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FREE CONVECTION AND MASS TRANSFER FLOW THROUGH A POROUS MEDIUM PAST AN INFINITE VERTICAL POROUS PLATE WITH TIME DEPENDENT TEMPERATURE AND CONCENTRATION

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A similarity analysis is made for a two dimensional incompressible viscous flow in a porous medium past an infinite vertical porous plate with time-dependent temperature and concentration. In the analysis the free-stream velocity and the suction velocity are also taken to be time-dependent. Similarity equations are then derived by the introduction of a similarity parameter, taken to be a function of time. The non-dimensional equation for the temperature and concentration are then solved analytically. But because of the complexity of the momentum equation, the solutions for the velocity distribution are obtained numerically. The numerical results for the velocity distribution are then shown graphically for various values of the parameters entering into the problem. The corresponding skin friction coefficients are also shown in tabular form.

1. INTRODUCTION

There are many transport processes in nature and in industry where flows with free convection currents caused by the temperature difference are affected by the differences in concentration or material constitution. In a number of engineering applications, foreign gases are injected to attain more efficiency, the advantage being the reduction in wall shear stress, the mass transfer conductance or the rate of heat transfer. Gases such as H, H_2, O, H_2O, CO_2 etc. are usually used as foreign gases in air flowing past bodies. So the problems of heat and mass transfer past vertical bodies in boundary layer flows have been studied by many of whom the names of Somers[22], Gill *et al.*[4]. Adams and Lowell[1] and Gebhart and Para[5,6] are worth mentioning. The mass transfer phenomenon in unsteady free convection flow past infinite vertical porous plate was also studied by Soundalgekar and Wavre[19,20] and Hossain and Begum[7]. All these studies have been confined to flows in nonporous medium. However, flows in a porous medium have several applications in geophysical, geothermal and oil reservoir engineering etc.

Many researchers have studied the combined heat and mass transfer flow in a porous medium[2, 3, 10, 11, 12]. These studies did not take into account the effect of the free stream velocity or in other words the pressure gradient effect. Raptis[23], however, took a constant free stream velocity to study the effects of free-convection mass transfer flow in a porous medium past an infinite porous plate with time-dependent temperature. A similar study of a steady flow with viscous dissipation effects was later made by Raptis and Perdikis[10].

In the absence of porous medium, the effects of free convection with a free stream, oscillating in time about a non-zero constant mean, were studied respectively by Soundalgekar[17], Soundalgekar and Gupta[18] and Soundalgekar and Hiremath[21].

In consequence to the above studies, the present study addresses the situation in which free stream is taken to be a function of time. Taking this time-dependent free stream velocity the sutdy, thus, considers the case of an unsteady free convection and mass transfer flow, through a very porous medium, past an infinite vertical porous plate with variable suction and time dependent temperature and concentration. The present study, therefore, addresses a situation which is completely different from those mentioned above.

2. Mathematical Formulation

The physical configuration of the problem considred here is shown in Fig. 1 in which we have taken the unsteady free convection and mass transfer flow in a porous medium bounded by a semi infinite vertical porous plate at $y = 0$ subjected to time dependent temperature and concentration to the plate. The level of foreign mass in

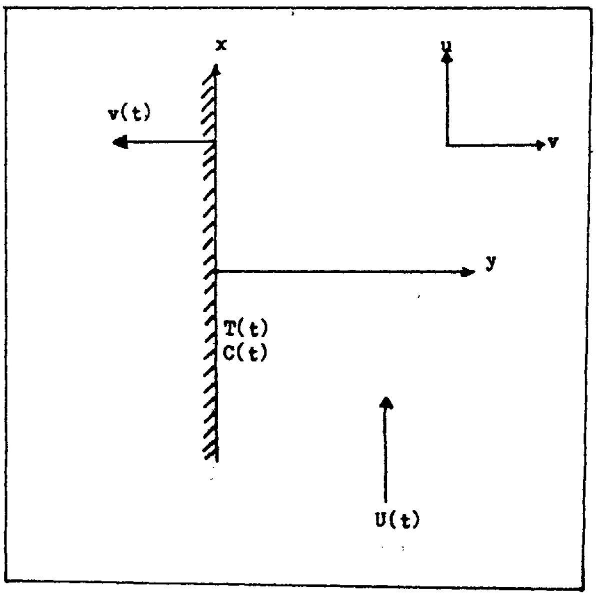

FIG. 1. Flow configuration.

the fluid is assumed to be very low and hence the Soret-Dufour effects are neglected. It is also assumed that there is no chemical reaction taking place between the foreign mass and the fluid. Initially it is assumed that the plate and the fluid are at the same temperature and the concentration level everywhere in the fluid is same. At time $t > 0$, the plate temperature and concentration level of the fluid are instantly raised, respectively, to $T(t)$ and $C(t)$. The boundary layer so obtained is subjected to an unsteady pressure gradient which refers to a time dependent external flow. We thus consider the external flow velocity or the free stream velocity as $U(t)$ which also describes the slip velocity at the surface of the plate by the purely inviscid theory. Now taking the x-axis along the plate, the y-axis normal to it and neglecting the viscous dissipation effects, the governing equations under the Boussinesq's approximation are

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu}{K}(U - u) + \nu\frac{\partial^2 u}{\partial y^2} \quad \ldots (1)$$

$$\frac{\partial v}{\partial y} = 0 \quad \ldots (2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad \ldots (3)$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \quad \ldots (4)$$

where (u, v) is the velocity vector, T and C are, respectively, the temperature and concentration of the fluid, g is the acceleration due to gravity, ν the kinematic coefficient of viscosity, β the coefficient of volume expansion, β^* the volumetric coefficient of expansion with concentration, k the thermal conductivity, ρ the density, C_p the specific heat at constant pressure, D the chemical molecular diffusivity and K the permeability coefficient.

The boundary conditions corresponding to the problem are

$$u = 0,\ v = v(t),\ T = T(t),\ C = C(t) \text{ at } y = 0$$

$$u = U(t),\ v = 0,\ T = T_\infty, C = C_\infty \text{ as } y \to \infty. \quad \ldots (5)$$

where $v(t)$ indicates the time dependent suction at the plate and T_∞ and C_∞ are, respectively, the temperature and concentration of the free stream flow.

In order to obtain similarity solutions of the problem, a similarity parameter σ is now introduced as $\sigma = \sigma(t)$ such that σ is a length scale.

With this similarity parameter, a similarity variable is then introduced as

$$\eta = y/\sigma. \quad \ldots (7)$$

In terms of σ, now, a convenient solution of eqn. (2) can be taken as

$$v = v(t) = -\frac{\nu}{\sigma}v_0 \quad \ldots (8)$$

where v_0 denotes non-dimensional suction or injection parameter according as $v_0 > 0$ or $v_0 < 0$.

The free stream velocity $U(t)$, the plate temperature $T(t)$ and the concentration $C(t)$ on the plate are now assumed to have the following forms :

$$U(t) = U_0 \, \sigma_*^{2m+2}$$

$$T(t) = T_\infty + (T_0 - T_\infty) \, \sigma_*^{2m} \qquad \text{... (9)}$$

$$C(t) = C_\infty + (C_0 - C_\infty) \, \sigma_*^{2m}$$

where m is an integer, $\sigma_* = \sigma/\sigma_0$, U_0, T_0 and C_0 are, respectively, the mean velocity, the mean temperature and the mean concentration all being constants.

To make the momentum, energy and concentration eqns. (1), (3) and (4) dimensionless, the following similarity transformations are introduced :

$$\left.\begin{aligned} u &= U(t)\,F(\eta) = U_0\,\sigma_*^{2m+2}\,F(\eta) \\ T &= T_\infty + (T_0 - T_\infty)\,\sigma*^{2m}\,\theta(\eta) \\ C &= C_\infty + (C_0 - C_\infty)\,\sigma^{*2m}\,\Phi(\eta). \end{aligned}\right\} \qquad \text{... (10)}$$

Equations (1), (3) and (4) in their dimensionless form then become

$$\frac{\sigma}{\nu}\frac{d\sigma}{dt}\left[(2m+2)(F-1) - \eta F'\right] - v_0 F' = F'' + G_r\,\theta + G_c\,\Phi - \gamma\,(1-F) \qquad \text{... (11)}$$

$$\frac{\sigma}{\nu}\frac{d\sigma}{dt}\left[2m\,\theta - \eta\,\theta'\right] - v_0\,\theta' = \frac{1}{p_r}\,\theta'' \qquad \text{... (12)}$$

$$\frac{\sigma}{\nu}\frac{d\sigma}{dt}\left[2m\,\Phi - \eta\,\Phi'\right] - v_0\,\Phi' = \frac{1}{s_c}\,\Phi'' \qquad \text{... (13)}$$

where a prime denotes differentiation with respect to the argument and G_r and G_c are the free convection parameters, γ is the permeability parameter, P_r and S_c are the Prandtl and Schmidt numbers, respectively, defined as

$$G_r = \frac{g\beta\,(T_0 - T_\infty)\,\sigma_0^2}{U_0\,\nu}$$

$$G_c = \frac{g\beta^*\,(C_0 - C_\infty)\,\sigma_0^2}{U_0\,\nu}$$

$$\gamma = \frac{\sigma^2}{K}$$

$$P_r = \frac{\nu\,\rho\,C_p}{k} \text{ and } S_c = \frac{\nu}{D}\cdot$$

Equation (11)-(13) are similar accept for the term $\frac{\sigma}{\nu}\frac{d\sigma}{dt}$ where t appears explicitly. Thus the similarity condition requires that $\frac{\sigma}{\nu}\frac{d\sigma}{dt}$ must be a constant. Hence following Sattar and Hossain[14], one can try a class of solutions of the eqns. (11)-(13) by assuming that

$$\frac{\sigma}{\nu}\frac{d\sigma}{dt} = c \text{ (a constant).} \qquad \text{... (14)}$$

Now integrating (14), one obtains

$$\sigma = \sqrt{2c\,\nu t} \qquad \text{... (15)}$$

where the constant of integration is determined through the conditions that $\sigma = 0$ when $t = 0$. It has appears from (15) that the length scale σ is consistent with the usual length scale considered for various non-steady flows[16].

Now, making a realistic choice of c to be equal to 2 in (14), eqns. (11)-(13) finally become

$$F'' + 2(\eta + \alpha_0) F' + (4m + 4 - \gamma)(1 - F) = -G_r\,\theta - G_c\,\Phi \qquad \text{... (16)}$$

$$\theta'' + 2P_r(\eta + \alpha_0)\,\theta' - 4m P_r\,\theta = 0 \qquad \text{... (17)}$$

$$\Phi'' + 2S_c(\eta + \alpha_0)\,\Phi' - 4m S_c\,\Phi = 0 \qquad \text{... (18)}$$

where $\alpha_0 = v_0/2$.

The corresponding boundary conditions from (5) then become

$$\left.\begin{aligned} F = 0,\ \theta = 1,\ \Phi = 1 \quad &\text{at} \quad \eta = 0 \\ F = 1,\ \theta = 0,\ \Phi = 0 \quad &\text{as} \quad \eta \to \infty \end{aligned}\right\} \qquad \text{... (19)}$$

The problem under consideration is now reduced to the system of equations (16)-(19), the solutions of which are obtained in the following section.

3. Solutions

In order to solve eqns. (16), (17) and (18) under the boundary conditions (19), it is convenient to solve eqns. (17) and (18) first and then using these solutions eqn. (16) can be solved effectively.

Thus the solutions of eqns. (17) and (18) under the boundary conditions (19) are, respectively

$$\theta = \frac{Hh_{2m}(\sqrt{2P_r}\,\xi)}{Hh_{2m}(\sqrt{2P_r}\,\alpha_0)} \qquad \text{... (20)}$$

$$= \frac{Hh_{2m}(\sqrt{2S_c}\,\xi)}{Hh_{2m}(\sqrt{2S_c}\,\alpha_0)} \qquad \text{... (21)}$$

where $\xi = \eta + \alpha_0$ and the function $Hh_n(x)$ is defined as

$$Hh_n(x) = \int_x^\infty \frac{(S - x)}{n\,!} \exp(-S^2/2)\, dS$$

Other properties of this function are discussed by Jeffreys and Jeffreys[8].

Now substituting the values of θ and Φ from (20) and (21) in eqn. (16), its solution under the boundary condition (19) are obtained numerically by the method of superposition[9]. By this method the boundary value problem has been reduced to an initial value problem which has been solved by an initial value solver namely Merson Integration Scheme (MIS). The use of this Scheme has recently been made by Sattar[15].

The results of the numerical integration are shown in Figs. 2 and 3 for the velocity profiles and in Table I for the skin friction coefficients for different values of G_r, G_c, γ, and ν_0. The values of the parameters P_r and S_c are, however, taken to be fixed so as to represent specific conditions of the flow.

TABLE I

Skin friction coefficients for $P_r = 0.71$ and $S_c = 0.24$

G_r	G_c	ν_0	γ	$F'(0)$
1.0	2.0	1.0	1.0	4.31718
5.0	2.0	1.0	1.0	5.34380
5.0	4.0	1.0	1.0	5.99317
5.0	6.0	1.0	1.0	6.64254
5.0	2.0	2.0	1.0	6.07587
5.0	2.0	1.0	2.0	5.26778
10.0	2.0	1.0	1.0	6.62706

4. DISCUSSION

In the calculations, the Prandtl number P_r is taken to be equal to 0.71 which corresponds to air and the Schmidt number S_c is taken to be equal to 0.24 which corresponds to the Hydrogen gas. Under these physical conditions, the velocity profiles for the case of cooling ($G_r > 0$) of the plate are shown in Fig. 2. This figure also shows the effects of suction at the plate. From Fig. 2, it is observed that for fixed values of γ and G_c, the velocity increases with the increase of the free convection current (increase in G_r), which is usually expected. As for the effects of suction parameter, it is observed from Fig. 2 that greater suction velocity increases the velocity of the fluid very slightly in a region close to the plate and then decreases throughout the whole boundary layer. In Fig. 3 the effects of the modified Grashof number G_c and the premeability parameter on the velocity field are shown. Figure 3 thus shows that velocity increases when the concentration difference between the mean and free stream values increases. But it is important to note that the permeability of the porous medium has little effect on the velocity field. Raptis[23, 24] and Raptis and Perdikis[10] in their problems have shown that larger permeability causes larger velocity. Thus from the present study one may conclude that by taking non-uniform free stream velocity and also non-uniform temperature and concentration

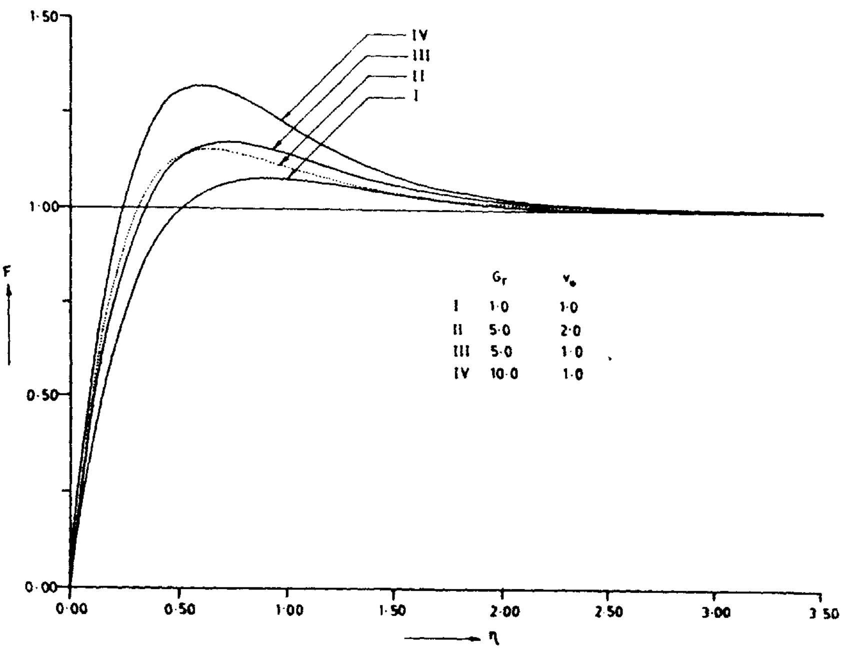

FIG. 2. Velocity distributions for different values of G_r and v_0 and for fixed values of G_c and γ.

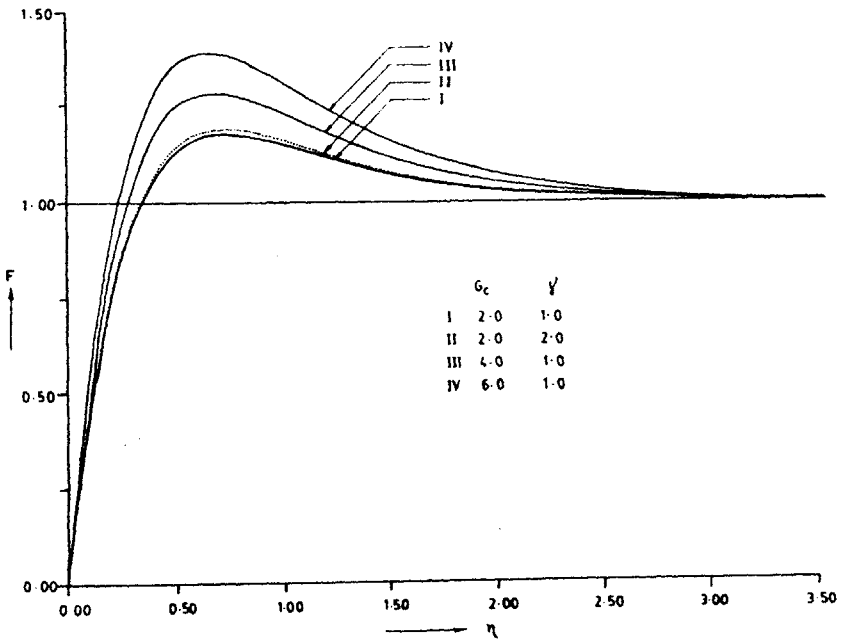

FIG. 3. Velocity distributions for different values of G_c and γ and for fixed values of G_r and v_0.

at the plate, the permeability effects can be reduced considerably. This theoretical result may be of considerable interest to the experimenters.

Finally Table I shows the effects of G_r, G_c, v_0 and γ on the skin friction coefficient. The conclusions and discussion regarding the behaviours of the above parameters on the skin friction coefficient are self evident from the above table and hence any further discussion about them seems to be redundant.

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