gpr-closed. Hence f is gpr-continuous.

(ii) Let U be closed in (Y, σ). Then $f^{-1}(U)$ is αg-closed in (X, τ) as f is αg-continuous. Since every αg-closed set is gpr-closed we have $f^{-1}(U)$ is gpr-closed. Thus f is gpr-continuous.

The converse claims are not true in general as it can be seen through the following examples.

Example 6.7 — Consider $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, X, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Define $f: (X, \tau) \to (X, \tau)$ by $f(a) = e$, $f(b) = d$, $f(c) = c$, $f(d) = a$ and $f(e) = b$. Here, inverse image of every closed set in (X, τ) is gpr-closed in (X, τ). Therefore f is gpr-continuous. But it is not rg-continuous since the inverse image of the closed set $\{e\}$ is not rg-closed in (X, τ).

Example 6.8 — Consider $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\phi, X, \{c\}\}$. Define $g : (X, \tau) \to (X, \sigma)$ by $g(a) = b$, $g(b) = a$ and $g(c) = c$. Then g is gpr-continuous. $g^{-1}(\{a, b\}) = \{a, b\}$ is not αg-closed. So g is not αg-continuous.

Theorem 6.9 — Let $f : (X, \tau) \to (Y, \sigma)$ be regular irresolute and preclosed. Then for every gpr-closed set A of (X, τ), $f(A)$ is gpr-closed in (Y, σ).

PROOF : Let A be gpr-closed in (X, τ). Let $f(A) \subset U$, where U is regular open in Y. Then $A \subset f^{-1}(U)$. Since f is regular irresolute and A is gpr-closed, pcl $A \subset f^{-1}(U)$. That is, $f(\text{pcl } A) \subset U$. Now pcl $(f(A)) \subset$ pcl $(f(\text{pcl } A)) = f(\text{pcl } A) \subset U$, since f is preclosed. Hence $f(A)$ is gpr-closed in (Y, σ).

The composition of two gpr-continuous functions need not be gpr-continuous. For, consider the following example

Example 6.10 — Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\phi, X, \{c\}, \{b, c\}\}$ and $\eta = \{\phi, X, \{b, c\}\}$. Define $f : (X, \tau) \to (X, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$, Define $g : (X, \sigma) \to (X, \eta)$ by $g(a) = b$, $g(b) = a$ and $g(c) = c$. Then f and g are gpr-continuous. $\{a\}$ is closed in (X, η). $(g \circ f)^{-1}(\{a\}) = f^{-1}(g^{-1}(\{a\}))$ $= f^{-1}(\{b\}) = \{a\}$ which is not gpr-closed in (X, τ). Hence $g \circ f$ is not gpr-continuous.

Theorem 6.11 — Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions. Then

(i) $g \circ f$ is gpr-continuous, if g is continuous and f is gpr-continuous.

(ii) $g \circ f$ is gpr-irresolute, if g is gpr-irresolute and f is gpr-irresolute.

(iii) $g \circ f$ is gpr-continuous, if g is gpr-continuous and f is gpr-irresolute.

PROOF : (i). Let V be closed in (Z, η). Then $g^{-1}(V)$ is closed in (Y, σ), since g is continuous. gpr-continuity of f implies $f^{-1}(g^{-1}(V))$ is gpr-closed in (X, τ). That is, $(g \circ f)^{-1}(V)$ is gpr-closed in (X, τ). Hence $g \circ f$ is gpr-continuous.

(ii) Let V be gpr-closed in (Z, η). Since g is gpr-irresolute, $g^{-1}(V)$ is gpr-closed in (Y, σ). As f is gpr-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpr-closed in (X, τ). Therefore $g \circ f$ is gpr-irresolute.

(iii) Let V be closed in (Z, η). Since g is gpr-continuous, $g^{-1}(V)$ is gpr-closed in (Y, σ). As f is gpr-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is gpr-closed in (X, τ). Therefore $g \circ f$ is gpr-continuous.

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