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SURFACE WAVE PROPAGATION IN ANISOTROPIC ELASTIC LAYER SANDWICHED BETWEEN A UNIFORM LAYER OF LIQUID AND HETEROGENEOUS SOLID ELASTIC HALF-SPACE

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The problem of surface wave propagation in a stratum of homogeneous transversely isotropic elastic solid overlying a heterogeneous isotropic elastic solid half-space and beneath a uniform layer of homogeneous liquid is discussed. The frequency equation in the form of eighth-order determinant is obtained. The problem of Gogna[7] has been reduced as a special case by removing inhomogeneity factor. The effect of heterogeneity on the dispersion curves for the phase velocity for a particular model is observed and depicted graphically for fundamental and first modes only.

INTRODUCTION

Surface waves dispersion analysis is concerned with the relationship between phase/group velocity and the wave number. Since the velocities of elastic waves depend on the material moduli through which they travel. Though the assumption of isotropy and homogeneity is often approximately satisfied in practice but certain disagreements exist between theory and observation, indicating the necessity of discussing the problem of dispersion under more realistic assumptions.

Many investigators Stoneley[13], Biot[4], Tolstoy[14] studied the propagation of elastic waves in a system consisting of a liquid layer of finite depth lying over an isotropic homogeneous half-space. Stoneley[13] considered the case of oceanic layering. Abubaker and Hudson[1], Anderson[2,3] studied the dispersion of surface waves by taking the anisotropy of the medium into account. Gogna[7] and Sharma *et al.*[12] incorporated the anisotropy into their studies of surface wave propagation.

Practically speaking the earth structure is not homogeneous isotropic throughout. Reasonable grounds are there for the assumption of existence of anisotropy and heterogeneity in the continents. Since the material deposited in water are settled in

specific directions. We have, therefore, made an attempt to study the problem (two-dimensional) of surface wave propagation in a homogeneous anisotropic layer lying over a heterogeneous semi-infinite isotropic half-space and under a uniform layer of homogeneous liquid.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

A medium H_1 consisting of a liquid layer of thickness h, density ρ_1 and Bulk modulus λ_1, lying over a homogeneous anisotropic solid layer H_2 of thickness h', having density ρ_2 is considered. Below H_2, the region is occupied by an isotropic inhomogeneous half-space H with elastic parameters λ, μ. The geometry of the problem is shown in Fig. 1. We consider two-dimensional problem in x-z plane so that wave front is parallel to y-z plane and displacement components parallel to x and z axis are independent of y.

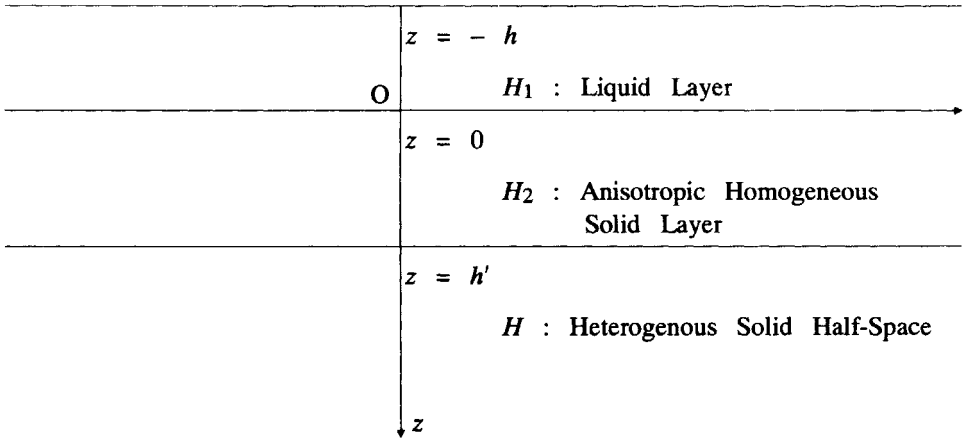

FIG. 1. Geometry of the problem.

DISPLACEMENTS AND STRESSES

For the medium H_1, the equation of motion is given by

$$\nabla^2 \Phi = \frac{1}{c_1^2} \frac{\partial^2 \Phi}{\partial t^2} \qquad \ldots (1)$$

where Φ is the displacement potential and ∇^2 is two-dimensional Laplacian operator defined by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

and $c_1^2 = \dfrac{\lambda_1}{\rho_1}$ is the velocity of dilatational wave in liquid.

We assume the solution of (1) in the form

$$\Phi = \Phi_1(z) \exp\{ik(x - ct)\}. \quad \ldots (2)$$

Equations (1) and (2) yield

$$\frac{d^2\Phi_1(z)}{dz^2} + k_1^2 \Phi_1(z) = 0 \quad \ldots (3)$$

where

$$k_1^2 = k^2\left(\frac{c^2}{c_1^2} - 1\right), \quad \ldots (4)$$

k is the wave number and c the phase velocity.
Solution of (3) is given by

$$\Phi_1(z) = A_1 \cos(k_1 z) + B_1 \sin(k_1 z) \quad \ldots (5)$$

where A_1 and B_1 are arbitrary constants.

Thus solution of (1) is given by

$$\Phi = [A_1 \cos(k_1 z) + B_1 \sin(k_1 z)] \exp\{ik(x - ct)\}. \quad \ldots (6)$$

The displacements and pressure in this medium are given by[5]

$$u_1 = \frac{\partial\Phi}{\partial x}, \quad w_1 = \frac{\partial\Phi}{\partial z}, \quad \ldots (7)$$

$$p = -\rho_1 \frac{\partial^2\Phi}{\partial t^2}. \quad \ldots (8)$$

In medium H_2, the strain energy volume density function for two-dimensional problem in x-z plane reduces to (Love[11])

$$2W_2^* = A_2 e_{xx}^2 + C_2 e_{zz}^2 + 2F_2 e_{xx} e_{zz} + L_2 e_{zx}^2 \quad \ldots (9)$$

where for W_2^* to be positive definite $A_2 > 0$, $L_2 > 0$, $C_2 > 0$ and $A_2C_2 - F_2^2 > 0$. The components of stress can be obtained from (9) by using the formula

$$\tau_{ij} = \frac{\partial W_2^*}{\partial e_{ij}} \qquad (i, j = x, z) \quad \ldots (10)$$

Thus (10) gives

$$\left.\begin{aligned} \tau_{zz} &= F_2 \frac{\partial u_2}{\partial x} + C_2 \frac{\partial w_2}{\partial z} \\ \tau_{zx} &= L_2 \left(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x} \right) \end{aligned}\right\} \quad \ldots (11)$$

where u_2 and w_2 are the displacement components.

The equations of motion in the absence of body forces are

$$A_2 \frac{\partial^2 u_2}{\partial x^2} + L_2 \frac{\partial^2 u_2}{\partial z^2} + (F_2 + L_2) \frac{\partial^2 w_2}{\partial x \, \partial z} = \rho_2 \frac{\partial^2 u_2}{\partial t^2} \quad \ldots (12)$$

and

$$L_2 \frac{\partial^2 w_2}{\partial x^2} + C_2 \frac{\partial^2 w_2}{\partial z^2} + (F_2 + L_2) \frac{\partial^2 u_2}{\partial x \, \partial z} = \rho_2 \frac{\partial^2 w_2}{\partial t^2} . \quad \ldots (13)$$

We seek the solution of (12) and (13) of the form

$$(u_2, w_2) = [U(z), W(z)] \exp \{ik(x - ct)\} \quad \ldots (14)$$

where c and k are defined earlier.

The displacement components given by (14) satisfy (12) and (13) if

$$-A_2 k^2 U(z) + L_2 U''(z) + ik (F_2 + L_2) W'(z) = -\rho_2 k^2 c^2 U(z) \quad \ldots (15)$$

$$-L_2 k^2 W(z) + C_2 W''(z) + ik (F_2 + L_2) U'(z) = -\rho_2 k^2 c^2 W(z) \quad \ldots (16)$$

where primes denote differentiation with respect to z.

We assume

$$\left.\begin{aligned} U(z) &= U \exp (mz) \\ W(z) &= iW \exp (mz) \end{aligned}\right\} \quad \ldots (17)$$

where U and W are arbitrary constants.

Using (17), eqns. (15) and (16) reduce to

$$-A_2 k^2 U + L_2 m^2 U - km (F_2 + L_2) W = -\rho_2 k^2 c^2 U \quad \ldots (18)$$

$$-L_2 k^2 W + C_2 m^2 W + k (F_2 + L_2) mU = -\rho_2 k^2 c^2 W. \quad \ldots (19)$$

These equations given us the ratio $U : W$ which makes (14) a solution of the equation of motion. For the existence of non-zero solution of (18) and (19), we must have

$$(L_2 m^2 - A_2 k^2 + \rho_2 k^2 c^2) (C_2 m^2 - L_2 k^2 + \rho_2 k^2 c^2) + k^2 (F_2 + L_2) m^2 = 0. \quad \ldots (20)$$

This equation is quadratic in m^2. Two values of m^2 are given by

$$m_i^2 = \frac{k^2(-G \pm \sqrt{G^2 - 4L_2 C_2 H})}{2L_2 C_2} \qquad (i = 1, 2) \qquad \ldots (21)$$

where

$$\left.\begin{aligned} G &= (F_2 + L_2)^2 - (A_2 - \rho_2 c^2) C_2 - L_2 (L_2 - \rho_2 c^2) \\ H &= (A_2 - \rho_2 c^2)(L_2 - \rho_2 c^2). \end{aligned}\right\} \qquad \ldots (22)$$

The ratio of displacement components U_i, W_i corresponding to these m_i's is given as

$$\frac{U_i}{W_i} = \frac{k m_i (F_2 + L_2)}{L_2 m_i^2 - A_2 k^2 + \rho_2 k^2 c^2} = \frac{C_2 m_i^2 - L_2 k^2 + \rho_2 k^2 c^2}{k(F_2 + L_2) m_i} = \frac{1}{\varepsilon_i} \text{ (say).} \qquad \ldots (23)$$

Thus our solutions can be written as

$$\begin{aligned} u_2 &= [U_1 \sinh(m_1 z) + U_2 \cosh(m_1 z) + U_3 \sinh(m_2 z) \\ &\qquad + U_4 \cosh(m_2 z)] \exp\{ik(x - ct)\}, \\ w_2 &= [i\varepsilon_1 U_1 \cosh(m_1 z) + i\varepsilon_1 U_2 \sinh(m_1 z) \\ &\quad + i\varepsilon_2 U_3 \cosh(m_2 z) + i\varepsilon_2 U_4 \sinh(m_2 z)] \exp\{ik(x - ct)\}. \end{aligned} \qquad \ldots (24)$$

Also (11) gives the stresses in the form

$$\begin{aligned} \tau_{zz} &= i[U_1(\varepsilon_1 m_1 C_2 + kF_2) \sinh(m_1 z) + U_2(\varepsilon_1 m_1 C_2 + kF_2) \cosh(m_1 z) \\ &\qquad + U_3(\varepsilon_2 m_2 C_2 + kF_2) \sinh(m_2 z) \\ &\qquad + U_4(\varepsilon_2 m_2 C_2 + kF_2) \cosh(m_2 z)] \exp\{ik(x - ct)\}, \\ \tau_{zx} &= L_2[(m_1 - \varepsilon_1 k) U_1 \cosh(m_1 z) + (m_1 - \varepsilon_1 k) U_2 \sinh(m_1 z) \\ &\qquad + (m_2 - \varepsilon_2 k) U_3 \cosh(m_2 z) \\ &\qquad + (m_2 - \varepsilon_2 k) U_4 \sinh(m_2 z)] \exp\{ik(x - ct)\}. \end{aligned} \qquad \ldots (25)$$

In the half-space H, we have considered the quadratic inhomogeneity varying with the depth, the elastic parameters for which are given by

$$\left.\begin{aligned} \lambda &= \lambda_0 (1 + bz)^2 \\ \mu &= \mu_0 (1 + bz)^2 \end{aligned}\right\} \qquad \ldots (26)$$

where λ_0, μ_0 are constants. Let ρ denotes the density of the medium. The longitudinal and shear wave velocities are given by

$$\alpha^2 = \alpha_0^2 (1 + bz)^2,$$

$$\beta^2 = \beta_0^2 (1 + bz)^2, \text{ respectively}$$

where

$$\alpha_0^2 = \frac{\lambda_0 + 2\mu_0}{\rho},$$

$$\beta_0^2 = \frac{\mu_0}{\rho}.$$

The equation of motion in x-z plane is given by Hook[10], and Gupta[8,9]

$$\gamma \nabla (\mu \nabla \cdot \vec{S}) - \nabla \times (\mu \nabla \times \vec{S}) + 2\mu' \left[\frac{\partial \vec{S}}{\partial z} - \hat{i}_z \times (\nabla \times \vec{S}) \right] = \rho \frac{\partial^2 \vec{S}}{\partial t^2} \quad \ldots (27)$$

where

$$\left.\begin{aligned} \gamma &= \frac{\lambda + 2\mu}{\mu} \\ \mu' &= \frac{\partial \mu}{\partial z}, \text{ and the displacement vector} \\ \vec{S} &= \mu \nabla (\mu^{-1} \phi) + \mu^{-1} \nabla \times (\mu \vec{\psi}) \end{aligned}\right\} \quad \ldots (28)$$

where $\phi, \vec{\psi}$ are scalar and vector potentials respectively.

The wave equations for compressional and shear wave velocities with harmonic time dependence are given by

$$\nabla^2 \phi + \alpha_0^{-2} \omega^2 (1 + bz)^{-2} \phi = 0 \quad \ldots (29)$$

$$\nabla^2 \psi + \beta_0^{-2} \omega^2 (1 + bz)^{-2} \psi = 0; \ \omega = kc. \quad \ldots (30)$$

Solutions of (29) and (30) are given by

$$\phi = P_3 e^{-rz} \exp \{ik (x - ct)\} \quad \ldots (31)$$

$$\psi = Q_3 e^{-sz} \exp \{ik (x - ct)\} \quad \ldots (32)$$

where

$$r = k \sqrt{1 - c^2/\alpha_0^2 (1 + bz)^2}$$

$$s = k \sqrt{1 - c^2/\beta_0^2 (1 + bz)^2}.$$

The components of displacement in this medium are given by

$$u_3 = \left(\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} - \frac{2b\psi}{1+bz} \right). \quad \ldots (33)$$

$$w_3 = \left(\frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} - \frac{2b\phi}{1+bz} \right). \quad \ldots (34)$$

Putting values of ϕ and ψ from (31) and (32) in (33) and (34), we get

$$u_3 = \left[ik\, e^{-rz} P_3 + \left(s + z \frac{\partial s}{\partial z} - \frac{2b}{1+bz} \right) Q_3 e^{-sz} \right] \exp\{ik\,(x - ct)\} \quad \ldots (35)$$

$$w_3 = \left[ik\, e^{-sz} Q_3 - \left(r + z \frac{\partial r}{\partial z} + \frac{2b}{1+bz} \right) P_3 e^{-rz} \right] \exp\{ik\,(x - ct)\}. \quad \ldots (36)$$

Stress components for $\lambda = \mu$ are given by

$$\tau_{zz} = \mu \left[\frac{\partial^2 \phi}{\partial x\, \partial z} + 3 \frac{\partial^2 \phi}{\partial z^2} - \frac{6b}{1+bz} \frac{\partial \phi}{\partial z} + \frac{6b^2}{(1+bz)^2} \phi + 2 \frac{\partial^2 \psi}{\partial x\, \partial z} - \frac{2b}{1+bz} \frac{\partial \psi}{\partial x} \right] \quad \ldots (37)$$

$$\tau_{zx} = \mu \left[2 \frac{\partial^2 \phi}{\partial x\, \partial z} - \frac{2b}{1+bz} \frac{\partial \phi}{\partial x} + \frac{\partial^2 \psi}{\partial x^2} - \frac{2b}{1+bz} \frac{\partial \psi}{\partial z} + \frac{2b^2}{(1+bz)^2} \psi - \frac{\partial^2 \psi}{\partial z^2} \right]. \quad \ldots (38)$$

Now denoting μ by μ_3 in half-space H and putting values of ϕ and ψ in (37) and (38), we get

$$\tau_{zz} = \mu_3 \left[P_3 e^{-rz} \left\{ \frac{6b}{1+bz} \left(r + z \frac{\partial r}{\partial z} + \frac{b}{1+bz} \right) + 3 \left(\left(r + z \frac{\partial r}{\partial z} \right)^2 - 2 \frac{\partial r}{\partial z} - z \frac{\partial^2 r}{\partial z^2} \right) - k^2 \right\} - 2ik\, Q_3 e^{-sz} \left(s + z \frac{\partial s}{\partial z} + \frac{b}{1+bz} \right) \right] \times \exp\{ik(x - ct)\} \quad \ldots (39)$$

$$\tau_{zx} = \mu_3 \left[-2ik\, P_3 e^{-rz} \left(r + z \frac{\partial r}{\partial z} + \frac{b}{1+bz} \right) + Q_3 e^{-sz} \times \left\{ \frac{2b}{1+bz} \left(s + z \frac{\partial s}{\partial z} + \frac{b}{1+bz} \right) + 2 \frac{\partial s}{\partial z} + z \frac{\partial^2 s}{\partial z^2} - \left(s + z \frac{\partial s}{\partial z} \right)^2 - k^2 \right\} \right] \times \exp\{ik(x - ct)\}. \quad \ldots (40)$$

BOUNDARY CONDITIONS

$$\left.\begin{aligned}
&p = 0 && \text{at } z = -h,\\
&(\tau_{zz})_{H_2} = -p && \text{at } z = 0,\\
&(\tau_{zx})_{H_2} = 0 && \text{at } z = 0,\\
&(w_2)_{H_2} = (w_1)_{H_1} && \text{at } z = 0,\\
&(\tau_{zz})_{H_2} = (\tau_{zz})_H && \text{at } z = h',\\
&(\tau_{zx})_{H_2} = (\tau_{zx})_H && \text{at } z = h',\\
&(w_2)_{H_2} = (w_3)_H && \text{at } z = h',\\
&(u_2)_{H_2} = (u_3)_H && \text{at } z = h'.
\end{aligned}\right\} \qquad \ldots (41)$$

Making use of eqns. (7), (8), (24), (25), (35), (36), (39) and (40), we get from eqn. (41), eight homogeneous equations in eight unknowns namely $A_1, B_1, U_1, U_2, U_3, U_4, P_3$ and Q_3. The period equation is given by

$$\det (a_{ij}) = 0, \qquad (i, j = 1, 2, \ldots, 8). \qquad \ldots (42)$$

Putting $c/c_1 = x$ and $kh = y$, the elements a_{ij} in non-dimensional form are given as under :

$$a_{11} = a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = a_{21} = a_{23} = a_{25} = a_{26}$$

$$= a_{28} = a_{32} = a_{34} = a_{35} = a_{36} = a_{37} = a_{38} = a_{42} = a_{44} = a_{45}$$

$$= a_{46} = a_{47} = a_{57} = a_{58} = a_{67} = a_{68} = a_{77} = a_{78} = a_{87}$$

$$= a_{88} = 0,$$

$$a_{17} = \cos (y\sqrt{x^2 - 1}), \qquad a_{18} = \sin (y\sqrt{x^2 - 1}),$$

$$a_{22} = \frac{F_2}{C_2} + \frac{\varepsilon_1 m_1}{k}, \qquad a_{24} = \frac{F_2}{C_2} + \frac{\varepsilon_2 m_2}{k},$$

$$a_{27} = -\frac{\lambda_1}{C_2} x^2, \qquad a_{31} = \frac{L_2}{C_2}\left(\frac{m_1}{k} - \varepsilon_1\right),$$

$$a_{33} = \frac{L_2}{C_2}\left(\frac{m_2}{k} - \varepsilon_2\right), \qquad a_{41} = \varepsilon_1,$$

$$a_{43} = \varepsilon_2, \qquad a_{48} = -\sqrt{x^2 - 1},$$

$$a_{51} = \left(\varepsilon_1 \frac{m_1}{k} + \frac{F_2}{C_2} \right) \sinh (m_1 h'),$$

$$a_{52} = \left(\varepsilon_1 \frac{m_1}{k} + \frac{F_2}{C_2} \right) \cosh (m_1 h'),$$

$$a_{53} = \left(\varepsilon_2 \frac{m_2}{k} + \frac{F_2}{C_2} \right) \sinh (m_2 h'),$$

$$a_{54} = \left(\varepsilon_2 \frac{m_2}{k} + \frac{F_2}{C_2} \right) \cosh (m_2 h'),$$

$$a_{55} = \frac{\mu_3}{C_2} e^{-rh'} \frac{1}{k^2 h^2} \left[\frac{6bh}{1 + bh'} \left(rh + h' h r_z|_{z=h'} + \frac{bh}{1 + bh'} \right) + 3 \left(rh + h' h r_z|_{z=h'} \right)^2 - 2h^2 r_z|_{z=h'} - h' h^2 r_{zz}|_{z=h'} - h^2 k^2 \right],$$

$$a_{56} = 2 \frac{\mu_3}{C_2} \frac{e^{-sh'}}{kh} \left(sh + h' h s_z|_{z=h'} + \frac{bh}{1 + bh'} \right),$$

$$a_{61} = \frac{L_2}{C_2} \left(\frac{m_1}{k} - \varepsilon_1 \right) \cosh (m_1 h'),$$

$$a_{62} = \frac{L_2}{C_2} \left(\frac{m_1}{k} - \varepsilon_1 \right) \sinh (m_1 h'),$$

$$a_{63} = \frac{L_2}{C_2} \left(\frac{m_2}{k} - \varepsilon_2 \right) \cosh (m_2 h'),$$

$$a_{64} = \frac{L_2}{C_2} \left(\frac{m_2}{k} - \varepsilon_2 \right) \sinh (m_2 h'),$$

$$a_{65} = \frac{2\mu_3}{C_2} \frac{e^{-rh'}}{kh} \left(rh + h' h r_z|_{z=h'} + \frac{bh}{1 + bh'} \right),$$

$$a_{66} = -\frac{\mu_3}{C_2} e^{-sh'} \frac{1}{k^2 h^2} \left[\frac{2bh}{1 + bh'} \left(sh + h' h s_z|_{z=h'} + \frac{bh}{1 + bh'} \right) + 2h^2 s_z|_{z=h'} + h' h^2 s_{zz}|_{z=h'} - \left(sh + h' h s_z|_{z=h'} \right)^2 - k^2 h^2 \right],$$

$$a_{71} = \sinh (m_1 h'),$$

$$a_{72} = \cosh (m_1 h'),$$

$$a_{73} = \sinh (m_2 h'),$$

$$a_{74} = \cosh (m_2 h'),$$

$$a_{75} = -e^{-rh'},$$

$$a_{76} = -\frac{1}{kh}\left(sh + h'\, hs_z|_{z=h'} - \frac{2bh}{1+bh'} \right) e^{-sh'},$$

$$a_{81} = \varepsilon_1 \cosh(m_1 h'),$$

$$a_{82} = \varepsilon_1 \sinh(m_1 h'),$$

$$a_{83} = \varepsilon_2 \cosh(m_2 h'),$$

$$a_{84} = \varepsilon_2 \sinh(m_2 h'),$$

$$a_{85} = -\frac{1}{kh}\left(rh + h'\, hr_z|_{z=h'} + \frac{2bh}{1+bh'} \right) e^{-rh'},$$

$$a_{86} = -e^{-sh'},$$

where

$$h'\, hr_z|_{z=h'} = \frac{h'}{h}\frac{y\,bh}{(1+bh')^2}\left(\frac{c}{\alpha_0}\right)^2 \frac{1}{\sqrt{(1+bh')^2 - \left(\frac{c}{\alpha_0}\right)^2}},$$

$$h^2\, r_z|_{z=h'} = \frac{y\,bh}{(1+bh')^2}\left(\frac{c}{\alpha_0}\right)^2 \frac{1}{\sqrt{(1+bh')^2 - \left(\frac{c}{\alpha_0}\right)^2}},$$

$$h'\, h^2 r_{zz}|_{z=h'} = -\frac{h'}{h}(bh)^2\, kh\, \frac{c^2}{\alpha_0^2}\left[\frac{3(1+bh')^2 - 2(c/\alpha_0)^2}{(1+bh')^3\left[(1+bh')^2 - \left(\frac{c}{\alpha_0}\right)^2\right]^{3/2}} \right],$$

$$h'\, hs_z|_{z=h'} = \frac{h'}{h}\frac{y\,bh}{(1+bh')^2}\left(\frac{c}{\beta_0}\right)^2 \frac{1}{\sqrt{(1+bh')^2 - \left(\frac{c}{\beta_0}\right)^2}},$$

$$h^2\, s_z|_{z=h'} = \frac{y\,bh}{(1+bh')^2}\left(\frac{c}{\beta_0}\right)^2 \frac{1}{\sqrt{(1+bh')^2 - (c/\beta_0)^2}},$$

$$h'\, h^2 s_z|_{z=h'} = -\frac{h'}{h}(bh)^2\, kh \left(\frac{c}{\beta_0}\right)^2 \times \left[\frac{3(1+bh')^2 - 2(c/\beta_0)^2}{(1+bh')^3\,[(1+bh')^2 - (c/\beta_0)^2]^{3/2}} \right].$$

Equation (42) relates the phase velocity c to the wavelength $\frac{2\pi}{k}$. The wavelength is a multivalued function of the phase velocity, each value corresponding to different mode of propagation. Given any value of k, c can be determined from eqn. (42).

SPECIAL CASE

If we put the inhomogeneity factor $b = 0$ in (42), we get the same frequency equation as obtained by Gogna[7] for surface wave propagation in a homogeneous anisotropic layer lying over a homogeneous isotropic half-space and under an uniform layer of liquid.

NUMERICAL RESULTS AND DISCUSSION

For liquid layer H_1, following Ewing *et al.*[6] the values of elastic parameters are taken as :

$$\lambda_1 = 0.214 \times 10^{11} \text{ dynes/cm}^2,$$

$$\rho_1 = 1.01 \text{ gm/cm}^3.$$

For anisotropic solid layer H_2, following Love[11] the values of parameters are taken as :

$$A_2 = 26.94 \times 10^{11} \text{ dynes/cm}^2.$$

$$C_2 = 23.63 \times 10^{11} \text{ dynes/cm}^2,$$

$$F_2 = 6.61 \times 10^{11} \text{ dynes/cm}^2,$$

$$L_2 = 6.53 \times 10^{11} \text{ dynes/cm}^2,$$

$$\rho_2 = 2.7 \text{ gm/cm}^3.$$

For heterogeneous elastic half-space H, we take the values of relevant parameters $\rho_3 = 2.6$ gm/cm^3 and $\mu_3 = 3.3 \times 10^{11}$ dynes/cm^2.

The thickness of the liquid layer and anisotropic layer are taken 3 km and 2 km respectively.

We now investigate the possibility of the propagation of surface waves discussed above along the x-direction for the model considered. We have calculated the ratio of the phase velocity c of the surface wave to the velocity c_1 of the dilatational wave in liquid for $kh = 0.5$ to 3.50; and $bh = 0.0$, 0.5. The variation of phase velocity of the surface wave versus the wave number, for fundamental and first modes are shown graphically in Figs. 2 and 3 respectively.

We observe from Fig. 2 (fundamental mode) that the ratio c/c_1 takes the value 2.14 for homogeneous half-space ($bh = 0.0$) and 3.04 for inhomogeneous half-space ($bh = 0.5$) at $kh = 0.75$ and $kh = 0.50$ respectively; and Fig. 3 (first mode) the ratio c/c_1 takes the values 2.22 ($bh = 0.0$) and 3.04 ($bh = 0.5$) at $kh = 2.30$ and $kh = 0.55$ respectively. It is clear that the effect of inhomogeneity on the dispersion curve is significant for smaller values of kh but it is negligible for large values of kh, for which it becomes asymptotic to 1 for both the cases ($bh = 0.0$ and $bh = 0.5$). Thus we conclude that the phase velocity in the absence of inhomogeneity is less as compared to the case when heterogeneity is taken into account.

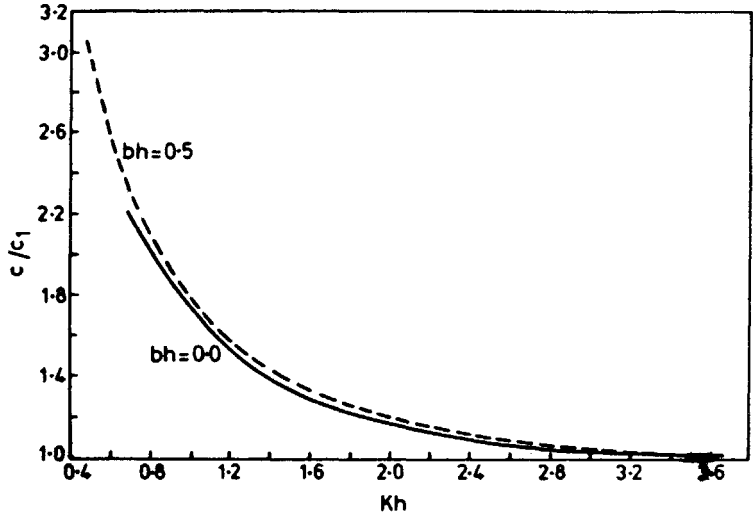

FIG. 2. Variation of phase velocity with wave number (fundamental mode).

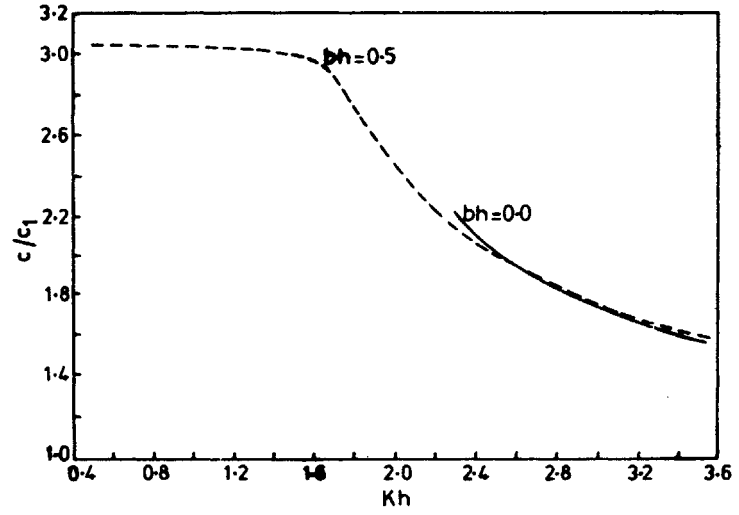

FIG. 3. Variation of phase velocity with wave number (first mode).

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