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PROPAGATION OF A CRACK DUE TO SHEAR WAVES IN A NON-HOMOGENEOUS MEDIUM OF MONOCLINIC TYPE

A. CHATTOPADHYAY*, A. KESHRI AND S. CHOUDHURY

Department of Applied Mathematics, Indian School of Mines, Dhanbad 826 004

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The paper discusses the propagation of a crack due to shear waves in a non-homogeneous medium having monoclinic symmetry. The stress intensity factor at the crack tip for a concentrated force of a constant intensity is calculated. The method developed by Matczynski[6] has been used to solve the problem. It has been shown that the stress intensity factor decreases as the length of the crack increases. It is also observed that due to the effect of anisotropy the stress intensity factor increases more as compared to the homogeneous case for a particular value of the length of the crack. The increase is relatively more when the medium is anisotropic and inhomogeneous simultaneously.

1. INTRODUCTION

The problem of the determination of the stress and strain fields in elastic solids containing cracks of finite dimensions has received considerable attention during recent years. The application of a sudden disturbance to the surface of an elastic body gives rise to elastic waves which encounter internal flaws such as cracks and a complicated pattern of diffracted waves is generated. Mal[1] has found the dynamic stress intensity factor for a non axisymmetric loading of the penny-shaped crack. He has also discussed the interaction of elastic waves with a penny-shaped crack[2]. Achenbach[3] has investigated the conditions for crack propagation upon diffraction of an incident wave by a crack. Singh and Dhaliwal[4] have studied the diffraction of SH waves by a moving crack. Tait and Moodie[5] have obtained the closed form solutions to dynamic crack and punch problems. Matczynski[6] has solved the quasi-static problem of a crack in an antiplane state of strain of an elastic strip.

Recently Chattopadhyay and Maugin[7] have studied the two-dimensional problem of diffraction of magnetoelastic shear waves by a rigid strip in an infinite elastic perfect conductor. Chattopadhyay and Bandyopadhyay[9] have discussed the propagation

**Present address* : Director, All India Council for Technical Education, Indira Gandhi Sports Complex, Indraprastha Estate, New Delhi 110 002.

of a crack due to shear waves in a medium of monoclinic type. The same authors[8] have also discussed the shear waves in an infinite monoclinic crystal plate.

The above mentioned authors have not discussed the propagation of a crack due to shear waves in a non-homogeneous medium of monoclinic type. The vibrations of the infinite rotated y-cut quartz plate was first established by Ekstein[10] and subsequently explored in detail by Newman and Mindlin[11] and Kaul and Mindlin[12]. Chattopadhyay and Bandyopadhyay[13] have discussed the diffraction of shear waves by a rigid strip in a medium of monoclinic type.

In the present paper the propagation of a crack due to shear waves in a non-homogeneous medium having monoclinic symmetry is investigated. The Wiener-Hopf technique (cf. Noble[15]) has been used to solve the problem. The stress intensity factor at the crack tip is computed for concentrated force of constant intensity. The effect of anisotropy and non-homogeneity parameter is distinctly marked on the stress intensity factor.

The laminated structures are both anisotropic and inhomogeneous in nature. Such structures are being used in different occasions. The study will provide a method to find out the stress intensity factor and prediction of instability of the structure.

2. Formulation of the Problem

We consider a strip of monoclinic type $-\infty < z_1 < \infty$, $-h \leq y_1 \leq h$ weakend in the middle at $y_1 = 0$ by a semi-infinite crack $z_1 < 0$. The surfaces $y_1 = \pm h$ are rigidly clamped. The surface of the crack are subjected to the action of forces $T_6 = p(z_1)$. The crack and the load propagate at a constant speed s in the positive direction along the z_1 axis (Fig. 1).

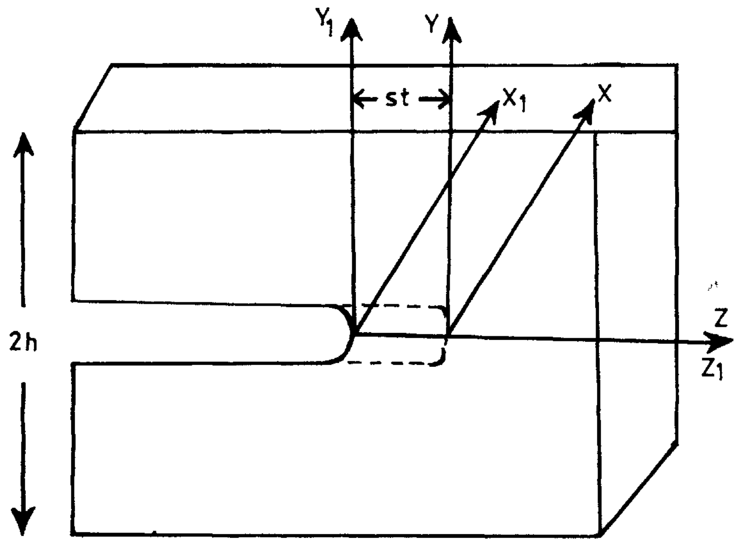

Fig. 1. Geometry of the problem.

The strain-displacement relation for a crystal plate are

$$\left.\begin{aligned} S_1 &= \frac{\partial u}{\partial x_1}, \quad S_2 = \frac{\partial v}{\partial y_1}, \quad S_3 = \frac{\partial w}{\partial z_1} \\ S_4 &= \frac{\partial w}{\partial y_1} + \frac{\partial v}{\partial z_1}, \quad S_5 = \frac{\partial u}{\partial z_1} + \frac{\partial w}{\partial x_1} \quad S_6 = \frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial y_1} \end{aligned}\right\} \quad \dots (2.1)$$

where u, v, w are the displacement components in the directions x_1, y_1, z_1 and S_i (i = 1, 2, ..., 6) are the strain components.

The stress-strain relations for a rotated y-cut quartz plate which exhibits monoclinic symmetry with x_1 being the diagonal axis are :

$$\left.\begin{aligned} T_1 &= C_{11} S_1 + C_{12} S_2 + C_{13} S_3 + C_{14} S_4, \\ T_2 &= C_{21} S_1 + C_{22} S_2 + C_{23} S_3 + C_{24} S_4, \\ T_3 &= C_{31} S_1 + C_{32} S_2 + C_{33} S_3 + C_{34} S_4, \\ T_4 &= C_{41} S_1 + C_{42} S_2 + C_{43} S_3 + C_{44} S_4, \\ T_5 &= C_{55} S_5 + C_{56} S_6, \quad T_6 = C_{56} S_5 + C_{66} S_6, \end{aligned}\right\} \quad \dots (2.2)$$

where T_i (i = 1, 2, 3) are the normal stresses, T_i (i = 4, 5, 6) are the shearing stresses and C_{ij} (i, j = 1, 2, ..., 6) are the elastic constants.

For propagation of a plane transverse wave in the positive z_1-direction and polarized parallel to the plane we may take

$$v = w = 0 \text{ and } u = u(y_1, z_1, t). \quad \dots (2.3)$$

The equations of motion without body forces are :

$$\left.\begin{aligned} \frac{\partial T_1}{\partial x_1} + \frac{\partial T_6}{\partial y_1} + \frac{\partial T_5}{\partial z_1} &= \rho \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial T_6}{\partial x_1} + \frac{\partial T_2}{\partial y_1} + \frac{\partial T_4}{\partial z_1} &= \rho \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial T_5}{\partial x_1} + \frac{\partial T_4}{\partial y_1} + \frac{\partial T_3}{\partial z_1} &= \rho \frac{\partial^2 w}{\partial t^2}. \end{aligned}\right\} \quad \dots (2.4)$$

Using (2.1) and (2.3) the relations (2.2) becomes

$$\left.\begin{aligned} T_1 &= T_2 = T_3 = T_4 = 0, \\ T_5 &= C_{55} \frac{\partial u}{\partial z_1} + C_{56} \frac{\partial u}{\partial y_1}, \\ T_6 &= C_{56} \frac{\partial u}{\partial z_1} + C_{66} \frac{\partial u}{\partial y_1}. \end{aligned}\right\} \quad \dots (2.5)$$

The elastic constants and the density are assumed to vary according to the relation

$$\frac{C_{55}}{C'_{55}} = \frac{C_{56}}{C'_{56}} = \frac{C_{66}}{C'_{66}} = \frac{\rho}{\rho'} = e^{\varepsilon y_1} \quad \dots (2.5a)$$

where ε is the inhomogeneity parameter.

Inserting (2.5) and (2.5a) in (2.4) the only equation of motion in the absence of body forces is

$$\frac{\partial}{\partial y_1}\left\{C'_{56}\; e^{\varepsilon y_1}\frac{\partial u}{\partial z_1}+C'_{66}\; e^{\varepsilon y_1}\frac{\partial u}{\partial y_1}\right\}$$

$$+\frac{\partial}{\partial z_1}\left\{C'_{55}\; e^{\varepsilon y_1}\frac{\partial u}{\partial z_1}+C'_{56}\; e^{\varepsilon y_1}\frac{\partial u}{\partial y_1}\right\} = \rho'\, e^{\varepsilon y_1}\frac{\partial^2 u}{\partial t^2}\,. \quad \text{... (2.6)}$$

Now let us replace the system (x_1, y_1, z_1) by the convective system (x, y, z) according to the Galilean transformation law

$$x_1 = x,\; y_1 = y,\; z_1 = z + st \quad \text{... (2.7)}$$

where s is the velocity of the motion of the system $(x,\; y,\; z)$. Substituting the transformation (2.7) in equations (2.6) and (2.5) we have,

$$C'_{66}\,\frac{\partial^2 u}{\partial y^2}+\varepsilon\, C'_{56}\,\frac{\partial u}{\partial z}+\varepsilon\, C'_{66}\,\frac{\partial u}{\partial y}+2\, C'_{56}\,\frac{\partial^2 u}{\partial y\,\partial z}$$

$$+\;(C'_{55}-\rho'\, s^2)\,\frac{\partial^2 u}{\partial z^2} = 0, \quad \text{... (2.8)}$$

$$T_6 = e^{\varepsilon y}\left\{C'_{56}\,\frac{\partial u}{\partial z}+C'_{66}\,\frac{\partial u}{\partial y}\right\}. \quad \text{... (2.9)}$$

Our boundary condition may then be stated as

$$\left.\begin{aligned} &u(y, z) = 0 \text{ for } |\,z\,| < \infty, y = h\\ &u(y, z) = 0 \text{ for } z > 0, y = 0\\ &T_6(y, z) = p(z) \text{ for } z < 0, y = 0.\end{aligned}\right\} \quad \text{... (2.10)}$$

3. Solution of the Problem

We shall now use the two-sided Fourier integral transform defined by

$$\left.\begin{aligned} F(y,\alpha) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(y,z)\, e^{i\alpha z}\, dz\,,\\ f(y,z) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty+i\gamma}^{\infty+i\gamma} F(y,\alpha)\, e^{-i\alpha z}\, d\alpha \end{aligned}\right\} \quad \text{... (3.1)}$$

where $i\gamma$ is the imaginary number lying in the common strip of regularity of all transforms appearing in the solution and α is the transform parameter which is complex. We shall use the following representation by Titchmarsh[14]

$$F(y,\alpha) = F^-(y,\alpha) + F^+(y,\alpha) \quad \text{... (3.2)}$$

where

$$\left.\begin{aligned} F^-(y,\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} f(y,z)\, e^{i\alpha z}\, dz, \\ F^+(y,\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} f(y,z)\, e^{i\alpha z}\, dz \end{aligned}\right\} \quad \ldots (3.3)$$

are regular functions in the respective half plane Imag $\alpha < \gamma_2$ and Imag $\alpha < \gamma_1$. Applying the Fourier integral transform (3.3) to eqns. (2.8) and (2.9) we obtain

$$\frac{d^2U(y,\alpha)}{dy^2} + a\,\frac{dU(y,\alpha)}{dy} + b\,U(y,\alpha) = 0, \quad \ldots (3.4)$$

$$T_6(y,\alpha) = e^{\varepsilon y}\left[C'_{66}\,\frac{dU(y,\alpha)}{dy} - i\,\alpha\, C'_{56}\; U(y,\alpha)\right] \quad \ldots (3.5)$$

where

$$a = \varepsilon - \frac{2i\,\alpha\, C'_{56}}{C'_{66}}$$

$$b = \frac{\left\{\alpha^2\,(C'_{55} - \rho' s^2) - \varepsilon\, i\,\alpha\, C'_{56}\right\}}{C'_{66}}.$$

The solution of eqn. (3.4) is

$$U(y,\alpha) = e^{my}\,[A(\alpha)\sinh(ny) + B(\alpha)\cosh(ny)] \quad \ldots (3.6)$$

where

$$m = \frac{i\,\alpha\, C'_{56}}{C'_{66}} - \varepsilon/2,$$

$$n = \frac{1}{2}\left\{\varepsilon^2 + 4\alpha^2\left(\frac{C'_{55} - \rho' s^2}{C'_{66}} - \frac{C'^2_{56}}{C'^2_{66}}\right)\right\}^{1/2},$$

and $A(\alpha)$, $B(\alpha)$ are unknown functions.

Substituting from eqn. (3.6) into eqn. (3.5) we have

$$\begin{aligned} T_6(y,\alpha) = e^{\varepsilon y}\,[A(\alpha)\{m\, C'_{66}\;(\sinh(ny) + n\cosh(ny)) \\ - i\,\alpha\, C'_{56}\;\sinh(ny)\} + B(\alpha)\;\{C'_{66}\;(\cosh(ny) + n\sinh(ny)) \\ - i\,\alpha\, C'_{56}\;\cosh(ny)\}]. \end{aligned} \quad \ldots (3.7)$$

Now, applying the Fourier integral transform (3.1) to the three boundary conditions (2.10) and using eqns. (3.6) and (3.7) we have the following Wiener-Hopf equation

$$U^-(0,\alpha) = -\tanh(nh)\left[\frac{p(\alpha) + T_6^+(\alpha)}{n\, C'_{66} + (\varepsilon/2)\, C'_{66}\;\tanh(nh)}\right], \quad \ldots (3.8)$$

where

$$p(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} p(z)\, e^{i\alpha z}\, dz. \qquad \text{... (3.9)}$$

The region of existence of eqn. (3.8) is

$$-\frac{\pi}{2nh} < -\eta < \text{Imag}\,\alpha < 0.$$

The Kernel of the eqn. (3.8) is

$$\frac{\tanh(nh)}{\alpha} = H(\alpha)$$

which can be written in terms of Γ-functions as (Noble[15], p.41)

$$H(\alpha) = \frac{nh}{\pi} \frac{\Gamma\left(\frac{1}{2} - \frac{inh}{\pi}\right)\Gamma\left(\frac{1}{2} + \frac{inh}{\pi}\right)}{\Gamma\left(1 - \frac{inh}{\pi}\right)\Gamma\left(1 + \frac{inh}{\pi}\right)}$$

$$= \frac{nh}{\pi} H^{+}(\alpha) H^{-}(\alpha) \qquad \text{... (3.10)}$$

where

$$H^{+}(\alpha) = \frac{\Gamma\left(\frac{1}{2} - \frac{inh}{\pi}\right)}{\Gamma\left(1 - \frac{inh}{\pi}\right)}, \quad H^{+}(-\alpha) = H^{-}(\alpha). \qquad \text{... (3.11)}$$

The functions

$$\left\{\Gamma\left(1 + \frac{inh}{\pi}\right), \Gamma\left(\frac{1}{2} + \frac{inh}{\pi}\right)\right\} \text{ and } \left\{\Gamma\left(1 - \frac{inh}{\pi}\right), \Gamma\left(\frac{1}{2} - \frac{inh}{\pi}\right)\right\}$$

are regular and non-zero in the half planes Imag $\alpha < \frac{\pi}{2nh}$ and Imag $\alpha > \frac{-\pi}{2nh}$.

Using eqn. (3.10) eqn. (3.8) takes the form

$$-\frac{C'_{66}}{h}\left[\frac{\pi + \frac{\varepsilon h}{2} H^{+}(\alpha) H^{-}(\alpha)}{H^{-}(\alpha)}\right] U^{-}(0, \alpha)$$

$$= H^{+}(\alpha)\, T_6^{+}(\alpha) + E(\alpha) \qquad \text{... (3.12)}$$

where

$$E(\alpha) = H^{+}(\alpha) \cdot p(\alpha) \qquad \text{... (3.13)}$$

has to decompose into the expressions regular in the corresponding half planes Imag $\alpha > -\eta$ and Imag $\alpha < 0$, i.e.,

$$E(\alpha) = E^{+}(\alpha) - E^{-}(\alpha) \quad \text{... (3.14)}$$

where

$$E^{+}(\alpha) = \frac{1}{2\pi i} \int_{-\infty - i\delta_1}^{\infty - i\delta_1} \frac{E(\zeta)}{\zeta - \alpha} d\zeta$$

and

$$E^{-}(\alpha) = \frac{1}{2\pi i} \int_{-\infty + i\delta_2}^{\infty + i\delta_2} \frac{E(\zeta)}{\zeta - \alpha} d\zeta \quad \text{... (3.15)}$$

in which $-\delta_1 <$ Imag $\alpha < \delta_2$ and $i\delta_1$, $i\delta_2$ lie within the required strip of regularity and $E^{+}(\alpha)$, $E^{-}(\alpha)$ are regular in the half planes Imag $\alpha > -\pi/nh$ and Imag $\alpha < 0$ respectively. Using relation (3.14), we have from eqn. (3.12)

$$-\frac{C'_{66}}{h}\left[\frac{\pi + \frac{\varepsilon h}{2} H^{+}(\alpha) \cdot H^{-}(\alpha)}{H^{-}(\alpha)}\right] U^{-}(0, \alpha) + E(\alpha)$$

$$= H^{+}(\alpha)\, T_6^{+}(\alpha) + E^{+}(\alpha). \quad \text{... (3.16)}$$

Since the both sides of the above equation represent regular and non-zero functions in the respective half planes Imag $\alpha > \frac{-\pi}{2nh}$ and Imag $\alpha < 0$ by Liouville's theorem we have,

$$U^{-}(0, \alpha) = \frac{2h}{C'_{66}} \frac{E^{-}(\alpha) \cdot H^{-}(\alpha)}{2\pi + \varepsilon h H^{+}(\alpha) \cdot H^{-}(\alpha)} \text{ regular in Imag } \alpha < 0 \text{... (3.17)}$$

and

$$T_6^{+}(\alpha) = \frac{-E'(\alpha)}{H^{+}(\alpha)} \text{ regular in Imag } \alpha > -\frac{\pi}{2nh}. \quad \text{... (3.18)}$$

The function $E^{+}(\alpha)$ and $E^{-}(\alpha)$ are assumed to be analytic function for $-\pi/2nh <$ Imag $\alpha < 0$, so that they may be represented in the form

$$E^{\pm}(\alpha) = -\frac{1}{\alpha}\left[I - \frac{1}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} \frac{\zeta E(\zeta)}{\zeta - \alpha} d\zeta\right] \quad \text{... (3.19)}$$

where

$$I = \frac{1}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} E(\zeta)\, d\zeta, \quad \delta_1 < \delta < \delta_2. \quad \text{... (3.20)}$$

Using the properties of $E^{+}(\alpha)$, the functions $U^{-}(0, \alpha)$ and $T_6^{+}(\alpha)$ from eqns. (3.17) and (3.18) becomes

$$U^{-}(0,\alpha)=-\frac{2I}{C'_{66}\,\alpha}\cdot\frac{\sqrt{nh/i\alpha}}{(\varepsilon+2n)} \quad \text{... (3.21)}$$

and

$$T_6^{+}(\alpha)=(I/\alpha)\sqrt{-inh/\pi}. \quad \text{... (3.22)}$$

Equations (3.21) and (3.22) by means of Abels theorem (cf. Noble[15], p.41), give

$$U(z)=\frac{2S}{\beta}\left[-\sqrt{-z}+\frac{\varepsilon(-z)^{3/2}}{3i\beta}+\frac{\varepsilon^2(-z)^{5/2}}{20\beta^2}\right] \text{ for } z\to-\eta,\quad \eta\to 0 \quad \text{... (3.23)}$$

and

$$T_6(z)=S\left[\frac{1}{\sqrt{z}}-\frac{\varepsilon^2}{12\beta^2}z^{3/2}\right] \text{ for } z\to+\eta,\ \eta\to 0 \quad \text{... (3.24)}$$

where

$$\beta^2=\left[\frac{C'_{55}}{C'_{66}}-\left(\frac{C'_{56}}{C'_{66}}\right)^2\right]-\frac{s^2}{c_T^2},$$

$$c_T^2=C'_{66}/\rho',$$

and $S=-iI\sqrt{2h\,\beta/\pi}$ is the stress intensity factor and has great importance for crack stability.

4. Stress Intensity Factor for Concentrated Force

Let us consider the edge $z = 0$ of the crack are loaded at $y = -l$ by a force of intensity P which is constant.

Let,

$$\mathrm{P}(z) = P_0\,\delta(z+l). \quad \text{... (4.1)}$$

Substituting eqn. (4.1) into eqn. (3.3) we have

$$p(\alpha)=\frac{P_0}{\sqrt{2\pi}}e^{-i\alpha l}. \quad \text{... (4.2)}$$

Now using eqns. (3.10), (3.13), (3.20) and (4.1), we obtain

$$S=-\frac{P_0}{\sqrt{h}}\frac{1}{\beta^{1/2}}\exp(-l\pi/2\beta h)\frac{1}{2\pi i}\int_{\eta-i\infty}^{\eta+i\infty}\frac{\Gamma(p)}{\Gamma(p+1/2)}\exp(\pi lp/\beta h)\,dp$$

$$+\frac{P_0\,\varepsilon^2 h^{3/2}}{8\pi^2\beta^{1/2}}\cdot\exp\left(-\frac{l\pi}{2\beta h}\right)\times$$

(*Equation continued on p.* 1203)

$$\times \frac{1}{2\pi i} \int_{\eta - i\infty}^{\eta + i\infty} \frac{\Gamma(p)}{\Gamma(p+1/2)} \exp\,(\pi l p/\beta h) \frac{dp}{(p-1/2)^2}$$

$$- \frac{P_0\,\varepsilon^2 \sqrt{h}\, l}{8\pi\,\beta^{3/2}} \cdot \exp\left(-\frac{l\pi}{2\beta h}\right)$$

$$\times \frac{l}{2\pi\, i} \int_{\eta - i\infty}^{\eta + i\infty} \frac{\Gamma(p)}{\Gamma(p+1/2)} \exp\,(\pi l p/\beta h) \frac{dp}{(p-1/2)} \qquad \ldots (4.3)$$

where $0 < \eta < 1/2$. Integrating (cf. Erdelyi *et al.*[16], p. 261), we obtain

$$S = -\frac{P_0}{\sqrt{\pi h}} \frac{1}{\beta^{1/2}} \exp\left(-\frac{l\pi}{2\,\beta h}\right) \frac{1}{\left\{1-\exp\left(-\frac{\pi\, l}{\beta h}\right)\right\}^{1/2}}$$

$$\frac{P_0\,\varepsilon^2\, h^{3/2}}{8\pi^2\,\beta^{1/2}} \left\{\frac{1}{2} + \frac{\pi\, l}{\beta h} - \mathrm{Erf}\left(\sqrt{\frac{\pi\, l}{\beta h}}\right)\right\} - \frac{P_0\,\varepsilon^2\, l\sqrt{h}}{8\pi\,\beta^{3/2}}\, \mathrm{Erf}\left(\sqrt{\frac{\pi\, l}{\beta h}}\right). \qquad \ldots (4.4)$$

We have calculated $-S\sqrt{\pi h}/P_0$, the stress intensity factor, against (l/h) for different values of s/c_T (0, 0.5, 0.8) and for different values of inhomogeneity parameter εh (0.0, 0.2, 0.4, 0.6).

5. Numerical Calculations and Discussions

For numerical calculations the following values have been used (Tiersten[17]) :

$$C'_{55} = 68.81\, N/m^2,$$

$$C'_{56} = 2.53\, N/m^2,$$

$$C'_{66} = 29.01\, N/m^2,$$

$$\rho' = 2649\, \mathrm{kg}/m^2.$$

When $s/c_T = 0$, .5, .8 for different values of inhomogeneity parameter the stress intensity factor have been calculated against the l/h. It is clear from Figs. 2, 3, 4 that the stress intensity factor decreases as l/h increases. But the rate of decrease is more as s/c_T increases. When $s/c_T = 0$ the stress intensity factor increases by 23 percent for $l/h = 1.0$ as the inhomogeneity parameter increases from $\varepsilon h = 0$ to $\varepsilon h = 0.2$. The stress intensity factor increases by 67 percent for $l/h = 1.0$ as the inhomogeneity parameter increases from $\varepsilon h = 0$ to $\varepsilon h = 0.6$. Therefore we can conclude that the inhomogeneity parameter is playing a very important role.

It is clear from Fig. 2 (Curve 1 and 5). That the stress intensity factor in case of anisotropic homogeneous case (curve 1) increases by 50 percent compared to homogeneous case (Curve 5) when $s/c_T = 0$. The increase is relatively more (curve 2, 3, 4) when the medium is anisotropic and heterogeneous simultaneously. The rate of increase of the stress intensity factor is more as s/c_T also increases.

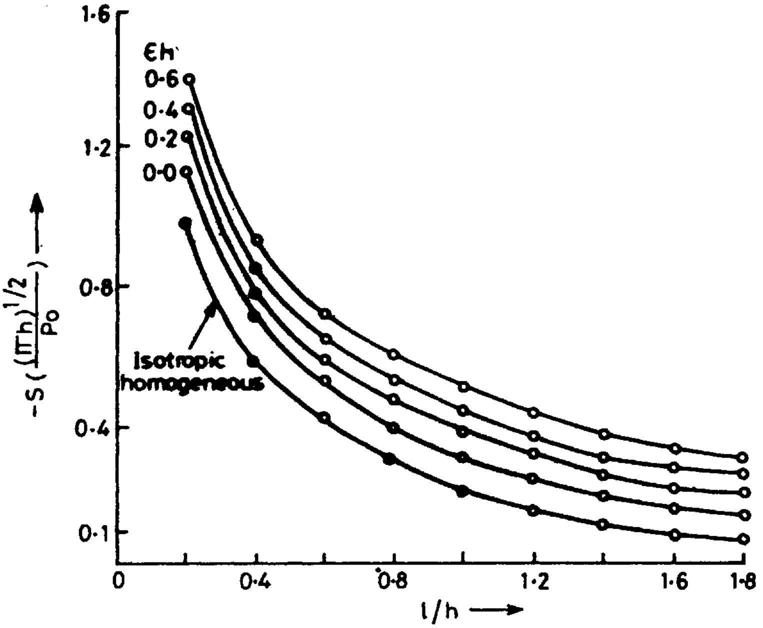

FIG. 2. Stress intensity factor against l/h for different values of εh at $s/c_T = 0.0$.

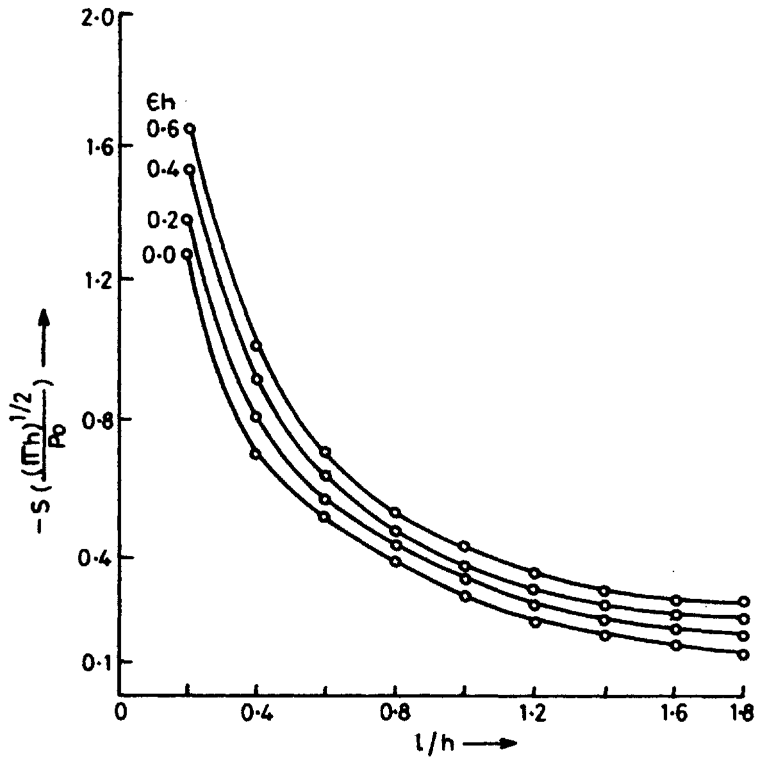

FIG. 3. Stress intensity factor against l/h for different values of εh at $s/c_T = 0.5$.

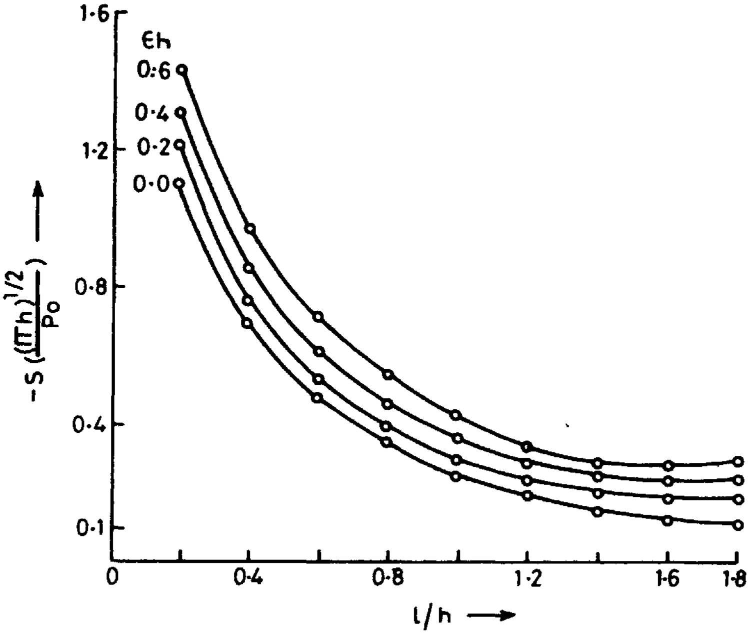

FIG. 4. Stress intensity factor against l/h for different values of εh at s/c_T = 0.8.

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