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ON SPECIAL TYPES OF SEMIGROUP COMPACTIFICATIONS

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Some special types of congruences of a topological semigroup S are obtained. The idea of these congruences are applied to get some special types of semigroup compactifications of S. Moreover, the results can be used to study 'lattices' of semigroup compactifications.

1. INTRODUCTION AND DEFINITION

By definition[1], a topological semigroup S is a Hausdorff space with continuous associative multiplication $(x, y) \mapsto xy : S \times S \to S$.

Associated with each topological semigroup S, there is a compact semigroup called the Bohr compactification of S which is universal over the compact semigroups containing dense continuous homomorphic images of S.

Definition 1 — Bohr compactification : If S is a topological semigroup, then Bohr compactification of S is a pair (β, B) such that B is a compact semigroup, $\beta : S \to B$ is a continuous homomorphism with $B = \overline{\beta(S)}$ and if $g : S \to T$ is a continuous homomorphism of S into compact semigroup T, then there exists a unique continuous homomorphism $f : B \to T$ such that

$$f\beta = g. \qquad \ldots (1)$$

Definition 2 — By a semigroup compactification (g, T) of a topological semigroup S, we mean T is a compact semigroup and $g : S \to T$ is a dense continuous homomorphism of S into T (here g is dense means $g(S)$ is dense in T) (Kripalini[4]).

The significant observation made was that the semigroup compactifications of S are precisely the quotients of the Bohr compactification B of S under closed congruence[4]. Also it is known[4] that $K_1(S)$ family of all semigroup compactifications of S is a complete lattice, under the ordering $(\alpha, A) \geq (\gamma, C)$ if there exists a continuous surmorphism $f : A \to C$ such that $f\alpha = \gamma$.

In this paper, we describe some special types of congruences and prove that :

(i) A topological semigroup S with Bohr compactification (β, B) has a semigroup compactification (α, A) determined by n disjoint closed proper weak ideals (ideals) of B, at least one of which is non-degenerate only if S has a semigroup compactification strictly bigger than (α, A).

(ii) A topological semigroup S with Bohr compactification (β, B) has an n-point compactification determined by n non-empty subsets of B does not imply that it has an $(n-1)$-point compactification, nor does it imply that there is a semigroup compactification strictly bigger than (α, A) different from (β, B).

(iii) If a topological semigroup S with (β, B) has an n-point compactification (α, A) determined by n non-empty weak ideals (ideals) of B, then there exists semigroup compactification strictly bigger than (α, A), but it does not imply that S has an $(n-1)$-point compactification.

2. Special Types of Congruences

In this section we introduce weak ideals, joint weak ideals and complementary joint ideals of a semigroup S and describe special types of congruences on S.

For a semigroup S, we give $S \times S$ the coordinate multiplication and let Δ denote the diagonal of $S \times S$.

Definition 2.1 — We say that a subset $\omega \subseteq S$ is a weak ideal of S provided that $(\omega \times \omega) \bigcup \Delta$ is an ideal of $S \times S$.

Definition 2.2 — A family Ω of disjoint non-empty subsets of a semigroups S is said to be a family of joint [weak] ideals provided that

$$\bigcup_{\omega \in \Omega} (\omega \times \omega) \left[\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta \right] \text{ is an ideal of } S \times S.$$

Definition 2.3 — A family Ω of joint [weak] ideals of a semigroup S is said to be a family of complementary joint (weak) ideals provided that $S = \bigcup_{\omega \in \Omega} \omega$.

Lemma 2.4 — Let S be a topological semigroup and Ω be a finite family of disjoint non-empty subsets of S. Then $\bigcup_{\omega \in \Omega} (\omega \times \omega) \left[\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta \right]$ is a closed congruence on S if and only if Ω is a family of closed complementary joint [joint weak] ideals of S, at least one of which is non-degenerate.

Proof : Suppose S has a closed congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega) \left[\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta \right]$, where Ω be a finite family of disjoint non-empty subsets of S.

Assume that $| \Omega | = n$

i.e., $\Omega = \{\omega_i\}_{i = 1, 2, \ldots, n}$ where ω_i's are non-empty subsets of S.

i.e., $$\bigcup_{\omega \in \Omega} (\omega \times \omega) \left[\bigcup_{\omega \in \Omega} (\omega \times \omega) \cup \Delta \right]$$

$$= \bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right] = R.$$

Then

(1) ω_i's are proper subsets of S, at least one of which is non-degenerate.

(2) ω_i's are complementary.

For all $a \in S$

$$(a, a) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i) \quad [\because R \text{ is a congruence}]$$

i.e., a is an element of any one of ω_i.

i.e., $a \in \bigcup_{i=1}^{n} \omega_i$, for all $a \in S$

i.e., $S = \bigcup_{i=1}^{n} \omega_i$.

(3) ω_i's are joint [weak] ideals.

For all both $a, b \in \bigcup_{i=1}^{n} \omega_i \left[\bigcup_{i=1}^{n} \omega_i \text{ or for } a = b \right]$

and for all $x \in S$ both

$$(ax, bx), (xa, xb) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$$

$(\because R \text{ is compatible})$

i.e., $\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$ is an ideal of $S \times S$.

(4) ω_i's are closed.

Consider ω_1. If ω_1 is degenerate, then clearly it is closed. If ω_1 is non-degenerate, we proceed as follows :

Let (x_α) be a net in ω_1, $(x_\alpha) \to x \in S$. Since ω_1 is non-degenerate, there exists $y (\neq x) \in \omega_1$, $y \notin \omega_i$, $i = 2; \ldots, n$.

$\therefore$ (x_α, y) be a net in $\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$ which is closed.

$\therefore$ The limit (x, y) of (x_α, y) belongs to

$$\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$$

i.e., $(x, y) \in \omega_1 \times \omega_1$ ($\because$ $y \in \omega_1$)

$\therefore$ $x \in \omega_1$. Thus ω_1 is closed.

Similarly ω_i, $i = 1, 2, \ldots, n$ are closed. Hence the result.

On the other hand, assume that a finite family Ω of disjoint non-empty subsets of a topological semigroup S are closed complementary joint (weak) ideals of S at least one of which is non-degenerate.

Claim : $\bigcup_{\omega \in \Omega} (\omega \times \omega) \left[\bigcup_{\omega \in \Omega} (\omega \times \omega) \cup \Delta \right]$ is a closed congruence on S.

Assume that $|\Omega| = n$

i.e., $\Omega = \{\omega_i\}_{i = 1, 2, \ldots, n}$ at least one of which is non-degenerate.

Since ω_i's are disjoint non-empty closed complementary joint [weak] ideals of S at least one of which is non-degenerate, then

$$R = \bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$$

is a closed subset of $S \times S$. ($\because$ ω_i's are closed in S and Δ is closed in $S \times S$).

R is an equivalence. For all $a \in S$, a is an element of any one of $\{\omega_i\}_{i = 1, 2, \ldots, n}$ ($\because$ $\bigcup_{i=1}^{n} \omega_i = S$ and $\bigcup_{i=1}^{n} \omega_i = \phi$).

$\therefore$ $$(a, a) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i)$$

i.e., $$\Delta \subset \bigcup_{i=1}^{n} (\omega_i \times \omega_i).$$

Hence R is reflexive. Clearly R is symmetric.

Again R is transitive.

For, let $(a, b), (b, c) \in \bigcup_{i \in 1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i \in 1}^{n} (\omega_i \times \omega_i) \cup \Delta \right]$.

i.e., both $a, b \in \bigcup_{i=1}^{n} \omega_i$ and both $b, c \in \bigcup_{i=1}^{n} \omega_i$

$$[\text{both } a, b \in \bigcup_{i=1}^{n} \omega_i \text{ or } a = b \text{ and both } b, c \in \bigcup_{i=1}^{n} \omega_i \text{ or } b = c].$$

Since ω_i's are disjoint, the possible cases are

both $\{a, b\}, \{b, c\} \in$ any one of ω_i's

then, $\{a, c\} \in$ any one of ω_i's.

$$\therefore \quad (a, c) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i).$$

R is compatible. Since ω_i's are joint (weak) ideals, by the definition of joint (weak) ideals

$$\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \bigcup \Delta \right] \text{ is an ideal of } S \times S.$$

i.e., for all $x \in S$ and for all $(a, b) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i)$

We have both

$$(ax, bx), (xa, xb) \in \bigcup_{i=1}^{n} (\omega_i \times \omega_i) \left[\bigcup_{i=1}^{n} (\omega_i \times \omega_i) \bigcup \Delta \right].$$

Hence the result.

Corollary 2.5 — A topological semigroup S has a closed congruence of the form $(\omega \times \omega) \bigcup \Delta$ if and only if ω is a closed non-degenerate weak ideal of S.

Corollary 2.6 — Let S be a topological semigroup with Bohr compactification (β, B). If S has a semigroup compactification determined by ω if and only if ω is a closed non-degenerate weak ideal of B.

Corollary 2.7 — Let S be a topological semigroup with Bohr compactification (β, B). If S has an n-point compactification if and only if its Bohr compactification has 'n' disjoint closed complementary joint ideals, at least one of which is non-degenerate.

Remark 2.8 : A semigroup S has a finite family Ω of joint weak ideals does not imply that any of the family is a weak ideal, nor does it imply that a proper subset of Ω forms joint weak ideals.

Example — $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ with multiplication modulo 8 is a semigroup

(1) $\{\omega_1, \omega_2\} = \{\{1, 5\}, \{3, 7\}\}$ is a set of joint weak ideals, but neither $\{1, 5\}$ nor $\{3, 7\}$ is a weak ideal of Z_8.

(2) $\{\omega_1, \omega_2, \omega_3\} = \{\{1, 7\}, \{2, 6\}, \{3, 5\}\}$ is a set of joint weak ideals, but $\{\{1, 7\}, \{3, 5\}\}$ is not a set of joint weak ideals. Also $\{1, 7\}, \{3, 5\}$ are not weak ideals.

Result 2.9 — A semigroup S has a congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$ does not imply

(1) $(\omega_i \times \omega_i) \bigcup \Delta$ is a congruence on S for any $\omega_i \in \Omega$

(2) $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ is a congruence on S for some proper subset $\{\omega_j\}$ of Ω.

Remark 2.10 : A semigroup S has a congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$ with ω's are weak ideals (ideals), then S has a congruence of the form $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ where $\{\omega_j\}$'s are any proper subset of Ω, contained in $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$.

Result 2.11 — Let S be a topological semigroup with closed congruence $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$ where ω_i's are either ideals or weak ideals. Then S has a closed congruence of the form $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ contained in

$$\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta \text{ where } \{\omega_j\} \text{ is any proper subset of } \Omega.$$

PROOF : Given $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$ is a closed congruence with ω's are weak ideals (ideals).

i.e., ω's are closed disjoint proper weak ideals, at least one of which is non-degenerate.

i.e., $(\omega_j \times \omega_j) \bigcup \Delta$ is a closed congruence for each $\omega_j \in \Omega$

i.e., $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ where $\{\omega_j\}$ is any proper subset of Ω is a congruence contained in $\bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta$ (Result 2.9).

Also $R = \bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ is closed, since ω's are closed in S and Δ is closed in $S \times S$.

We obtained the following theorem about the lattice $K_1(S)$ of family of semigroup compactification of a given topological semigroup S with Bohr compactification (β, B).

Theorem 2.12 — Let S be a topological semigroup with Bohr compactification (β, B). If S has a semigroup compactification (α, A) determined by a finite family

Ω of disjoint closed proper weak ideals (ideals) of B, at least one of which is non-degenerate, then there is a semigroup compactification in $K_1(S)$ strictly bigger than (α, A).

PROOF : Since Ω is a finite family of disjoint closed proper weak ideals (ideals) of B, at least one of which is non-degenerate, then B has a closed congruence of the form

$$R = \bigcup_{\omega \in \Omega} (\omega \times \omega) \bigcup \Delta.$$

Let (α, A) denote the semigroup compactification determined by R.

i.e., $(\beta, B) > (\alpha, A) \in K_1(S)$.

Again since ω's are weak ideals (ideals) for each $\omega \in \Omega$, by Remark 2.10, B has a closed congruence of the form

$$R' = \bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$$

where $\{\omega_j\}$'s are proper subset of Ω and R' is contained in R.

Let (α_1, A_1) denotes semigroup compactification determined by R' and $(\beta, B) > (\alpha_1, A_1) > (\alpha, A)$.

Hence the result.

Remark 2.13 : Theorem 2.12 need not be true, if (α, A) is determined by a finite family Ω of disjoint closed proper weak ideals, at least one of which is non-degenerate.

For example, let S be a topological semigroup with (β, B), where $B = Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ with discrete topology and multiplication modulo 8.

(1) Here $\{\omega_1, \omega_2, \omega_3\} = \{\{1, 7\}, \{2, 6\}, \{3, 5\}\}$ set of joint weak ideals and Z_8 has a congruence

$$\{1, 7\} \times \{1, 7\} \bigcup \{2, 6\} \times \{2, 6\} \bigcup \{3, 5\} \times \{3, 5\} \bigcup \Delta$$

but $\{1, 7\} \times \{1, 7\} \bigcup \Delta$ and $\{3, 5\} \times \{3, 5\} \bigcup \Delta$ are not congruences.

(2) $\{\{1, 5\}, \{3, 7\}\}$ set of joint weak ideals and $\{1, 5\} \times \{1, 5\} \bigcup \{3, 7\} \times \{3, 7\} \bigcup \Delta$ is a congruence but $\{1, 5\} \times \{1, 5\} \bigcup \Delta$ and $\{3, 7\} \times \{3, 7\} \bigcup \Delta$ are not congruences.

Remark 2.14 : A semigroup S has a finite family Ω of complementary joint ideals does not imply that any of the ω's is a weak ideal, nor does it imply that a proper subset of Ω forms joint weak ideals, or complementary joint ideals.

Example — (1) Let $S = \{e, a, f, b\}$ with multiplication defined below is a semigroup

	e	a	f	b
e	e	a	f	b
a	a	e	b	f
f	f	b	f	b
b	b	f	b	f

Here $\{\omega_1, \omega_2\} = \{\{e, f\}, \{a, b\}\}$ is a set of complementary joint ideals, but neither $\{e, f\}$ nor $\{a, b\}$ is a weak ideal.

(2) $Z_6 = \{0, 1, 2, 3, 4, 5\}$ with multiplication modulo 6 is a semigroup.

Here $\{\omega_1, \omega_2, \omega_3\} = \{\{2, 5\}, \{1, 4\}, \{0, 3\}\}$ is a set of complementary joint ideals but $\{\{2, 5\}, \{1, 4\}\}$ are not sets of joint weak ideals, nor complementary joint ideals.

From the above remark, we have the following result.

Result 2.15 — A semigroup S has a congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega)$ does not imply

(1) $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ is a congruence on S, where $\{\omega_j\}$'s are any proper subset of Ω.

(2) $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j)$ is a congruence on S, where $\{\omega_j\}$'s are any proper subset of Ω.

Theorem 2.16 — A semigroup S has a non-trivial congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega)$ with ω's are weak ideals (ideals) then S has a congruence of the form $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$, where $\{\omega_j\}$'s are any proper subset of Ω. But $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j)$ is not a congruence.

PROOF : Given $\bigcup_{\omega \in \Omega} (\omega \times \omega)$ is a non-trivial congruence with ω's are weak ideals (ideals).

i.e., ω's are disjoint proper weak ideals at least one of which is non-degenerate.

$\therefore$ $(\omega \times \omega) \bigcup \Delta$ is a congruence for each $\omega \in \Omega$

i.e., $(\omega_j \times \omega_j) \bigcup \Delta$ is a congruence for each $\omega_j \in \Omega$

$\therefore$ $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ is a conguence contained in $\bigcup_{\omega \in \Omega} (\omega \times \omega)$.

But

$\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j)$ is not a congruence, since it is not reflexive.

Theorem 2.17 — Let S be a topological semigroup with closed congruence $\bigcup_{\omega \in \Omega} (\omega \times \omega)$, where ω's are weak ideals (ideals), then S has a closed congruence of the form $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \bigcup \Delta$ contained in $\bigcup_{\omega \in \Omega} (\omega \times \omega)$, where $\{\omega_j\}$'s are any proper subset of Ω.

PROOF : This is immediate from Lemma 2.4 and Theorem 2.16.

Result 2.18 — A topological semigroup S with (β, B) has an n-point compactification does not imply that it has an $(n-1)$-point compactification, nor does it imply that there is a semigroup compactification strictly bigger than (α, A) and different from (β, B).

For example, let S be a topological semigroup with (β, B) where $B = \{e, a, f, b\}$ with discrete topology and multiplication defined below :

	e	a	f	b
e	e	a	f	b
a	a	e	b	f
f	f	b	f	b
b	b	f	b	f

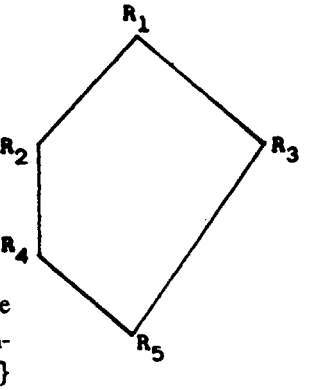
Closed congruences on B are

$R_1 = \Delta$, $R_2 = (\{f, b\} \times \{f, b\}) \bigcup \Delta$

$R_3 = (\{e, f\} \times \{e, f\}) \bigcup (\{a, b\} \times \{a, b\})$

$R_4 = (\{e, a\} \times \{e, a\}) \bigcup (\{f, b\} \times \{f, b\})$

$R_5 = B \times B$.



R_3 determines a 2-point compactification say (α, A), where $\{\{e, f\}, \{a, b\}\}$ is a set of closed proper disjoint non-degenerate complementary joint ideals but $\{e, f\} \times \{e, f\}$ and $\{a, b\} \times \{a, b\}$ are not closed congruences on B.

Therefore two point compactification does not imply the existence of one-point compactification. Also,

$$\{e, f\} \times \{e, f\} \cup \Delta \text{ and } \{a, b\} \times \{a, b\} \cup \Delta$$

are not closed congruences contained in R_3. So 2-point compactification does not imply there exist a semigroup compactification strictly bigger than (α, A) and different from (β, B).

Next theorem shows that if a topological semigroup S with (β, B) has an n-point compactification (α, A) determined by 'n' weak ideals (ideals) of B, then there exists semigroup compactification strictly bigger than (α, A). And in this case also it does not imply that S has an $(n-1)$-point compactification.

Theorem 2.19 — A topological semigroup S with (β, B) has an n-point compactification (α, A) determined by n weak ideals (ideals) of B, then there exists semigroup compactification strictly bigger than (α, A). And in this case also it does not imply that S has an $(n-1)$-point compactification.

PROOF : Since (α, A) is an n-point compactification of S, (α, A) is determined by a closed congruence of the form $\bigcup_{\omega \in \Omega} (\omega \times \omega)$, where ω's are closed complementary joint ideals of B, at least one of which is non-degenerate and $|\Omega| = n$. Also given that ω's are weak ideals (ideals) : i.e. ω's are closed disjoint proper weak ideals (ideals) of B, at least one of which is non-degenerate.

By Lemma 2.4, B has a closed congruence of the form $\bigcup_{\omega_j \in \Omega} (\omega_j \times \omega_j) \cup \Delta$, where $\{\omega_j\}$'s are any propet subset of Ω.

Also it determines a semigroup compactification (α_1, A_1) such that $(\beta, B) \geq (\alpha_1, A_1) > (\alpha, A)$. i.e., there is a semigroup compactification strictly bigger than (α, A).

But it does not imply that S has an $(n-1)$-point compactification.

For example, let S be a topological semigroup with (β, B), where $B = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, 1\right\}$ with discrete topology and multiplication defined by $xy = \max\left\{\frac{1}{2}, xy\right\}$. Here S has a 2-point compactification determined by

$$R = \left\{\frac{1}{2}, 1\right\} \times \left\{\frac{1}{2}, 1\right\} \cup \left\{\frac{1}{3}, \frac{1}{4}\right\} \times \left\{\frac{1}{3}, \frac{1}{4}\right\}$$

where $\left\{\left\{\frac{1}{2}, 1\right\}, \left\{\frac{1}{3}, \frac{1}{4}\right\}\right\}$ is a set of disjoint proper closed non-singleton complementary joint ideals.

Also $\left\{\frac{1}{2}, 1\right\}$ and $\left\{\frac{1}{3}, \frac{1}{4}\right\}$ are weak ideals.

But $\left\{\frac{1}{2}, 1\right\} \times \left\{\frac{1}{2}, 1\right\}$ and $\left\{\frac{1}{3}, \frac{1}{4}\right\} \times \left\{\frac{1}{3}, \frac{1}{4}\right\}$

are not congruences.

Therefore 2-point compactifications determined by weak ideals does not imply existence of one-point compactification.

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