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EFFECT OF PERTURBED PORTNTIALS ON TRIANGULAR SOLUTIONS AT CRITICAL MASS

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In this note we have investigated the effect of perturbed potentials on the characteristic roots of the triangular solutions and the eccentricity of the retrograde elliptic periodic orbits around the triangular points in the restricted three-body problem at critical mass.

1. INTRODUCTION

Sharma and Subbarao[2] have shown that the triangular solution in the restricted three-body problem at critical mass is in general unstable due to the presence of secular terms codming through the double roots of the characteristic equation. They have shown that suitable choice of velocity components eliminates the secular terms and the oslution represents retrograde elliptic periodic motion. They have observed that unlike the classical case[3], the characteristic roots and the eccentricity are functions of the oblateness co-efficient of the more massive primary of oblate spheroidal shape whose equatorial plane coincides with the plane of motion.

Here we have investigated the effect of perturbed potentials as defined by Bhatnagar and Hallan[1] on the characteristic roots of the triangular solutions and the eccentricity of the retrograde elliptic periodic orbits around the triangular points in the restricted three-body problem at critical mass. We have applied the theory in the following cases :

(i) There are no perturbations in the potentials (classical problem)

(ii) Only the more massive primary is an oblate spheroid whose equatorial plane coincides with the plane of motion

(iii) Only the two primaries are oblate spheroids whose equatorial planes coincide with the plane of motion

(iv) All the three bodies are oblate spheroids whose equatorial planes coincide witht he plane of motion.

(v) The more massive primary is the source of radiation.

2. Discussion

Taking the results from Bhatnagar and Hallan[1] and proceeding as Sharma and Subbarao[2], we can establish the existence of retrograde elliptic orbits around the triangular libration points whose eccentricity is given by

$$e = (1 - m^2)^{\frac{1}{2}} \quad \text{... (1)}$$

where

$$m = (k + \overline{\lambda}_2^*)/(k + \overline{\lambda}_1^*) \quad \text{... (2)}$$

$$\overline{\lambda}_{1,2}^* = (2n^2 - k) \pm 2n\,(n^2 - k)^{\frac{1}{2}} \quad \text{... (3)}$$

$$k = (4n^2 - \Omega_{xx}^0 - \Omega_{yy}^0)/2$$

$$= \frac{1}{2}\Big[1 - \alpha_{12}\,U_{12}'(1) + \alpha_{13}\,(2U_{13}'(1) + U_{13}''(1))$$

$$+\ \mu\{\alpha_{13}\,(2U_{13}'(1) + U_{13}''(1)) - \alpha_{23}\,(2U_{23}'(1) + U_{23}''(1))\}\Big]. \quad \text{... (4)}$$

Hence

$$e = 2^{\frac{1}{2}}\,(2^{\frac{1}{2}} - 1)^{\frac{1}{2}}\,[1 + (2 - 2^{\frac{1}{2}})\ \{\alpha_{13}\,(2U_{13}'(1) + U_{13}''(1))$$

$$-\ \mu\{\alpha_{13}\,(2U_{13}'(1) + U_{13}''(1)) - \alpha_{23}\,(2U_{23}'(1) + U_{23}''(1))\}]. \quad \text{... (5)}$$

3. Applications

(i) There are no perturbations in the potentials (classical problem).

In this case, $\alpha_{ij} = 0$

$$k = 1/2$$

$$e = 2^{\frac{1}{2}}\,(2^{\frac{1}{2}} - 1)^{\frac{1}{2}}$$

(ii) Only the more massive primary is a oblate spheroid whose equatorial plane coincides with the plane of motion.

In this case, $\alpha_{12} = A_1,\ \alpha_{13} = A_1,\ \alpha_{23} = 0$

$$k = \frac{1}{2}\,\big[1 - 3(1 - 2\mu)\,A_1/2\,\big]$$

$$e = 2^{\frac{1}{2}} (2^{\frac{1}{2}} - 1)^{\frac{1}{2}} [1 + 3 (2 - 2^{\frac{1}{2}}) (1 - \mu) A_1/4].$$

(iii) Only the two primaries are oblate spheroids whose equatorial planes coincide with the plane of motion.

In this case, $\alpha_{12} = A_1 + A_2$, $\alpha_{13} = A_1$, $\alpha_{23} = A_2$

$$U_{ij} (r_{ij}) = 1/2r_{ij}^3 .$$

$$k = \frac{1}{2} \left[1 - 3 (1 - 2\mu) (A_1 + A_2)/2 \right]$$

$$e = 2^{\frac{1}{2}} (2^{\frac{1}{2}} - 1)^{\frac{1}{2}} [1 + 3 (2 - 2^{\frac{1}{2}}) \ \{(1 - \mu) A_1 - \mu A_2\}/4.$$

(iv) All the three bodies are oblate spheroids whose equatorial planes coincide with the plane of motion.

In this case, $\alpha_{12} = A_1 + A_2$, $\alpha_{13} = A_1 + A_3$, $A_{23} = A_2 + A_3$

(where A_3 is the oblateness co-efficient of the third body)

$$U_{ij} (r_{ij}) = 1/2r_{ij}^3 .$$

$$k = \frac{1}{2} \left[1 - 3 (1 - 2\mu) \ \{ (A_1 + A_2) - 3A_3\}/2\right]$$

$$e = 2^{1/2} (2^{1/2} - 1)^{1/2} [1 + 3(2 - 2^{1/2}) \ \{(1 - \mu) A_1 - \mu A_2 + (1 - 2\mu) A_3\}/4];$$

(v) The more massive primary is the source of radiation pressure.

In this case, $\alpha_{12} = 0 = \alpha_{23}$, $\alpha_{13} = 1 - q$

$$U_{12} (r_{12}) = U_{23} (r_{23}) = 0, \ U_{13} (r_{13}) = - 1/r_{13}$$

$$k = q/2$$

$$e = 2^{\frac{1}{2}} (2^{\frac{1}{2}} - 1)^{\frac{1}{2}}.$$

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