Indian J. pure appl. Math., 25(4); 443-447, April 1994

THE MOTION OF (2 + ν) CHARGED PARTICLES WHEN PRIMARIES ARE TAKEN AS TWO MAGNETIC DIPOLES ASSOCIATED WITH TWO CARRIER STARS

B. D. PRASAD

Research Centre, C-2/106A, Lawrence Road, New Delhi 110 035

(Received 22 *January* 1993; *after revision* 19 *May* 1993; *accepted* 8 *July* 1993*)*

The motion of (2 + ν) charged particles are studied under the influence of two magnetic dipoles as primaries with their two carrier stars. The first integral analogous to Jacobian integral of the restricted three body problem has been determined and surface of zero velocity is discussed.

1. INTRODUCTION

In 1907, Stormer[3] has done the problem of motion of single charged particle in the field of one magnetic dipole. The problem of motion of a single charged particle in the field of two rotating magnetic dipoles was done by Mavraganis[2] in 1978. He studied the areas of motion in planar magnetic binary case. The same problem has been done by Goudas and Petsagourakis[1]. They considered the direction of the dipole moments as perpendicular to the plane of motion of the Primaries, but Mavraganis has not fixed this direction of the dipole's moment in his problem.

The option to consider the two dipoles stationary or miving, is important in view of its astronomical and engineering applications.

The simplest motion that the two dipoles should perform is circular motion about a fixed point "O" resting anywhere along a straight line connecting the dipoles. If these dipoles are associated with two stars S_1 and S_2 then their centre of mass is the centre of rotation of the dipoles. In such a case, the formulation of the problem can follow the lines of the elliptical or circular restricted problem.

We have generalized Goudas and Petsagourakis problem as in place of a single charged particle. We have taken P_j (j = 1, 2, ..., ν) charged particles with charge q_j and mass m_j, moving in the plane of motion of the primaries under the action of Lorentz and Coulomb's forces. The present problem is formulated on the lines of the circular three body restricted problem. The criteria for the restricted problem has been discussed in detail by Szebehely[5]. It will have applications to the magnetosphere, ionosphere, polar aurora, etc. in the stellar systems.

2. Equations of Motion

Let two magnetic dipoles of magnetic moments $\overline{M}_1$ and $\overline{M}_2$ respectively participate in the circular motion of their carrier stars S_1 and S_2 around their centre of mass "O". We further assume that there are p_j (j = 1, 2, ..., ν) charged particles of charge q_j and mass m_j which are moving under the influence of the two dipoles but not influencing the dipoles.

Let there be a rotating frame of reference O (x, y, z) which rotates in the same direction and with the same angular velocity $\overline{\omega}(0, 0, \omega)$ as the stars S_1 and S_2, which in the frame of reference, stay at rest on the OX-axis (Fig. 1).

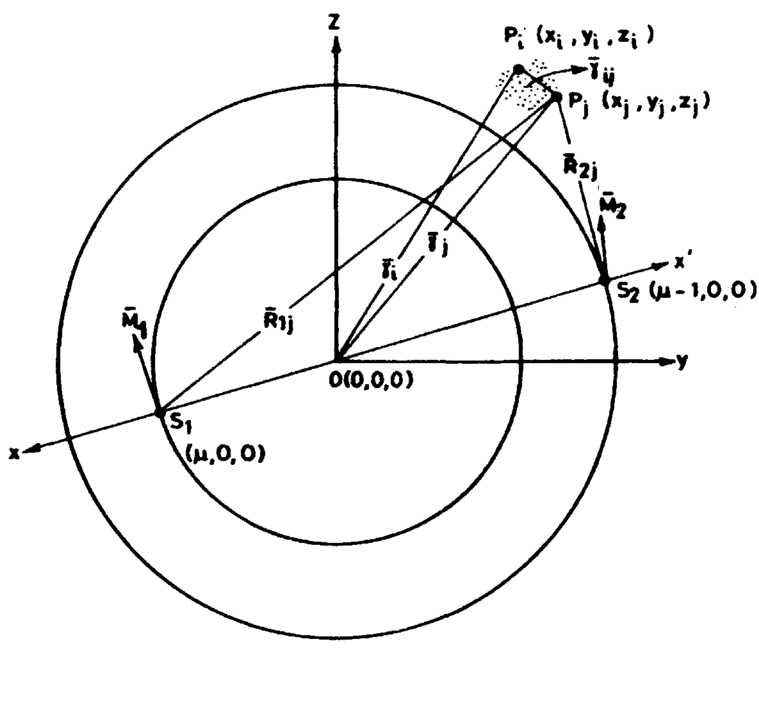

Fig. 1. Motion of Charged particles P_j in the presence of dipoles at the primeries S_1 and S_2.

Now if the coordinates of P_j (j = 1, 2, ..., ν) charged particles at time t be (x_j, y_j, z_j) and its position vector $\overline{r}_j$, then equations of motion of charged particles are given by (William[4])

$$m_j \left[\frac{\partial^2 \overline{r}_j}{\partial t^2} + 2\overline{\omega} \times \frac{\partial \overline{r}_j}{\partial t} + \overline{\omega} \times (\overline{\omega} \times \overline{r}_j) \right] = \overline{F}_{ij} + \overline{F}_{2j} \qquad \dots (1)$$

Where

$\overline{F}_{ij} = \overline{F}_{ej} + \overline{F}_{mj}$ (= Lorentz force)

$\overline{F}_{ej} = q_j \overline{E}_j$ (= electronic force due to dipoles)

$\overline{E}_j = -\frac{1}{c}\frac{\partial \overline{A}_j}{\partial t}$ (= electric field strength)

$C_. =$ velocity of light

$\overline{A}_j =$ Vector potential

$$= \frac{\overline{M}_1 \times \overline{R}_{1j}}{|\overline{R}_{1j}|^3} + \frac{\overline{M}_2 \times \overline{R}_{2j}}{|\overline{R}_{2j}|^3} \quad \text{... (2)}$$

$\overline{R}_1 = \overline{S_1 P_j}$

$\overline{R}_{2j} = \overline{S_2 P_j}$

$\overline{F}_{mj} = q_j \overline{\nabla}_j \times \overline{B}_j$ (= magnetic force due to dipoles)

$\overline{V}_j = \frac{d\overline{r}_j}{dt}$ (= velocity of P_j)

$\overline{B}_j = \frac{1}{C}\left(\overline{\nabla}_j \times \overline{A}_j\right)$ (= magnetic induction vecotr)

$\overline{F}_{2j} = \frac{1}{4\pi \in_0} \sum_{\substack{i=1 \\ i \neq j}}^{\nu} \frac{q_i q_j \overline{r}_{ij}}{|\overline{r}_{ij}|^3}$ (= Culomb's force due to eletrstatic mutual interaction between the charged particles).

$\varepsilon_0 =$ Permitivity of the medium between the charged particles.

Now if we choose the distance between two stars S_1 and S_2 as the unit of length, the sum of their masses as the unit of mass and in addition denote the mass of star S_2 as $\mu = M_2/(M_1 + M_2)$, then the two stars rest at the positions (μ, 0, 0) and (μ − 1, 0, 0) respectively.

Further, we assume that

$$\overline{M}_i = M_{ix}\, i + M_{iy}\, j + M_{iz}\, k.$$

Thus equations of motion of P_j (j = 1, 2, ..., ν) charged particles in cartesian form may be written as

$$\ddot{x}_j - \dot{y}_j\left\{2\omega + \frac{q_j}{Cm_j}\left(\frac{\partial A_{y_j}}{\partial x_j} - \frac{\partial A_{x_j}}{\partial y_j}\right)\right\} + \frac{q_j}{Cm_j}\dot{z} \times \left(\frac{\partial A_{x_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial x_j}\right) = \frac{\partial T}{\partial x_j} \quad \text{... (3)}$$

$$\ddot{y}_j + \dot{x}_j\left\{2\omega + \frac{q_j}{Cm_j}\left(\frac{\partial A_{y_j}}{\partial x_j} - \frac{\partial A_{x_j}}{\partial y_j}\right)\right\} + \frac{q_j}{Cm_j}\dot{z}_j \times \left(\frac{\partial A_{y_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial y_j}\right) = \frac{\partial T}{\partial y_j} \quad \text{... (4)}$$

and

$$\ddot{z}_j - \frac{q_j}{Cm_j}\left\{\dot{x}_j\left(\frac{\partial A_{x_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial x_j}\right) + \dot{y}_j\left(\frac{\partial A_{y_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial y_j}\right)\right\} = \frac{\partial T}{\partial z_j} \qquad \text{... (5)}$$

where

$$T = \sum_{j=1}^{\nu}\left\{\frac{\omega^2}{2}\left(x_j^2 + y_j^2\right) + \frac{q_j\,\omega}{Cm_j}\left(x_j A_{y_j} - y_j A_{x_j}\right)\right\} - \frac{1}{4\,\pi\,\varepsilon_0}\sum_{\substack{i=1\\ i\neq j}}^{\nu}\frac{q_i\,q_j}{|\,\bar{r}_{ij}\,|}. \qquad \text{... (6)}$$

3. Integral of Equtions of Motion

By putting $\tau = \omega t$, equations of motion (3), (4) and (5) may be written as

$$x_j'' - y_j'\left\{2\omega + \frac{q_j}{C\mu_j}\left(\frac{\partial A_{y_j}}{\partial x_j} - \frac{\partial A_{x_j}}{\partial y_j}\right)\right\} + \frac{q_j}{C\mu_j} z_j' \times \left(\frac{\partial A_{x_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial x_j}\right) = \frac{\partial T}{\partial x_j} \qquad \text{... (7)}$$

$$y_j'' + x_j'\left\{2\omega + \frac{q_j}{C\mu_j}\left(\frac{\partial A_{y_j}}{\partial x_j} - \frac{\partial A_{x_j}}{\partial y_j}\right)\right\} + \frac{q_j}{C\mu_j} z_j' \times \left(\frac{\partial A_{y_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial y_j}\right) = \frac{\partial T}{\partial y_j} \qquad \text{... (8)}$$

and

$$z_j'' - \frac{q_j}{C\mu_j}\left\{x_j'\left(\frac{\partial A_{x_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial x_j}\right) + y_j'\left(\frac{\partial A_{y_j}}{\partial z_j} - \frac{\partial A_{z_j}}{\partial y_j}\right)\right\} = \frac{\partial T}{\partial z_j} \qquad \text{... (9)}$$

Where $\quad \mu_j = \dfrac{m_j}{M_1 + M_2}, \quad (j = 1, 2, ..., \nu).$

Here dashes denote differentiation with respect to τ. If eqns. (7), (8) and (9) are multiplied by x_j', y_j' and z_j' respectively and summed up then, we get

$$x_j'' x_j' + y_j'' y_j' + z_j'' z_j' = \frac{\partial T}{\partial x_j} x_j' + \frac{\partial T}{\partial y_j} y_j' + \frac{\partial T}{\partial z_j} z_j' \qquad \text{... (10)}$$

which is non-integrable.

This is due to the term $\dfrac{1}{4\,\pi\varepsilon_0}\displaystyle\sum_{\substack{i=1\\ i\neq j}}\frac{q_i\,q_j\,\bar{r}_{ij}}{|\,\bar{r}_{ij}\,|^3}$

appearing in equations of motion of P_j ($j = 1, 2, ..., \nu$) charged partiles. Thus we find that due to this term separate Jacobian integrals of the ν-minor bodies are lost but in place of them a new integral is obtained for the system of minor bodies.

If eqn. (1) is summed over $j = 1, 2, ..., \nu$ then a new integral is obtained as

$$\sum_{j=1}^{v} |\bar{r}_j'|^2 = 2T - K \qquad \text{... (11)}$$

where K is constant of integration and is determined by the initial conditions of the minor bodies.

4. Surface of Zero Velocity

Equation (11) may be written as

$$\sum_{j=1}^{v} |\bar{r}_j'|^2 + K = 2T$$

$$\Rightarrow \quad 2T \geq K \qquad \text{... (12)}$$

Since $\quad D|\bar{r}_j'|^2 \geq 0, \quad \forall \quad i \leq j \leq v.$

Thus inequality (12) defines a region in 3v dimensional configuration space of the minor bodies where motion is permitted for a given value of K. These regions are bounded by (3v − 1) dimensional hyper surface defined by the equation

$$2T = K \qquad \text{... (13)}$$

The hyper surface defined by eqn. (13) is analogous to the zero velocity surfaces when $v = 1$ which is studied by Goudas and Petsagourakis[1]. In that paper they constrain the possible configuration which minor bodies may assume for a given K.

It may be noted that the equations of motion, the first-integral and the regions of motion for $J = 1$ are exactly the same as given by Goudas and Petsagourakis[1].

Acknowledgement

We are thankful to Department of Science and Technology, which has funced this project.

References

1. C. L. Goudas and E. G. Petsagourakis, *Celestial Mech.* **37** (1985), 127-48.
2. A. G. Mavragnis, *Astr. Phy. Space Sci.* **54** (1978) 305-13.
3. C. F. Stormer, *Arch. Sci. Phys. Et. Nat. Geneva* **24** (1907), 350.
4. T. Scott William, *Physics of Electricity and Magnetism.* John Wiley & Sous INC, London, 1959.
5. V. Szebehely, *Theory of Orbits*, Academic Press, New York, 1967.