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MOTION OF TWO CHARGED PARTICLES IN THE FIELD OF TWO DIPOLES WITH THEIR CARRIER STARS

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The motion of two charged particles moving in the field of two rotating magnetic dipoles has been studied. The magnetic moments of the dipoles are taken perpendicular to the plane of the motion of the primaries. There exist fourteen equilibrium solutions; six solutions are collinear and eight solutions are found to be non-collinear. The stability of the equilibrium points has also been examined. It is further observed that all the collinear and non-collinear equilibrium points are unstable for given values of mass parameter μ and the dipoles moment parameter λ.

1. INTRODUCTION

Whipple[5] has studied the motion of two minor bodies P, Q of masses m_1 and m_2 under the gravitational forces of the two primaries of masses M_1 and M_2 which move on a circular Keplerian orbit about their centre of mass. These two primaries are attracting each other but are not perturbed by the minor bodies. We have modified the above problem as follows :

(a) We suppose that there are two dipoles of magnetic moments $\overline{M}_1$ and $\overline{M}_2$, which are participating in the circular motion of their carrier stars S_1 and S_2 moving around their centre of mass.

(b) There are two charged particle P and Q of charges q_1 and q_2 having masses m_1 and m_2 which are moving under the gravitational forces and the magnetic forces of S_1 and S_2.

(c) The motion is such that the dipoles S_1 and S_2 are not affected by P and Q but they do influence the motion of P and Q.

2. EQUATION OF MOTION

Let us consider two dipoles of magnetic moments $\overline{M}_1$ and $\overline{M}_2$. Suppose that the dipoles participate in the circular motion of their carrier stars S_1 and S_2 around their centre of mass O. Let there be two particles P and Q of charges q_1 and q_2 moving under the magnetic f(ces of dipoles and the gravitational forces of S_1 and S_2. Let the position vectors of P and Q at any time t be $\bar{r}_1$ and $\bar{r}_2$ respectively referred to a rotating frame of reference $O(x, y, z)$ which is rotating with the same angular velocity $\bar{\omega} = (0, 0, \omega)$ as the stars S_1 and S_2. Suppose initially the stars S_1 and S_2 lie on the x-axis. Since the angular velocity of the frame of reference $O(x, y, z)$ is the same as those of the stars S_1 and S_2, and as $t = 0$ they lie on the x-axis, they will

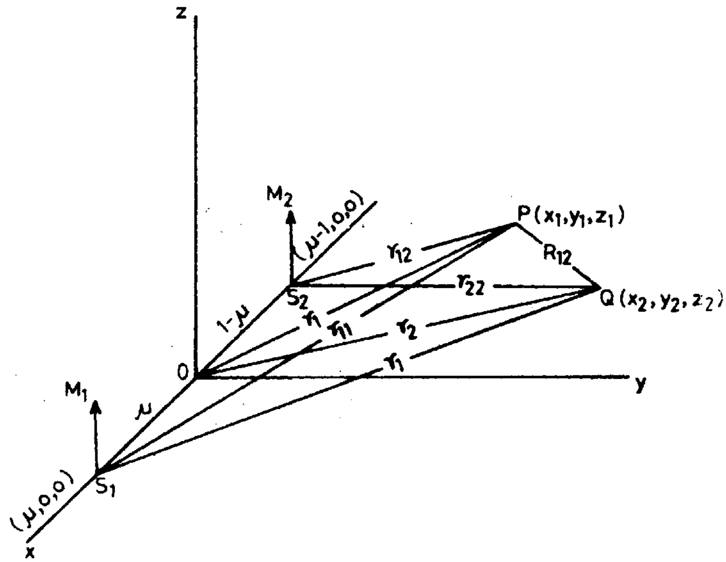

FIG. 1.

continue to lie on the x-axis. (Fig. 1). We choose the distance between S_1 and S_2 as the unit of distance and the sum of their masses as the unit of mass. If we take the mass of S_2 as μ, then the mass of S_1 will be $(1 - \mu)$. In this unit suppose the mass of P is μ_1 and of Q is μ_2. We choose the unit of time in such a way that the value of the gravitational constant G is unity.

Let $\quad \overline{S_1P} = \bar{r}_{11}, \ \overline{S_2P} = \bar{r}_{12}, \ \overline{S_1Q} = \bar{r}_{21}, \ \overline{S_2Q} = \bar{r}_{22}$

$$\overline{OP} = \bar{r}_1 = \overline{OQ} = \bar{r}_2 .$$

The equation of motion of the particles P and Q in the rotating frame is

$$\mu_i \left[\frac{\partial^2 \bar{r}_i}{\partial t^2} + 2\bar{\omega} \times \frac{\partial \bar{r}_i}{\partial t} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_i) \right] = - q_i/c \frac{d\bar{A}_i}{dt}$$

$$+ q_i/c \left[\left\{ \frac{\partial \bar{r}_i}{\partial t} + (\bar{\omega} \times \bar{r}_i) \right\} \times (\nabla_i \times \bar{A}_i) \right] - \frac{(1-\mu)\mu_i}{r_{i1}^3} \bar{r}_{i1}$$

$$- \frac{\mu_i \mu}{r_{i2}^3} \bar{r}_{i2} + (-1)^{i+1} \frac{\mu_1 \mu_2 \bar{R}_{12}}{R_{12}^3} + (-1)^i \frac{G' q_1 q_2 \bar{R}_{12}}{R_{12}^3}, \quad i = 1, 2, \quad \dots (1)$$

where

c = velocity of the light

$\bar{A}_i$ = vector potential = $\dfrac{\bar{M}_1 \times \bar{r}_{i1}}{r_{i1}^3} + \dfrac{\bar{M}_2 \times \bar{r}_{i2}}{r_{i2}^3}$, $(i = 1, 2)$

$\bar{R}_{12} = (x_2 - x_1)\bar{i} + (y_2 - y_1)\bar{j} + (z_2 - z_1)\bar{k}.$

G' = constant of proportionality of electrostatic force.

3. Stationary Solutions

For determining stationary solutions, we consider a particular case viz. when $\bar{M}_1 = (0, 0, 1)$ and $\bar{M}_2 = (0, 0, \lambda)$, then equation of motion (1) take the Cartesian component form :

$$\ddot{x}_i - \dot{y}_i \left\{ 2 - \left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3} \right) q_i/\mu_i c \right\} + 3 y_i z_i \dot{z}_i \left(\frac{1}{r_{i1}^5} + \frac{\lambda}{r_{i2}^5} \right) q_i/\mu_i c = - \frac{\partial U_i}{\partial x_i} \quad \dots (2)$$

$$\ddot{y}_i + \dot{x}_i \left\{ 2 - \left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3} \right) q_i/\mu_i c \right\} - 3 z_i \dot{z}_i \left(\frac{(x_i - \mu)}{r_{i1}^5} + \frac{\lambda (x_i + 1 - \mu)}{r_{i2}^5} \right) \times q_i/\mu_i c = - \frac{\partial U_i}{\partial y_i} \quad \dots (3)$$

and

$$\ddot{z}_i - 3 y_i z_i \dot{x}_i \left(\frac{1}{r_{i1}^5} + \frac{\lambda}{r_{i2}^5} \right) q_i/\mu_i c + 3 \dot{y}_i z_i \left\{ \frac{(x_i - \mu)}{r_{i1}^5} + \frac{\lambda (x_i + 1 - \mu)}{r_{i2}^5} \right\} \times q_i/\mu_i c = - \frac{\partial U_i}{\partial z_i} \quad \dots (4)$$

where

$$U_i = -(x_i^2 + y_i^2) \left\{ \frac{1}{2} + q_i/\mu_i c \left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3} \right) \right\} + q_i/\mu_i c \times \left\{ \frac{\mu}{r_{i1}^3} - \frac{\lambda (1 - \mu)}{r_{i2}^3} \right\} - \frac{(1 - \mu)}{r_{i1}} - \frac{\mu}{r_{i2}} - \frac{\mu_{3-i}}{R_{12}} + \frac{G' q_1 q_2}{R_{12} \mu_i}.$$

The stationary solutions are given by

$$\frac{\partial U_i}{\partial x_i} = \frac{\partial U_i}{\partial y_i} = \frac{\partial U_i}{\partial z_i} = 0. \qquad \dots (5)$$

So from eqn. (5) we get

$$q_i/\mu_i\, c \Bigg[\frac{(x_i-\mu)}{r_{i1}^3} + \frac{\lambda(x_i+1-\mu)}{r_{i2}^3}$$

$$+ x_i \left\{ \frac{-3(x_i-\mu)^2}{r_{i1}^5} - \frac{3\lambda(x_i+1-\mu)^2}{r_{i2}^5} + \frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3} \right\}$$

$$- y_i^2 \left\{ \frac{3(x_i-\mu)}{r_{i1}^5} + \frac{3\lambda(x_i+1-\mu)}{r_{i2}^5} \right\} \Bigg]$$

$$+ x_i - \frac{(1-\mu)(x_i-\mu)\mu_i}{r_{i1}^3\mu_i} - \frac{\mu_i\mu(x_i+1-\mu)}{r_{i2}^3\mu_i} + \frac{(-1)^{i+1}\mu_1\mu_2(x_2-x_1)}{R_{12}^3\mu_i}$$

$$- \frac{(-1)^i G'\, q_1 q_2 (x_2-x_1)}{R_{12}^3\mu_i} = 0, \qquad \dots (6)$$

$$q_i/\mu_i\, c \Bigg[\frac{y_i}{r_{i1}^3} + \frac{\lambda y_i}{r_{i2}^3} + \left\{ x_i - \frac{3y_i(x_i-\mu)}{r_{i1}^5} - y_i \frac{3\lambda(x_i+1-\mu)}{r_{i2}^5} \right\}$$

$$- y_i \left\{ \frac{3y_i^2}{r_{i1}^5} + \frac{3y_i^2\lambda}{r_{i2}^5} - \frac{1}{r_{i1}^3} - \frac{\lambda}{r_{i2}^3} \right\} \Bigg] - \frac{(1-\mu)y_i\mu_i}{r_{i1}^3\mu_i}$$

$$+ y_i - \frac{\mu y_i}{r_{i2}^3} + \frac{(-1)^{i+1}\mu_1\mu_2(y_2-y_1)}{R_{12}^3\mu_i} + \frac{(-1)^i G'\, q_1 q_2 (y_2-y_1)}{R_{12}^3\mu_i} = 0 \qquad \dots (7)$$

and

$$q_i/\mu_i\, c + \Bigg[x_i \left\{ \frac{-3(x_i-\mu)z_i}{r_{i1}^5} - \frac{3\lambda(x_i+1-\mu)z_i}{r_{i2}^5} \right\} - y_i$$

$$\times \left\{ \frac{3y_iz_i}{r_{i1}^5} + \frac{3y_iz_i\lambda}{r_{i2}^5} \right\} \Bigg] - \frac{(1-\mu)z_i\mu_i}{r_{i1}^3\mu_i} - \frac{\mu_1\mu_2 z_i}{r_{i2}^3\mu_i}$$

$$+ \frac{(-1)^{i+1}\mu_1\mu_2(z_2-z_1)}{R_{12}^3\mu_i} + \frac{(-1)^i G'\, q_1 q_2 (z_2-z_1)}{R_{12}^3\mu_i} = 0. \qquad \dots (8)$$

From the above equations we may consider the following three cases :

(1) $y = 0$, $z = 0$ (straight line solution)

(2) $y \neq 0$, $z = 0$ (solution in the x, y plane)

(3) $y = 0$, $z \neq 0$ (solution in the x, z plane).

Case I : When $y = 0$, $z = 0$ (solution around L_{1k}), $k = (1, 2, 3)$

In this case eqns. (7) and (8) are satisfied and the value of x_i are given by eqn. (6) after putting $y = 0$, $z = 0$. These correspond to collinear equilibrium points. The solution of this equation may be expressed as power series in the small parameters $\varepsilon_1, \varepsilon_2$ where

$$\varepsilon_i = \frac{\mu_i}{[\mu_1 + \mu_2]^{2/3}} << 1 \text{ for } i = 1, 2.$$

Let $$x_1 = L_{1k} + \sum_{j=1}^{\infty} a_{1j}\, \varepsilon_2^j \qquad \text{... (8a)}$$

and $$x_2 = L_{1k} + \sum_{j=1}^{\infty} a_{2j}\, \varepsilon_1^j \text{ for } k = 1, 2, 3.$$

L_{1k} corresponds to the values when the charge $q_2 = 0$, $\lambda = 1$ and certain values of μ. Here we have also assumed that the perturbation due to q_2 is small.

The values of L_{1k} have been worked out by Arif[2] for $\lambda = 1$ and certain values of μ.

Solution around the point L_{11} — Now from eqn. (6) after putting $y = 0$, $z = 0$ and $i = 1$ for x_1 and then putting $x_1 = L_{11} + a_{11}\, \varepsilon_2$, we have

$$-\frac{\mu_2 (x_2 - x_1)}{|x_2 - x_1|^3} - \frac{G' q_1 q_2 (x_2 - x_1)}{\mu_1 |x_2 - x_1|^3} + a_{11}\, \varepsilon_2 p_1 = 0, \qquad \text{... (9)}$$

where

$$P_1 = 1 + q_1/\mu_1 c \left\{ \frac{6L_{11}}{(L_{11} - \mu)^4} + \frac{6L_{11}\lambda}{(L_{11} + 1 - \mu)^4} - \frac{4}{(L_{11} - \mu)^3} - \frac{4\lambda}{(L_{11} + 1 - \mu)^3} \right\}$$

$$+ \frac{2(1-\mu)}{(L_{11} - \mu)^3} + \frac{2\mu}{(L_{11} + 1 - \mu)^3}.$$

Here, we have taken $r_{11} = L_{11} + a_{11}\, \varepsilon_2 - \mu$

and $r_{12} = L_{11} + a_{11}\, \varepsilon_2 + 1 - \mu$.

Thus for x_2 equation (6) takes the form as

$$-\frac{\mu_1 (x_2 - x_1)}{|x_2 - x_1|^3} + \frac{G' q_1 q_2 (x_2 - x_1)}{\mu_2 |x_2 - x_1|^3} + a_{21} P_2 = 0, \qquad \text{... (10)}$$

where

$$P_2 = 1 + q_2/\mu_2 c \left(\frac{6L_{11}}{(L_{11} - \mu)^4} + \frac{6L_{11}\lambda}{(L_{11} + 1 - \mu)^4} - \frac{4}{(L_{11} - \mu)^3} - \frac{4\lambda}{(L_{11} + 1 - \mu)^3} \right)$$

$$+ \frac{2(1-\mu)}{(L_{11} - \mu)^3} + \frac{2\mu}{(L_{11} + 1 - \mu)^3}.$$

Here

$$r_{11} = L_{11} + a_{21}\,\varepsilon_1 - \mu$$

and $$r_{12} = L_{11} + a_{21}\,\varepsilon_1 + 1 - \mu.$$

Now eqns. (9) and (10) may be combined to yield

$$\varepsilon_2\,\mu_1\,[a_{11} + a_{21}\,\lambda_1] = 0$$

where $$\lambda_1 = P_2/P_1.$$

But $$\varepsilon_2\,\mu_1 \neq 0 \text{ and so}$$

$$a_{11} = -\,a_{21}\,\lambda_1.$$

Equation (9) can now be written as

$$\frac{E_1}{|\,a_{11}\,|^3} = P_1\ ,$$

where $$E_1 = \frac{(\mu_1 + \mu_2\,\lambda_1)\,|\,(\mu_1 + \mu_2)^{2/3}\,\lambda_1\,|^3}{\lambda_1\,|\,\mu_1 + \mu_2\,\lambda_1\,|^3}\left[1 - \frac{G'\,q_1\,q_2}{\mu_1\,\mu_2}\right].$$

Thus

$$a_{11} = \pm\left[\frac{E_1}{P_1}\right]^{1/3},$$

and the stationary solution around L_{11} corresponds to

$$x_1 = L_{11} \pm \left(\frac{E_1}{P_1}\right)^{1/3} \frac{\mu_2}{(\mu_1 + \mu_2)^{2/3}}$$

and

$$x_2 = L_{11} \mp \left(\frac{(E_1/P_1)^{1/3}}{\lambda_1}\right)^{1/3} \frac{\mu_2}{(\mu_1 + \mu_2)^{2/3}}\,.$$

Similarly, we find the stationary solutions around L_{12} and L_{13} are respectively given by

$$x_1 = L_{12} \pm \left(\frac{E_2}{P_3}\right)^{1/3} \frac{\mu_2}{(\mu_1 + \mu_2)^{2/3}}$$

$$x_2 = L_{12} \mp \frac{(E_2/P_3)^{1/3}}{\lambda_2}\,\frac{\mu_1}{(\mu_1 + \mu_2)^{2/3}}$$

and

$$x_1 = L_{13} \pm \left(\frac{E_3}{P_5}\right)^{1/3} \frac{\mu_2}{(\mu_1 + \mu_2)^{2/3}}$$

$$x_2 = L_{13} \mp \frac{(E_3/P_5)^{1/3}}{\lambda_3} \frac{\mu_1}{(\mu_1 + \mu_2)^{2/3}}$$

where

$$E_i = \frac{(\mu_1 + \mu_2 \lambda_i) \mid (\mu_1 + \mu_2)^{2/3} \lambda_i \mid^3}{\lambda_i \{ \mu_1 + \mu_2 \lambda_i \}^3} \left[1 - \frac{G' q_1 q_2}{\mu_1 \mu_2} \right], \; i = (2, 3)$$

$$\lambda_2 = P_4/P_3$$

$$\lambda_3 = P_6/P_5$$

$$P_3 = 1 + q_1/\mu_1 c \left[-\frac{6L_{12}}{(\mu - L_{12})^4} + \frac{6L_{12}\lambda}{(L_{12} + 1 - \mu)^4} - \frac{4}{(\mu - L_{12})^3} - \frac{4\lambda}{(L_{12} + 1 - \mu)^3} \right] + \frac{2(1-\mu)}{(\mu - L_{12})^3} + \frac{2\mu}{(L_{12} + 1 - \mu)^3}$$

$$P_4 = 1 + q_2/\mu_2 c \left[-\frac{6L_{12}}{(\mu - L_{12})^4} + \frac{6L_{12}\lambda}{(L_{12} + 1 - \mu)^4} - \frac{4}{(\mu - L_{12})^3} - \frac{4\lambda}{(L_{12} + 1 - \mu)^3} \right] + \frac{2(1-\mu)}{(\mu - L_{12})^3} + \frac{2}{(L_{12} + 1 - \mu)^3}$$

$$P_5 = 1 + q_1/\mu_1 c \left[-\frac{6L_{13}}{(\mu - L_{13})^4} + \frac{6\lambda L_{13}\lambda}{(\mu - L_{13} - 1)^4} - \frac{4}{(\mu - L_{13})^3} - \frac{4\lambda}{(\mu - L_{13} - 1)^3} \right] + \frac{2(1-\mu)}{(\mu - L_{13})^3} + \frac{2\mu}{(\mu - L_{13} - 1)^3}$$

and

$$P_6 = 1 + q_2/\mu_2 c \left[-\frac{6L_{13}}{(\mu - L_{13})^4} + \frac{6\lambda L_{13}}{(\mu - L_{13} - 1)^4} - \frac{4}{(\mu - L_{13})^3} - \frac{4\lambda}{(\mu - L_{13} - 1)^3} \right] + \frac{2(1-\mu)}{(\mu - L_{13})^3} + \frac{2\mu}{(\mu - L_{13} - 1)^3}.$$

Figures 2, 3, 4 show the locations of the collinear equilibrium points L_1, L_2, L_3 respectively for various values of μ. The dotted lines indicate (in each case) the location of equilibrium points the Whipple case; that is when the dipoles at the primaries are not taken into account. The full line indicates the locations of equilibrium points in our case; that is when the dipoles at the primaries are also taken into account.

Here we have taken the unit of charge such that $\frac{1}{4\pi \varepsilon_0} = G' = 1$, $m_1 = \mu \times (10 \times$ mass of the sun) and $m_2 = (1 - \mu) \times (10 \times$ mass of the sun) (the mass of the sun $= 2 \times 10^{23}$ gm).

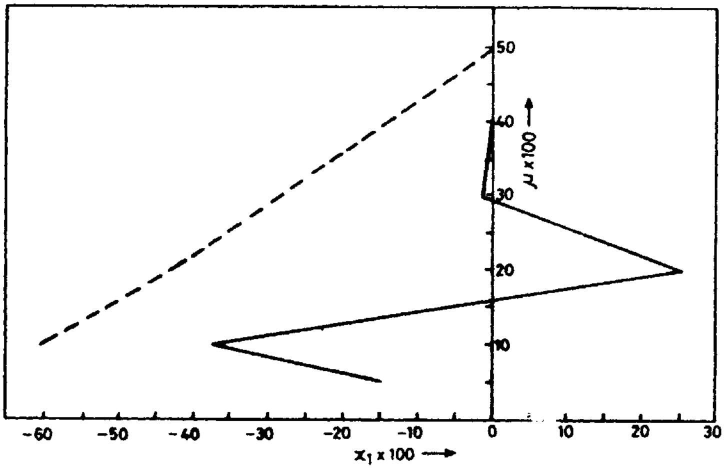

FIG. 2.

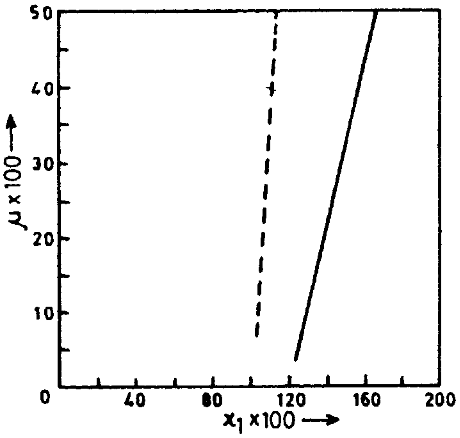

FIG. 3.

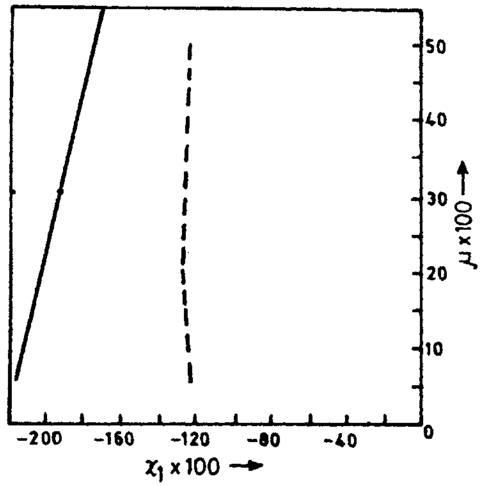

FIG. 4.

We have chosen the distance between the primaries as 4 light year and $q_1/\mu_1 c = \nu_1$ and $q_2/\mu_2 c = 1 - \nu_1$, $(\nu_1 < 1)$. Finally we define $\mu_1 = 0.01 \times \mu$ and $\mu_2 = 0.02 \times \mu$.

It is observed that due to dipoles at the primaries the equilibrium points L_1 and L_3 move away from the centre of mass. In the case of L_1 this deviation increases as μ increases. For $0.05 \leq \mu \leq 0.5$ this deviation lies between 0.2368 and 0.51935, in the case of L_3 this deviation decreases as μ increases. For $0.05 \leq \mu \leq 0.5$ this deviation lies between 0.9335 and 0.5112. The point L_2 moves towards the centre of mass for $0.05 \leq \mu \leq 0.5$ and this deviation decreases as μ increases.

Case II : $y_i \neq 0$, $z_i = 0$: Stationary solution around $L_{x_{1k}}$ and $L_{y_{1k}}$ for $k = 4, 5$

In this case eqn. (8) is satisfied and we may calculate the values of x_i and y_i from eqns. (6) and (7) after putting $z_i = 0$. The solution of eqns. (6) and (7) may be expressed as power series in the mass parameters ε_i $(i = 1, 2)$.

Let $$x_i = L_{x_{1k}} + \sum_{j=1}^{\infty} a_{ij}\, \varepsilon_{3-i}^{j},$$

$$y_i = L_{y_{1k}} + \sum_{j=1}^{\infty} b_{ij}\, \varepsilon_{3-i}^{j} \text{ for } i = 1, 2, \text{ and } k = 4, 5.$$

The values of $L_{x_{1k}}$ and $L_{y_{1k}}$ have been worked out by Arif[2] where $L_{x_{1k}}$ and $L_{y_{1k}}$ correspond to the values when the charge $q_2 = 0$.

Let us consider a particular case

$$q_1/\mu_1 c = q_2/\mu_2 c = \nu, \text{ say.}$$

Then, from eqn. (6), we have

$$a_{11}\, \varepsilon_2\, \psi_1 + b_{11}\, \varepsilon_2\, \psi_2 - \frac{G'\, \mu_2\, c^2\, \nu^2\, (x_2 - x_1)}{R_{12}^3} + \frac{\mu_2\, (x_2 - x_1)}{R_{12}^3} = 0, \quad \dots (11)$$

for $i = 1$,

and

$$a_{21}\, \varepsilon_1\, \psi_1 + b_{21}\, \varepsilon_1\, \psi_2 - \frac{G'\, \mu_1\, c^2\, \nu^2\, (x_2 - x_1)}{R_{12}^3} + \frac{\mu_1\, (x_2 - x_1)}{R_{12}^3} = 0, \quad \dots (12)$$

for $i = 2$,

where

$$\begin{aligned}
\psi_1 = {} & 2\,\nu\, s_{1k}^{-3/2} - s_{1k}^{-3/2}(1-\mu) - 3\,\nu\, s_{1k}^{-5/2}(L_{x_{1k}} - \mu)^2 \\
& - 3\,\nu\, s_{1k}^{-5/2} L_{x_{1k}} (L_{x_{1k}} - \mu) + 3\,(1-\mu)\, s_{1k}^{-5/2} (L_{x_{1k}} - \mu)^2 \\
& + 2\,\nu\, T_{1k}^{-3/2}\, \lambda - \mu\, T_{2k}^{-3/2} - 3\,(L_{x_{1k}} + 1 - \mu)^2\, T_{1k}^{-5/2} \\
& - 3\,\nu\, \lambda\, L_{x_{1k}}\, T_{1k}^{-5/2} (L_{x_{1k}} + 1 - \mu) + 3\,\mu\, T_{1k}^{-5/2} \\
& \times (L_{x_{1k}} + 1 - \mu)^2 - 6\,\nu\, L_{x_{1k}}\, s_{1k}^{-5/2} (L_{x_{1k}} - \mu) - 3\nu\, s_{1k}^{-5/2} (L_{x_{1k}} - \mu)^2 \\
& - 3\nu\, L_{y_{1k}}^2\, s_{1k}^{-5/2} + 15\,\nu\, L_{x_{1k}}\, s_{1k}^{-7/2} (L_{x_{1k}} - \mu)^3 + 15\,\nu\, L_{y_{1k}}^3 (L_{x_{1k}} - \mu)^2 \\
& \times s_{1k}^{-7/2} - 6\,\nu\, L_{x_{1k}}\, T_{1k}^{-5/2} \lambda\, (L_{x_{1k}} + 1 - \mu) - 3\,\nu\, T_{1k}^{-5/2} \lambda\, (L_{x_{1k}} + 1 - \mu)^2 \\
& - 3\,\nu\, L_{y_{1k}}^2\, T_{1k}^{-5/2} \lambda + 15\,\nu\, L_{x_{1k}}\, \lambda\, T_{1k}^{-7/2} (L_{x_{1k}} + 1 - \mu)^3\, \lambda \\
& + 15\,\nu\, L_{y_{1k}}^2\, (L_{x_{1k}} + 1 - \mu)^2\, T_{1k}^{-7/2} + 1,
\end{aligned}$$

$$\begin{aligned}\psi_2 = &-3\,\nu\, L_{y_{1k}}(L_{x_{1k}}-\mu)\, s_{1k} - 3\,\nu\, L_{x_{1k}} L_{y_{1k}}\, s_{1k}^{-5/2} \\ &+ 3(1-\mu)\, L_{y_{1k}}(L_{x_{1k}}-\mu)\, s_{1k}^{-5/2} - 3\,\nu\,(L_{x_{1k}}+1-\mu)\, T_{1k}^{-5/2} L_{y_{1k}} \\ &- 3\,\nu\,\lambda\, L_{x_{1k}} T_{1k}^{-5/2} L_{y_{1k}} + 3\,\mu\, T_{1k}^{-5/2}\,\lambda\,(L_{x_{1k}}+1-\mu)\, L_{y_{1k}} \\ &- 6\,\nu\, L_{y_{1k}}(L_{x_{1k}}-\mu)\, s_{1k}^{-5/2} + 15\,\nu\, L_{x_{1k}}(L_{x_{1k}}-\mu)^2 s_{1k}^{-7/2} L_{y_{1k}} \\ &+ 15\,\nu\, L_{y_{1k}}^3 s_{1k}^{-7/2}(L_{x_{1k}}-\mu) - 6\,\lambda\,\nu\, L_{y_{1k}}(L_{x_{1k}}+1-\mu)\, T_{1k}^{-5/2} \\ &+ 15\,\nu\,\lambda\, L_{x_{1k}}(L_{x_{1k}}+1-\mu)^2 L_{y_{1k}} T_{1k}^{-7/2} + 15\,\lambda\,\nu\, L_{y_{1k}}^3 (L_{x_{1k}}+1-\mu)\, T_{1k}^{-7/2},\end{aligned}$$

where $s_{1k} = (L_{x_{1k}}-\mu)^2 + y_{1k}^2$ and $T_{1k} = (L_{x_{1k}}+1-\mu)^2 + y_{1k}^2$.

Similarly, from eqn. (7), we have

$$a_{11}\,\varepsilon_2\,\psi_2 + b_{11}\,\varepsilon_2\,\psi_3 - \frac{G'\,\mu_2\,\nu^2 c^2 (y_2-y_1)}{R_{12}^3} + \frac{\mu_2 (y_2-y_1)}{R_{12}^3} = 0, \quad \text{for } i = 1 \qquad \dots (13)$$

and

$$a_{21}\,\varepsilon_1\,\psi_2 + b_{21}\,\varepsilon_1\,\psi_3 + \frac{G'\,\mu_1\,\nu^2 c^2 (y_2-y_1)}{R_{12}^3} - \frac{\mu_1 (y_2-y_1)}{R_{12}^3} = 0, \quad \text{for } i = 2 \qquad \dots (14)$$

where

$$\begin{aligned}\psi_3 = &\ 2\,\nu\, s_{1k}^{-3/2} - (1-\mu)\, s_{1k}^{-3/2} - 6\,\nu\,\lambda\, L_{y_{1k}}^2 s_{1k}^{-5/2} \\ &+ 3(1-\mu)\, L_{y_{1k}}^2 s_{1k}^{-5/2} + 2\,\nu\,\lambda\, T_{1k}^{-3/2} - \mu\, T_{1k}^{-3/2} - 6\,\nu\, L_{y_{1k}}^2 T_{1k}^{-5/2}\,\lambda \\ &+ 3\,\mu\, L_{y_{1k}}^2 T_{1k}^{-5/2} - 3\,\nu\, L_{x_{1k}}^2 s_{1k}^{-5/2} + 3\,\nu\, L_{x_{1k}}\,\mu\, s_{1k}^{-5/2} \\ &+ 15\, L_{x_{1k}}^2 L_{y_{1k}}^2 s_{1k}^{-7/2} - 15\,\nu\, L_{x_{1k}}\,\mu\, L_{y_{1k}}^2 s_{1k}^{-7/2} - 3\,\nu\, L_{y_{1k}}^2 s_{1k}^{-5/2} - 6 L_{y_{1k}}^2\,\nu\, s_{1k}^{-5/2} \\ &+ 15\,\nu\, L_{y_{1k}}^4 s_{1k}^{-7/2} - 3\,\nu\,\lambda\, T_{1k}^{-5/2} L_{x_{1k}}^2 - 3\,\nu\, L_{x_{1k}}(1-\mu)\,\lambda\, T_{1k}^{-5/2} \\ &+ 15\,\nu\,\lambda\, L_{x_{1k}}^2 L_{y_{1k}}^2 T_{1k}^{-7/2} + 15\,\nu\,\lambda\, L_{x_{1k}}(1-\mu)\, L_{y_{1k}}^2 T_{1k}^{-7/2} \\ &- 3\,\nu\, L_{y_{1k}}^2\,\lambda\, T_{1k}^{-5/2} - 6\,\lambda\, L_{y_{1k}}^2\,\nu\, T_{1k}^{-5/2} + 15\,\nu\, L_{y_{1k}}^4 T_{1k}^{-7/2} + 1.\end{aligned}$$

Combining eqn. (11) with (12) and (13) with (14) we have

$$\mu_1\,\varepsilon_2\,(a_{11}\,\psi_1 + b_{11}\,\psi_2) + \mu_2\,\varepsilon_1\,(a_{21}\,\psi_1 + b_{21}\,\psi_2) = 0, \qquad \dots (15)$$

$$\mu_1\,\varepsilon_2\,(a_{11}\,\psi_2 + b_{11}\,\psi_3) + \mu_2\,\varepsilon_1\,(a_{21}\,\psi_2 + b_{21}\,\psi_3) = 0. \qquad \dots (16)$$

Equations (15) and (16) lead to

$$(a_{11}+a_{21})(\psi_1\psi_3-\psi_2^2) = 0$$

and

$$(b_{11}+b_{21})(\psi_2^2-\psi_3\psi_1) = 0.$$

But $\quad \psi_1\psi_3-\psi_2^2 \neq 0$ and so $a_{11}=-a_{21}$.

Similarly $\quad b_{11}=-b_{21}$.

Now from eqns. (11) and (13) we have

$$a_{11}\varepsilon_2\psi_1+b_{11}\psi_2\varepsilon_2+\frac{\delta\varepsilon_2 a_{11}}{(a_{11}^2+b_{11}^2)^{3/2}} = 0 \qquad \dots (17)$$

and

$$a_{11}\varepsilon_2\psi_2+b_{11}\varepsilon_2\psi_3+\frac{\delta\varepsilon_2 b_{11}}{(a_{11}^2+b_{11}^2)^{3/2}} = 0, \qquad \dots (18)$$

where $\quad \delta = -1+G'\nu^2c^2$.

Further from eqns. (17) and (18), we get

$$a_{11}^2+\frac{a_{11}b_{11}(\psi_3-\psi_1)}{\psi_2}-b_{11}^2 = 0,$$

which leads to

$$(a_{11}+\alpha_1 b_{11})(a_{11}+\alpha_2 b_{11}) = 0,$$

where $\quad \alpha_1=\beta+\gamma\ ,\ \alpha_2=\beta-\gamma,$

$$\beta=\frac{\psi_3-\psi_1}{2\psi_2},\quad \gamma=\frac{\left\{(\psi_1-\psi_3)^2+4\psi_2^2\right\}^{1/2}}{4\psi_2^2}.$$

Thus there exist two cases :

(i) $a_{11}=-\alpha_1 b_{11}$

(ii) $a_{11}=-\alpha_2 b_{11}$.

From eqn. (18) we have

$$b_{11}=\pm\frac{\delta^{1/3}}{(\alpha_i\psi_2-\psi_3)^{1/3}(\alpha_i^2+1)^{1/2}} \text{ for } i = 1,\ 2,$$

while from eqn. (17) we get

$$a_{11}=\pm\frac{\delta^{1/3}}{\left(1+\left(\frac{1}{\alpha_i}\right)^2\right)^{1/2}\left(\frac{\psi_2}{\alpha_i}-\psi_1\right)^{1/3}}$$

Thus the equilibrium solutions around $L_{x_{1k}}$ and $L_{y_{1k}}$ are as follows :

Case (i)

$$x_{ik} = L_{x_{1k}} \pm \frac{\alpha_j (-1)^i \mu_{3-i}\, \delta^{1/3}}{[(\mu_1 + \mu_2)^2 (\alpha_j \psi_2 - \psi_3)]^{1/3} (1 + \alpha_j^2)^{1/2}}$$

and

$$y_{ik} = L_{y_{1k}} \pm \frac{(-1)^{i+1} \mu_{3-i}\, \delta^{1/3}}{[(\mu_1 + \mu_2)^2 (\psi_2 \alpha_j - \psi_3)]^{1/3} (1 + \alpha_j^2)},$$

when $k = 4$, and for $i = 1, 2$ and $j = 1$.

Case (ii)

$$x_{ik} = L_{x_{1k}} \pm \frac{(-1)^{i+1} \mu_{3-i}\, \delta^{1/3}}{\left[(\mu_1 + \mu_2)^2 \left(\frac{\psi_2}{\alpha_j} - \psi_1\right)\right]^{1/3} \left(1 + \frac{1}{\alpha_j^2}\right)^{1/2}}$$

and

$$y_{ik} = L_{y_{1k}} \pm \frac{(-1)^i \mu_{3-i}\, \delta^{1/3}}{\alpha_j \left[(\mu_1 + \mu_2)^2 \left(\frac{\psi_2}{\alpha_j} - \psi_1\right)\right]^{1/3} \left(1 + \frac{1}{\alpha_{down}} 10\ j\ jsup2\right)^{1/2}}$$

for $i = 1, 2$, and $j = 2$.

The equilibrium solutions around $L_{x_{15}}$ and $L_{y_{15}}$ can be found by putting $k = 5$, $j = 2$ in case (i) and $j = 1$, $k = 5$, $i = \pm 1$ in case (ii).

From Fig. 5 we observe that there exist two classes of non-collinear equilibrium solutions when we take $\mu_1 = 0.01 \times \mu$, $\mu_2 = 0.02 \times \mu$, $\mu = 0.05$, $q_1/\mu_1 c = q_2/\mu_2 c$ and the ratio of the magnetic moment $\lambda = 1$. One class of solution lies approximately along a line which passes through the centre of mass of the primaries while the second class of solution lies approximately along a line that is perpendicular to the first. Further we observe that if we change the value of λ then there exists only one class of solution which lies approximately along a line which passes through the centre of mass of the primaries for $\lambda = 2.5$ (Fig. 6).

4. Stability of the Equilibrium Points

Now we wish to study the stability of the equilibrium points. We have already determined the equation of motion. In two dimensions they are given by (2) and (3) by putting $z = 0$; thus we have

$$\ddot{x}_i - \dot{y}_i \left[2 - q_i/\mu_i c \left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3}\right)\right] = -\frac{\partial u_i}{\partial x_i} \qquad \text{... (19)}$$

and

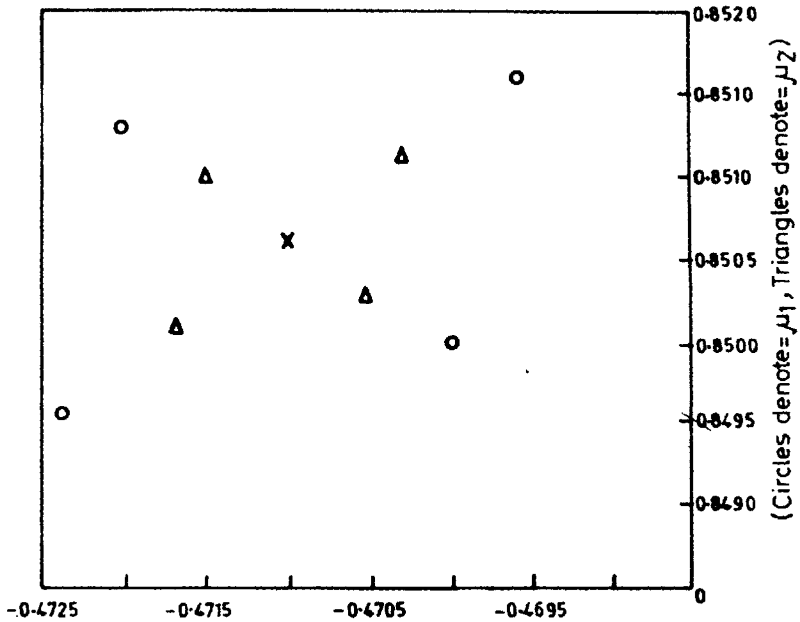

FIG. 5.

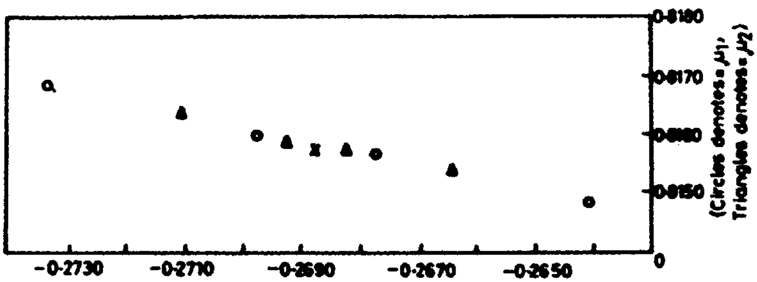

FIG. 6.

$$\ddot{y}_i + \dot{x}_i\left[2 - q_i/\mu_i\, c\left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3}\right)\right] = -\frac{\partial u_i}{\partial y_i}\,, \qquad \ldots (20)$$

where

$$u_i = -(x_i^2 + y_i^2)\left[\frac{1}{2} - q_i/\mu_i\, c\left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3}\right)\right] + q_i/\mu_i\, c x_i\left[\frac{\mu}{r_{i1}^3} - \frac{\lambda(1-\mu)}{r_{i2}^3}\right]$$

$$-\frac{(1-\mu)}{r_{i1}} - \frac{\mu}{r_{i2}} - \frac{\mu_{3-i}}{R_{12}} + \frac{G'\, q_1 q_2}{R_{12}\,\mu_i}\,, \quad (i = 1,\ 2).$$

Let (x_i^0, y_i^0) be the coordinates of any one of the equilibrium points.

Let $x_i = x_i^0 + \xi_i$ and $y_i = y_i^0 + \eta_i$, with $\xi_i, \eta_i << 1$.

Then the equations of motion become

$$\ddot{\xi}_i - \dot{\eta}_i R_i = -\xi_i \left(\frac{\partial^2 u_i}{\partial x_i^2}\right)^0 - \eta_i \left(\frac{\partial^2 u_i}{\partial x_i \partial y_i}\right)^0 ,$$

$$\ddot{\eta}_i + \dot{\xi}_i R_i = -\xi_i \left(\frac{\partial^2 u_i}{\partial x_i \partial y_i}\right)^0 - \eta_i \left(\frac{\partial^2 u_i}{\partial y_i^2}\right)^0 ,$$

where

$$\left(\frac{\partial^2 u}{\partial x_i^2}\right)^0 = -2\left[\frac{1}{2} + q_i/\mu_i c\left(\frac{1}{r_{i1}^3} + \frac{\lambda}{r_{i2}^3}\right)\right] + 12 x_i^0 q_i/\mu_i c$$

$$\times \left\{\frac{x_i^0 - \mu}{r_{i1}^5} + \frac{(x_i^0 + 1 - \mu)\lambda}{r_{i2}^5}\right\} - 15\,(x_i^{02} + y_i^{02})$$

$$\times \left[\left\{ q_i/\mu_i c \frac{(x_i^0 - \mu)^2}{r_{i1}^7} + \frac{\lambda (x_i^0 + 1 - \mu)^2}{r_{i2}^7}\right\}\right]$$

$$+ 3\,(x_i^{02} + y_i^{02}) + q_i/\mu_i c\left[\frac{1}{r_{i1}^5} + \frac{\lambda}{r_{i2}^5}\right]$$

$$- 6 q_i/\mu_i c\left[\frac{\mu (x_i^0 - \mu)}{r_{i1}^2} - \frac{\lambda (1 - \mu (x_i^0 + 1 - \mu)^2}{r_{i2}^5}\right]$$

$$+ 15 x_i^0\left[\frac{\mu (x_i^0 - \mu)^2}{r_{i1}^7} - \frac{\lambda (1 - \mu)(x_i^0 + 1 - \mu)^2}{r_{i2}^7}\right]$$

$$\times q_i/\mu_i c - 3 x_i^0 q_i/\mu_i c\left[\frac{\mu}{r_{i1}^5} - \frac{\lambda (1 - \mu)}{r_{i2}^5}\right]$$

$$+ \frac{(1 - \mu)}{r_{i1}^3} + \frac{\mu}{r_{i2}^3} + \frac{\mu_{3-i}}{R_{12}^3} - \frac{3(1 - \mu)(x_i^0 - \mu)^2}{R_{12}^5}$$

$$- \frac{3\mu (x_i^0 + 1 - \mu)^2}{r_{i2}^5} - \frac{3\mu_{3-i}(x_2^0 - x_1^0)^2}{R_{12}^5}$$

$$+ \frac{3 G' q_1 q_2 (x_2^0 - x_1^0)^2}{R_{12}^5 \mu_i} - \frac{G' q_1 q_2}{\mu_i R_{12}^3} ,$$

$$\left(\frac{\partial^2 u_i}{\partial x_i\,\partial y_i}\right)^0 = 6x_i^0\, q_i/\mu_i c\left(\frac{y_i^0}{r_{i1}^5}+\frac{y_i^0\lambda}{r_{i2}^5}\right)+6y_i^0$$

$$\times\left[\,q_i/\mu_i c\left[\frac{x_i^0-\mu}{r_{i1}^5}+\frac{\lambda\,(x_i^0+1-\mu)}{r_{i2}^5}\right]-15y_i^0\,(x_i^{0^2}+y_i^{0^2})\right.$$

$$\times\left[\,q_i/\mu_i c\left[\frac{(x_i^0-\mu)}{r_{i1}^7}+\frac{\lambda\,(x_i^0+1-\mu)}{r_{i2}^7}\right]-3\,y_i^0\,q_i/\mu_i c\right.$$

$$\times\left[\frac{\mu}{r_{i1}^5}-\frac{\lambda(1-\mu)}{r_{i2}^5}\right]+15\,x_i^0\,y_i^0\,\frac{\mu(x_i^0-\mu)}{r_{i1}^7}$$

$$-\frac{\lambda\,(1-\mu)\,(x_i^0+1-\mu)}{r_{i2}^7}-\frac{3\,(1-\mu)\,y_i^0\,(x_i^0-\mu)}{r_{i2}^5}-\frac{3\mu\,(x_i^0+1-\mu)\,y_i^0}{r_{i2}^3}$$

$$-\frac{3\mu_{3-i}\,(y_2^0-y_1^0)\,(x_2^0-x_1^0)}{R_{12}^5}+\frac{3\,G'\,q_1\,q_2}{R_{12\,i}^5}\,(y_2^0-y_1^0)\,(x_2^0-x_1)\ ,$$

$$\left(\frac{\partial^2 u_i}{\partial y_i^2}\right)^0 = -2\left[2q_i/\mu_i c\left\{\frac{1}{r_{i1}^3}+\frac{\lambda}{r_{i2}^3}\right\}\Big/+\frac{1}{2}\right]+12\,q_i/\mu_i c\,y_i^{0^2}$$

$$\times\left(\frac{1}{r_{i1}^5}+\frac{1}{r_{i2}^5}\right)+3\,(x_i^{0^2}+y_i^{0^2})\,q_i/\mu_i c\left(\frac{1}{r_{i1}^5}+\frac{\lambda}{r_{i2}^5}\right)$$

$$-15\,y_i^{0^2}\,(x_i^0+y_i^0)\,q_i/\mu_i c\left(\frac{1}{r_{i1}^7}+\frac{\lambda}{r_{i2}^7}\right)-3\,x_i^0\,q_i/\mu_i c\left[\frac{\mu}{r_{i1}^5}-\frac{\lambda\,(1-\mu)}{r_{i2}^5}\right]$$

$$+\ 15\,x_i^0\,y_i^{0^2}\,q_i/\mu_i c\left[\frac{\mu}{r_{i1}^7}-\frac{\lambda\,(1-\mu)}{r_{i2}^7}\right]+\frac{(1-\mu)}{r_{i1}^3}+\frac{\mu}{r_{i2}^3}-$$

$$-\frac{3(1-\mu)\,y_i^{0^2}}{r_{i1}^5}-\frac{3\,\mu\,y_i^{0^2}}{r_{i2}^5}-\frac{3\,\mu_{3-1}\,(y_2^0-y_1^0)^2}{R_{12}^5}+\frac{\mu_{3-i}}{R_{12}^3}$$

$$-\frac{G'\,q_1\,q_2}{\mu_i R_{12}^3}+\frac{3\,G'\,q_1\,q_2\,(y_2^0-y_1^0)^2}{R_{12}^5\,\mu_i}\ ,$$

$$\gamma_{i1}^2 = (x_i^0-\mu)^2+(y_i^0)^2\ ,$$

$$\gamma_{i2}^2 = (x_i^0+1-\mu)^2+(y_i^0)^2$$

$$R_i = \left[\,2-q_i/\mu_i c\left\{\frac{1}{\gamma_{i1}^3}+\frac{\lambda}{\gamma_{i2}^3}\right\}\right].$$

Let us suppose

$$\xi_i = A_i e^{\lambda t}$$

and

$$\eta_i = B_i e^{\lambda t}.$$

Then the characteristic equation is

$$\lambda^4 + \beta \lambda^2 + D_i = 0, \quad \ldots (21)$$

where

$$\beta_i = R_i^2 + \left(\frac{\partial^2 u_i}{\partial x_i^2}\right)^0 + \left(\frac{\partial^2 u_i}{\partial y_i^2}\right)^0,$$

$$D_i = \left(\frac{\partial^2 u_i}{\partial x_i^2}\right)^0 \left(\frac{\partial^2 u_i}{\partial y_i^2}\right)^0 - \left(\frac{\partial^2 u_i}{\partial x_i \partial y_i}\right)^{0^2}.$$

The equilibrium point is stable when all the roots of the characteristic equation are purely imaginary.

The roots of the characteristic equation are tabulated in Tables I-V none of the roots is found to be purely imaginary and so it is concluded that all the equilibrium points are unstable in the linear sense. In Tables I-V λ_{0i}, $i = 1, 4$ indicated the characteristic roots of eqn. (21)].

TABLE I

For $L_1 : \lambda_1 = 1$

μ	x_1	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	x_2	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.05	1.2576	$.603597 \times 10^{14}$	$-.120719 \times 10^{15}$	1.25626	$.301790 \times 10^{14}$	$-.150899 \times 10^{15}$
.15	1.361	$.56673 \times 10^{14}$	$-.113346 \times 10^{15}$	1.3569	$.283365 \times 10^{14}$	$-.141682 \times 10^{15}$
.25	1.4624	$.426406 \times 10^{14}$	$-.852812 \times 10^{15}$	1.4565	$.213203 \times 10^{14}$	$-.106601 \times 10^{15}$
.35	1.5654	$.3667109 \times 10^{14}$	$-.733421 \times 10^{14}$	1.555	$.183355 \times 10^{14}$	$-.916777 \times 10^{14}$
.45	1.6669	$.3667083 \times 10^{14}$	$-.733416 \times 10^{14}$	1.6357	$.183354 \times 10^{14}$	$-.916770 \times 10^{14}$
.5	1.7077	$.2767351 \times 10^{14}$	$-.55347 \times 10^{14}$	1.7027	$.138367 \times 10^{14}$	$-.6918377 \times 10^{14}$

TABLE II

For $L_2 : \lambda = 1$

μ	x_1	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	x_2	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.05	$-.15710$	$.146054 \times 10^{15}$	$-.292108 \times 10^{15}$	$-.158145$	$.73027 \times 10^{14}$	-365735×10^{15}
.25	$-.6619$	$.694099 \times 10^{14}$	$-.138819 \times 10^{15}$	.060361	$.347049 \times 10^{14}$	$-.173524 \times 10^{14}$
.35	$-.00407$	$.500732 \times 10^{14}$	$-.100146 \times 10^{15}$	$-.012909$	$.250366 \times 10^{14}$	$-.125183 \times 10^{15}$
.45	$-.00131$	$.355061 \times 10^{14}$	$-.710123 \times 10^{14}$	$-.01124$	$.177530 \times 10^{14}$	$-.887653 \times 10^{14}$
.5	$-.0100$	$.276734 \times 10^{14}$	-553469×10^{14}	$-.005$	$.138367 \times 10^{14}$	$-.691836 \times 10^{14}$

TABLE III

For $L_3 : \lambda = 1$

μ	x_1	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	x_2	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.15	– 2.0606	$.824056 \times 10^{14}$	$-.1648114 \times 10^{15}$	– 2.06435	$.412028 \times 10^{14}$	$-.26014 \times 10^{15}$
.25	– 1.9607	$.552649 \times 10^{14}$	$-.1105299 \times 10^{15}$	– 1.96725	$.276324 \times 10^{14}$	-138162×10^{15}
.35	– 1.8604	$.468103 \times 10^{14}$	-937220×10^{14}	– 1.87006	234305×10^{14}	$-.117152 \times 10^{15}$
.45	– 1.7599	$.323286 \times 10^{14}$	-646572×10^{14}	– 1.773	$.161643 \times 10^{14}$	$-.808215 \times 10^{15}$
.5	– 1.7095	$.564769 \times 10^{14}$	$-.1129539 \times 10^{14}$	– 1.7205	$.282384 \times 10^{14}$	$-.1411923 \times 10^{15}$

TABLE IV

Case I for L_4

μ	$x_{1,4}$	$y_{1,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	$x_{2,4}$	$y_{2,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.01	.3963	.9117	$.5295614\times10^{13}$	$-.105304\times10^{14}$	.3963	.9117	$.529561\times10^{13}$	$-.105304\times10^{14}$
.05	–.2734	.8169	$.124353\times10^{14}$	$-.232257\times10^{14}$	–.2734	.8169	$.1245353\times10^{14}$	$-.232257\times10^{14}$
.15	–8819	.6235	$.3296916\times10^{10}$	$-\ .659347\times10^{10}$	–.8819	.623	$.3296916\times10^{10}$	$-.659347\times10^{10}$

Case II for L_4

μ	$x_{1,4}$	$y_{1,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	$x_{2,4}$	$y_{2,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.01	.3997	.9115	$.1370079\times10^{16}$	$-.269951\times10^{16}$	.3997	.9115	$.1370079\times10^{16}$	$-.26995\times10^{16}$
.05	–.2678	.8158	$.2718954\times10^{15}$	$-.529538\times10^{15}$	–.2678	.8158	$.271895\times10^{15}$	$-.529538\times10^{15}$
.15	–.3771	.6205	$.922406\times10^{14}$	$-.184469\times10^{15}$	.3771	.620	$.922406\times10^{14}$	$-.184469\times10^{15}$

TABLE V

Case I for L_5

μ	$x_{1,4}$	$y_{1,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	$x_{2,4}$	$y_{2,4}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.01	.4013	–.9113	$.159276\times10^{14}$	$-.3130533\times10^{14}$	.4013	–.9113	$.1592766\times10^{14}$	$-.3130533\times10^{14}$
.05	–.2612	–.8149	$.466121\times10^{13}$	$-.908688\times10^{13}$	–.2612	–.8149	$.466121\times10^{13}$	$-.9086881\times10^{13}$
.5	.1673	–6175	$.277106\times10^{10}$	$-.5541879\times10^{10}$	.1673	–.617	$.2771064\times10^{10}$	$-.554187\times10^{10}$

Case II for L_5

μ	$x_{1,5}$	$y_{1,5}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$	$x_{2,5}$	$y_{2,5}$	$(\lambda_{0_{1,2}})^2$	$(\lambda_{0_{3,4}})^2$
.01	.3997	–.9115	$.647230\times 10^{15}$	$-.2609611\times10^{15}$	.3997	–.9115	$.6472306\times 10^{15}$	$-.2609611\times10^{15}$
.05	–2678	–.8157	$.263953\times 10^{15}$	-491343×10^{15}	–.2678	–.8157	$.263954\times 10^{15}$	-4913436×10^{15}
.15	–.3771	–.6205	$.922406\times 10^{14}$	$-.184469\times 10^{15}$	–.3771	–.6205	$.9224067\times 10^{14}$	$-.184469\times 10^{15}$

Conclusion

We observe that there exist fourteen equilibrium solutions to the given system. Six equilibrium solutions are collinear and eight solutions are non-collinear. We also have seen that the equilibrium points L_1 and L_3 go away from the centre of mass and L_2 move towards the centre of mass for certain values of mass parameter μ. We further observe that there exist two classes of non-collinear equilibrium solutions when $\lambda = 1$, one class of solutions lies approximately along a line which passes through the centre of mass of the primaries and second class of solutions lies approximately along a line that is perpendicular to the first, further we have seen that if we change the value of λ then there exists only one class of solutions which lies approximately along a line which passes through the centre of mass (for $\lambda = 2.5$). We further observe that the all colinear and non-colinear equilibrium points are linearly unstable for given values of μ and λ.

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