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ON INHOMOGENEOUS DEFORMATIONS OF A NONLINEAR ELASTIC DIELECTRIC MATERIAL

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We discuss the inhomogeneous shearing deformation of a slab and a cylindrical tube of elastic dielectric solids. We find that equations are highly nonlinear but admit solutions to the certain values of parameters. The results have been reduced to those of elastic materials. The effect of electric field is found to be quadratic.

1. INTRODUCTION

A very few genuinely nonlinear problems have been treated. Most of these problems are for incompressible solids and fall into the category of controllable states of elastic dielectrics. A controllable state is defined as that which can be maintained by the application of surface loads only. Here we discuss some non-homogeneous deformations in this category.

Recently, several inhomogeneous deformations of isotropic elastic materials[1-15] have been studied. In this paper, we study inhomogeneous shearing deformations of slabs and solid circular cylindrical tube of an isotropic elastic dielectric material. The material is taken to be incompressible. The semi-inverse method is used to study such inhomogeneous deformations. We assume a specified form of deformation and see whether the material under consideration can go such deformations. In section 2, we recapitulate the basic equations for incompressible dielectric solids[15]. In section 3, we formulate the problem of inhomogeneous deformation of slab and in section 4, problem of slab is solved. In section 5, we study inhomogeneous deformation of cylindrical tube. We, however, obtain the highly nonlinear differential equations but we are able to find the solution in the form of integral equations, which can be solved numerically. In case the results are reduced to those of elastic materials, we are able to solve the equation explicitly which are in agreement to those of Zhang and Rajagopal[15].

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2. Basic Equations

We recapitulate here the basic equations from Eringen and Mougin[16].

(i) *Constitutive Equations*

The constitutive equations for isotropic incompressible dielectric solids are

$$\overset{t}{E} = -p\,\mathbf{1} + b_{-1}\,\mathbf{c}^{-1} + b_1\,\mathbf{c} - \chi_1\,[\mathbf{E}\otimes(\mathbf{c}^{-1}\,\mathbf{E}) + (\mathbf{c}^{-1}\,\mathbf{E})\otimes\mathbf{E}]$$
$$-\ 2\chi_2\,[\mathbf{E}\otimes(\mathbf{c}^{-2}\,\mathbf{E}) + (\mathbf{c}^{-2}\,\mathbf{E})\otimes\mathbf{E}] \qquad \text{... (2.1)}$$

$$\mathbf{P} = \chi\,\mathbf{E} + \chi_1\,\mathbf{c}^{-1}\,\mathbf{E} + \chi_2\,\mathbf{c}^{-2}\,\mathbf{E} \qquad \text{... (2.2)}$$

where $p(x, t)$ is an unknown pressure and

$$b_{-1} = +2\frac{\partial\Sigma}{\partial I_1},$$

$$b_1 = -2\frac{\partial\Sigma}{\partial I_2},$$

$$\chi = -2\frac{\partial\Sigma}{\partial I_4},$$

$$\chi_1 = -2\frac{\partial\Sigma}{\partial I_5},$$

$$\chi_2 = -2\frac{\partial\Sigma}{\partial I_6},$$

$$I_1 = \text{tr}\cdot c^{-1},$$

$$I_2 = \frac{1}{2}[(\text{tr}\cdot c^{-1})^2 - \text{tr}\cdot c^{-2}],$$

$$I_4 = \mathbf{E}\cdot\mathbf{E},$$

$$I_5 = \mathbf{E}\,\mathbf{c}^{-1}\,\mathbf{E},$$

$$I_6 = \mathbf{E}\,\mathbf{c}^{-2}\,\mathbf{E},$$

$$\overset{-1}{c_{kl}} = x_{k,K}\,x_{l,K},$$

$$c_{kl} = X_{K,k}\,X_{K,l},$$

$$\Sigma = \Sigma(I_1, I_2, I_4, I_5, I_6). \qquad \text{... (2.3)}$$

In eqns. (2.1) to (2.3) $\overset{t}{E}$ is the symmetric stress tensor. $\mathbf{c}^{-1}$ is the Finger strain tensor. $\mathbf{E}$ is the electric field and $\mathbf{P}$ the polarization vector.

(ii) *Equations of Balance*

$$E^{t}kl,l + (q_e - \nabla \cdot \mathbf{P}) E_k + \rho f_k = 0 \quad \ldots (2.4)$$

$$\nabla \cdot \mathbf{D} = q_e \quad \ldots (2.5)$$

$$\nabla \times \mathbf{E} = 0 \quad \ldots (2.6)$$

In eqns. (2.4) to (2.6), q_e is the free charge density per unit volume. f_k is the mechanical body force density per unit mass and **D** the dielectric displacement vector given by

$$\mathbf{D} = \mathbf{P} + \mathbf{E}. \quad \ldots (2.7)$$

(A) INHOMOGENEOUS SHEARING OF A SLAB

3. FORMATION OF THE PROBLEM

We consider an infinite slab bounded by the planes $Z = 0$ and $Z = H$ in the undeformed state. The material of the slab is taken to be isotropic elastic dielectric incompressible solid. The slab is supposed to be bounded at both $Z = 0$ and $Z = H$ to rigid surfaces due to the existence of pressure gradient. The mechanical body force is neglected. Our problem is to study inhomogeneous shearing deformation of the slab. We assume a form for the deformation and investigate whether the material under consideration can undergo such deformation or not.

Let (X, Y, Z) denote the reference coordinates, and (x, y, z) the current coordinates of a body.

We consider an inhomogeneous deformation of the form :

$$x = X + f(Z),\ y = Y,\ z = Z. \quad \ldots (3.1)$$

The electric field has components

$$\mathbf{E} = [0,\ 0,\ E_3(Z)]. \quad \ldots (3.2)$$

The appropriate boundary conditions on the surfaces are

$$f(Z) = 0 \text{ at } Z = 0 \quad \ldots (3.3)$$

$$f(Z) = 0 \text{ at } Z = H \quad \ldots (3.4)$$

$$E_3 + P_3 = \mathbf{E}_3^{+} + w_e. \quad \ldots (3.5)$$

In eqn. (3.5), w_e is the surface charge and $\mathbf{E}^{+}$ is the external electric field.

4. SOLUTION OF THE PROBLEM

It follows from (3.1) and (2.3) that Finger strain tensor $\mathbf{c}^{-1}$ and the deformation gradient **F** take the form :

$$\mathbf{c}^{-1} = \begin{bmatrix} 1+(f')^2 & 0 & f' \\ 0 & 1 & 0 \\ f' & 0 & 1 \end{bmatrix} \quad \text{... (4.1)}$$

$$\mathbf{c} = \begin{bmatrix} 1 & 0 & -f' \\ 0 & 1 & 0 \\ -f' & 0 & 1+(f')^2 \end{bmatrix} \quad \text{... (4.2)}$$

$$\mathbf{c}^{-2} = \begin{bmatrix} 1+3f'^2+f'^4 & 0 & 2f'+f'^3 \\ 0 & 1 & 0 \\ 2f'+f'^3 & 0 & 1+f'^2 \end{bmatrix} \quad \text{... (4.3)}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & f' \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (4.4)}$$

where prime denotes the derivative with respect to the argument Z. We notice that eqn. (4.4) satisfies the condition of incompressibility.

Within the slab, free charge density q_e is taken as zero.

Using eqns. (3.1), (3.2) and (4.1) to (4.4), eqn. (2.1) after some simplification gives

$$E^{t}_{11} = -p + b_{-1}(1+f'^2) + b_1 \quad \text{... (4.5)}$$

$$E^{t}_{22} = -p + b_{-1} + b_1 \quad \text{... (4.6)}$$

$$E^{t}_{33} = -p + b_{-1} + b_1(1+f'^2) - 2\chi_1 E_3^2 - 4\chi_2(1+f'^2)E_3^2 \quad \text{... (4.7)}$$

$$E^{t}_{13} = E^{t}_{31} = (b_{-1}-b_1)f' - \chi_1 f' E_3^2 - 2\chi_2(2f'+f'^3)E_3^2 \quad \text{... (4.8)}$$

$$E^{t}_{12} = E^{t}_{21} = E^{t}_{23} = E^{t}_{32} = 0. \quad \text{... (4.9)}$$

With the help of eqns. (2.5), (2.7), (3.2), (4.1), (4.3) and (4.4), eqn. (2.2) gives

$$\left.\begin{aligned} P_1 &= \chi_1 f' E_3 + \chi_2(2f'+f'^3)E_3, \\ P_2 &= 0, \\ P_3 &= (\chi+\chi_2+\chi_3)E_3 + \chi_2 f'^2 E_3, \end{aligned}\right] \quad \text{... (4.10)}$$

$$E_3 = \alpha(\beta + \chi_2 f'^2)^{-1}. \quad \text{... (4.11)}$$

In eqn. (4.11), α is an arbitrary constant and

$$\beta = 1 + \chi + \chi_1 + \chi_2. \quad \text{... (4.12)}$$

To be specific, we consider the energy density function Σ of the form :

$$\Sigma = \alpha_1 (I_1 - 3) + \alpha_2 (I_2 - 3) + \alpha_4 (I_4 - 3) + \alpha_5 (I_5 - 3) + \alpha_6 (I_6 - 3). \quad \text{... (4.12a)}$$

With the help of eqns. (4.9), (4.10), (4.11) and (4.12a), the equations of equilibrium, in the absence of body forces are simplified to

$$-\frac{\partial p}{\partial X} + \frac{d}{dZ}[\,2(\alpha_1 + \alpha_2) f' + 2\alpha_5 f' E_3^2 + 4\alpha_6 (2f' + f'^3) E_3^2\,] = 0 \quad \text{... (4.13)}$$

$$-\frac{\partial p}{\partial Y} = 0 \quad \text{... (4.14)}$$

$$-\frac{\partial p}{\partial Z} + \frac{d}{dZ}\left[2(\alpha_1 - \alpha_2) - 2\alpha_2 f'^2 + \left\{ \frac{1}{2} + 4\alpha_5 + 8\alpha_6 (1 + f'^2) \right\} E_3^2 \right] = 0. \quad \text{... (4.15)}$$

If we define a function

$$G(Z) = 2(\alpha_1 - \alpha_2) - 2\alpha_2 f'^2 + \left\{ \frac{1}{2} + 4\alpha_5 + 8\alpha_6 (1 + f'^2) \right\} E_3^2 \quad \text{... (4.16)}$$

and introduce a function $\hat{p}(Z)$ through

$$\hat{p}(X, Y, Z) = p(X, Y, Z) - G(Z). \quad \text{... (4.17)}$$

Then eqns. (4.13) to (4.15) can be rewritten as

$$-\frac{\partial \hat{p}}{\partial X} + \frac{d}{dZ}[\,2(\alpha_1 + \alpha_2) f' + 2\left\{ (\alpha_5 + 4\alpha_6) + 2\alpha_6 f'^2 \right\} f' E_3^2\,] = 0 \quad \text{... (4.18)}$$

$$-\frac{\partial \hat{p}}{\partial Y} = 0 \quad \text{... (4.19)}$$

$$-\frac{\partial \hat{p}}{\partial Z} = 0. \quad \text{... (4.20)}$$

It follows from eqns. (4.18) to (4.20) that

$$\hat{p} = cX + c_1, \quad \text{... (4.21)}$$

where c and c_1 are constants.

Substituting (4.21) into (4.18), we obtain

$$\frac{d}{dZ}[\,2(\alpha_1 + \alpha_2) f' + 2\left\{ (\alpha_5 + 4\alpha_6) + 2\alpha_6 f'^2 \right\} f' E_3^2\,] = c. \quad \text{... (4.22)}$$

Since we are considering the shearing of a slab of thickness H which is bonded at both $Z = 0$ and $Z = H$ to rigid surfaces, due to the existence of a pressure gradient. Such a pressure gradient exists by virtue of the applied tractions at

$X = \pm\infty$. The value of the pressure gradient in the X-direction is given by c.

Integration of eqn. (4.22) gives

$$2(\alpha_1 + \alpha_2) f' + 2\{\alpha_5 + 4\alpha_6 + 2\alpha_6 f'^2\} f' E_3^2 = cZ + D, \qquad \ldots (4.23)$$

where D is a constant of integration.

Then eqn. (4.23) gives

$$4\alpha_6 E_3^2 f'^3 + 2\{(\alpha_1 + \alpha_2) + (\alpha_5 + 4\alpha_6) E_3^2\} f' - (cZ + D) = 0. \qquad \ldots (4.24)$$

This is a cubic equation in f'. Solving eqn. (4.24) by wellknown algebraic method, we obtain

$$f' = \left[\frac{cZ + D}{8\alpha_6 E_3^2} + \frac{1}{2}\xi(Z)\right]^{1/3} + \left[\frac{cZ + D}{8\alpha_6 E_3^2} - \frac{1}{2}\xi(Z)\right]^{1/3} \qquad \ldots (4.25)$$

where

$$\xi(Z) = \left[\frac{(cZ + D)^2}{16\alpha_6^2 E_3^4} + \frac{1}{54\alpha_6^3 E_3^6}\left\{(\alpha_1 + \alpha_2) + (\alpha_5 + 4\alpha_6) E_3^2\right\}^3\right]^{1/2} \cdot \qquad \ldots (4.26)$$

Integration of eqn. (4.25) gives

$$f(Z) = \int \left[\frac{cZ + D}{8\alpha_6 E_3^2} + \frac{1}{2}\xi(Z)\right]^{1/3} dZ + \int \left[\frac{cZ + D}{8\alpha_6 E_3^2} - \frac{1}{2}\xi(Z)\right]^{1/3} dZ + K \qquad \ldots (4.27)$$

where K is a constant of integration.

Equation (4.27) can be solved numerically subject to the appropriate boundary conditions.

(B) INHOMOGENEOUS SHEARING OF A CIRCULAR CYLINDER

5. Formation and Solution of the Problem

Next, we consider the inhomogeneous shearing of a cylindrical tube whose axis is directed along the Z-direction.

Let us consider the deformation of the type

$$r = R, \ \theta = \Theta, \ z = Z + f(R) \qquad \ldots (5.1)$$

where (R, Θ, Z) and (r, θ, z) represent the undeformed and deformed cylindrical co-ordinates respectively.

The electric field has components

$$\mathbf{E} = [E_1(R), 0, 0]. \qquad \ldots (5.2)$$

It follows from (5.1) and (2.3), that Finger strain tensor c^{-1} and deformation gradient take the form

$$c^{-1} = \begin{bmatrix} 1 & 0 & f' \\ 0 & 1 & 0 \\ f' & 0 & 1+f'^2 \end{bmatrix} \qquad \text{... (5.3)}$$

$$c = \begin{bmatrix} 1+f'^2 & 0 & -f' \\ 0 & 1 & 0 \\ -f' & 0 & 1 \end{bmatrix} \qquad \text{... (5.4)}$$

$$c^{-2} = \begin{bmatrix} 1+f'^2 & 0 & 2f'+f'^3 \\ 0 & 1 & 0 \\ 2f'+f'^3 & 0 & 1+3f'^2+f'^4 \end{bmatrix} \qquad \text{... (5.5)}$$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ f' & 0 & 1 \end{bmatrix} \qquad \text{... (5.6)}$$

where the prime denotes the derivative with respect to the argument R. We notice that eqn. (5.6) satisfies the condition of incompressibility. Within the tube, free charge density q_e is taken as zero.

With the help of eqns. (5.1) to (5.6), eqn. (2.1) gives

$$E^{t_{11}} = -p + b_{-1} + b_1(1+f'^2) - 2[\chi_1 + 2\chi_2(1+f'^2)]E_1^2 \qquad \text{... (5.7)}$$

$$E^{t_{22}} = -p + b_{-1} + b_1 \qquad \text{... (5.8)}$$

$$E^{t_{33}} = -p + b_{-1} + b_1 + b_{-1}f'^2 \qquad \text{... (5.9)}$$

$$E^{t_{13}} = E^{t_{31}} = (b_{-1} - b_1)f' - \chi_1 f' E_1^2 - 2\chi_2(2f'+f'^3)E_1^2 \qquad \text{... (5.10)}$$

$$E^{t_{12}} = E^{t_{21}} = E^{t_{23}} = E^{t_{32}} = 0. \qquad \text{... (5.11)}$$

Using eqns. (2.5), (2.7), (5.2), (5.3), (5.5), (5.6), eqn. (2.2) gives

$$\left.\begin{aligned} P_1 &= (\chi + \chi_1 + \chi_2)E_1 + x_2 f'^2 E_1 \\ P_2 &= 0 \\ P_3 &= [\chi_1 f' + \chi_2(2f'+f'^3)]E_1 \end{aligned}\right] \qquad \text{... (5.12)}$$

$$E_1 = \frac{\alpha}{R}(\beta + \chi_2 f'^2)^{-1}, \; E_2 = E_3 = 0 \qquad \text{... (5.13)}$$

where $\beta = 1 + \chi + \chi_2 + \chi_3.$ ... (5.14)

Again to be specific, we consider the energy density function Σ of the form :

$$\Sigma = \alpha_1 (I_1 - 3) + \alpha_2 (I_2 - 3) + \alpha_4 (I_4 - 3) + \alpha_5 (I_5 - 3) + \alpha_6 (I_6 - 3). \quad \text{... (5.14a)}$$

With the help of eqns. (5.7) to (5.14a), the equations of equilibrium in the absence of body forces reduce to

$$-\frac{\partial p}{\partial R} + \frac{d}{dR}[2(\alpha_1 - \alpha_2) - 2\alpha_2 f'^2 + \{4[\alpha_5 + 2\alpha_6(1 + f'^2)] + R\}\ E_1^2$$
$$- \int_R \frac{1}{R}[2\alpha_2 f'^2 - 4\{\alpha_5 + 2\alpha_6(1 + f'^2)\}\ E_1^2 + RE_1 E_1'] \, dR] = 0 \quad \text{... (5.15)}$$

$$-\frac{\partial p}{\partial \Theta} = 0 \quad \text{... (5.16)}$$

$$-\frac{\partial p}{\partial Z} + \frac{d}{dR}[2(\alpha_1 + \alpha_2) f' + 2\{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2]$$
$$+ \frac{1}{R}[2(\alpha_1 + \alpha_2) f' + 2\ \{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2] = 0. \quad \text{... (5.17)}$$

If we define a function

$$G(R) = 2(\alpha_1 - \alpha_2) - 2\alpha_2 f'^2 + \{4[\alpha_5 + 2\alpha_6(1 + f'^2)] + R\} E_1^2$$
$$- \int_R \frac{1}{R}[2\alpha_2 f'^2 - 4(\alpha_5 + 2\alpha_6) E_1^2 + RE_1 E_1'] \, dR \quad \text{... (5.18)}$$

and introduce another function $\hat{p}$ through

$$\hat{p}(R, \Theta, Z) = p(R, \Theta, Z) - G(R) \quad \text{... (5.19)}$$

then eqns. (5.15) to (5.17) reduce to

$$-\frac{\partial \hat{p}}{\partial R} = 0 \quad \text{... (5.20)}$$

$$-\frac{\partial \hat{p}}{\partial \Theta} = 0 \quad \text{... (5.21)}$$

$$-\frac{\partial \hat{p}}{\partial Z} + \frac{d}{dR}[2(\alpha_1 + \alpha_2) f' + 2\{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2]$$
$$+ \frac{1}{R}[2(\alpha_1 + \alpha_2) f' + 2\{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2] = 0. \quad \text{... (5.22)}$$

We conclude that

$$\hat{p} = cZ + c' \qquad \text{... (5.23)}$$

where c and c' are constants.

Substituting (5.23) in eqn. (5.22), we obtain

$$\frac{d}{dR}[2(\alpha_1 + \alpha_2)f' + 2\{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2\ ]$$

$$+ \frac{1}{R}[2(\alpha_1 + \alpha_2)f' + 2\{\alpha_5 f' + 2\alpha_6(2f' + f'^3)\}\ E_1^2\ ] = 0. \qquad \text{... (5.24)}$$

Since we are considering the deformation taking place in the annular region between the inner and outer cylinders of radii R_1 and R_2. We suppose that the material is bounded to the cylinders at $R = R_1$ and $R = R_2$ with the shearing taking place due to the applied pressure gradient. The appropriate boundary conditions are

$$f(R_1) = 0 \text{ and } f(R_2) = 0. \qquad \text{... (5.25)}$$

Integration of eqn. (5.24) gives

$$4\alpha_6 E_1^2 f'^3 + 2[(\alpha_1 + \alpha_2) + (\alpha_5 + 4\alpha_6)E_1^2]f' - \left(\frac{cR^2 + 2D}{R}\right) = 0. \qquad \text{... (5.26)}$$

Solving eqn. (5.26), we obtain

$$f' = \left[\frac{(cR^2 + 2D)}{8R\alpha_6 E_1^2} + \frac{1}{2}\xi(R)\right]^{1/3} + \left[\frac{(cR^2 + 2D)}{8R\alpha_6 E_1^2} - \frac{1}{2}\xi(R)\right]^{1/3} \qquad \text{... (5.27)}$$

where

$$\xi(R) = \left[\frac{(cR^2 + 2D)^2}{16R^2\alpha_6^2 E_1^4} - \frac{1}{108\alpha_6^2 E_1^6}\left\{(\alpha_1 + \alpha_2) + (\alpha_5 + 4\alpha_6)E_1^2\right\}\right]^{1/2} \cdot \qquad \text{... (5.28)}$$

From eqn. (5.27), we obtain

$$f(R) = \int \left[\left\{\frac{(cR^2 + 2D)}{8R\alpha_6 E_1^2} + \frac{1}{2}\xi(R)\right\}^{1/3} + \left\{\frac{(cR^2 + 2D)}{8R\alpha_6 E_1^2} - \frac{1}{2}\xi(R)\right\}^{1/3}\right] dR + K$$

$$\text{... (5.29)}$$

where K is a constant of integration.

It is possible to solve eqn. (5.29) numerically.

In case the electric field is absent, the solution reduces to the elastic solution and is given by

$$f(R) = \frac{c}{8(\alpha_1 + \alpha_2)\log\frac{R_2}{R_1}}\left[(R^2 - R_2^2)\log\frac{R_2}{R_1} + (R_1^2 - R_2^2)\log\frac{R}{R_2}\right]. \qquad \text{... (5.30)}$$

REFERENCES

1. S. S. Antman, *A family of semi-inverse problems in non-linear elasticity, contemporary developments in continuum mechanics and partial differential equations*, 1978, pp. 1-24.
2. S. S. Antman and Z. H. Guo, *J. Elasticity* **14** (1984), 249-62.
3. M. M. Carroll, *J. Elasticity* **8** (1978), 323-28.
4. P. K. Currie and M. Hayes, In : *Proc. IUTAM Symp. on Finite Elasticity*, 1981, pp. 143-50.
5. K. R. Rajagopal and R. S. Winemann, *Int. J. Engng. Sci.* **23** (1985), 217-34.
6. K. R. Rajagopal, W. Troy, and A. S. Winemannn, *Proc. R. Irish Acad.* **86A** (1986), 107-14.
7. C. B. Senseing, *Commun. Pure Appl. Math.* **18** (1965), 147-56.
8. D. FU, K. R. Rajagopal and A. Szeri, *Int. J. Non-linear Mech.* **25** (1990), 1-13.
9. L. Tao and K. R. Rajagopal, *Arch. Mech.* **42** (1990), 729-34.
10. J. B. McLeon, K. R. Rajagopal and A. S. Winemann, *Proc. R. Irish Acad.* **88A** (1988), 91-101.
11. D. T. Chung, C. O. Horgan and R. Abeyaratne, *Int. J. Solids Struct* **22** (1986), 1557-70.
12. P. Boulanger and M. A. Hayes, *Proc. R. Irish Acad.* **90A** (1990), 1-12.
13. D. A. Isherwood and R. W. Ogden, *Int. J. Solids Struct.* **13** (1977), 105-23.
14. E. Varley and E. Cumberbatch, 'The finite deformation of an elastic material surrounding an elliptic hole', *Finite Elasticity* (1977), 51-64.
15. J. P. Zhang and K. R. Rajagopal, *Int. J. Engng. Sci.* **7** (1992), 919-38.
16. A. C. Eringen and G. A. Mougin, *Electrodynamics of Continua I*, Springer-Verlag, New York, 1990.